



東南大學
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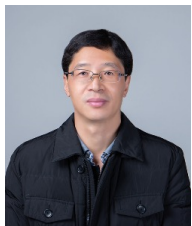
复旦大学-现代物理研究所

Imprints of Confinement in the Hadron Spectrum: From the Cornell Model to the DeepQuark

Lu Meng (孟璐)

Southeast University

Apr. 6, 2026



Shi-Lin Zhu Wei-Lin Wu Yao Ma Yan-Ke Chen

Based on: [PRD108, 114016 \(2023\)](#), [PRD110, 034030 \(2024\)](#),
[PRD109, 054034 \(2024\)](#), [PRL136, 071901 \(2026\)](#)

History of the multiquark states



Phys.Lett. 8 (1964) 214-215

Volume 8, number 3

PHYSICS LETTERS

1 February 1964

A SCHEMATIC MODEL OF BARYONS AND MESONS *

M. GELL-MANN

California Institute of Technology, Pasadena, California

Received 4 January 1964

...
A simpler and more elegant scheme can be constructed if we allow non-integral values for the charges. We can dispense entirely with the basic baryon b if we assign to the triplet t the following properties: spin $\frac{1}{2}$, $z = -\frac{1}{3}$, and baryon number $\frac{1}{3}$. We then refer to the members $u^{\frac{2}{3}}$, $d^{-\frac{1}{3}}$, and $s^{-\frac{1}{3}}$ of the triplet as "quarks" q and the members of the anti-triplet as anti-quarks $\bar{q}$. Baryons can now be constructed from quarks by using the combinations (qqq) , $(qqq\bar{q})$, etc., while mesons are made out of $(q\bar{q})$, $(qq\bar{q}\bar{q})$, etc. It is assuming that the lowest baryon configuration (qqq) gives just the represen-



8419/TH.412

21 February 1964

AN SU_3 MODEL FOR STRONG INTERACTION SYMMETRY AND ITS BREAKING

II *)

G. ZWEIF

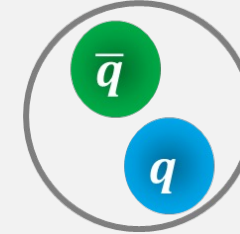
CERN---Geneva

*) Version I is CERN preprint 8182/TH.401, Jan. 17, 1964.

...

- 6) In general, we would expect that baryons are built not only from the product of three aces, AAA , but also from $\bar{A}AAAA$, $\bar{A}AAAAA$, etc., where $\bar{A}$ denotes an anti-ace. Similarly, mesons could be formed from $\bar{A}A$, $\bar{A}AAA$ etc. For the low mass mesons and baryons we will assume the simplest possibilities, $\bar{A}A$ and AAA , that is, "deuces and treys".

Meson

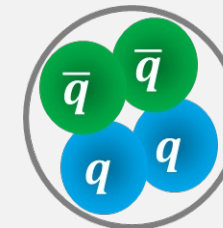


Baryon

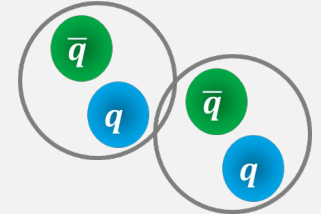


Conventional hadrons

Compact type



Molecular type



Multiquark states

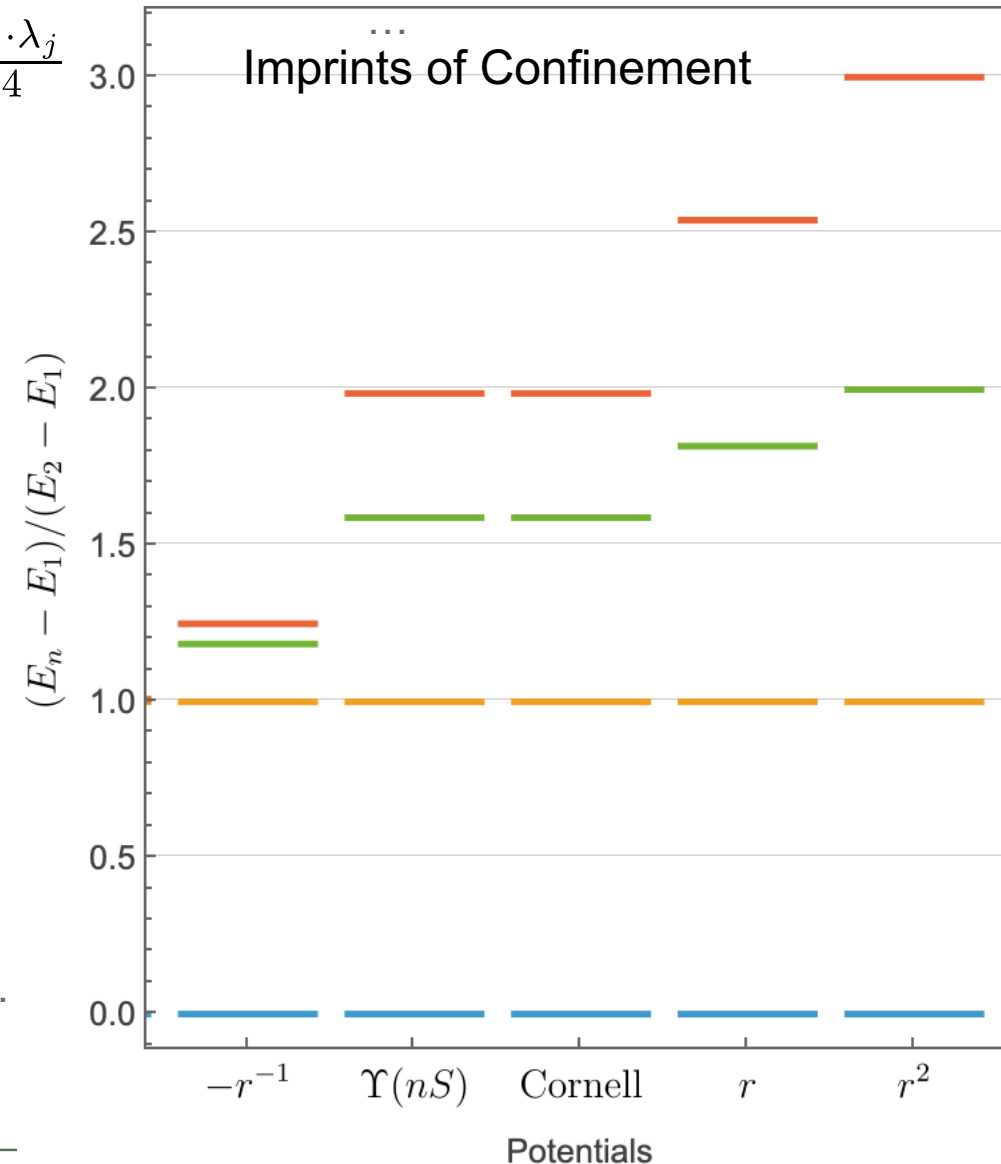
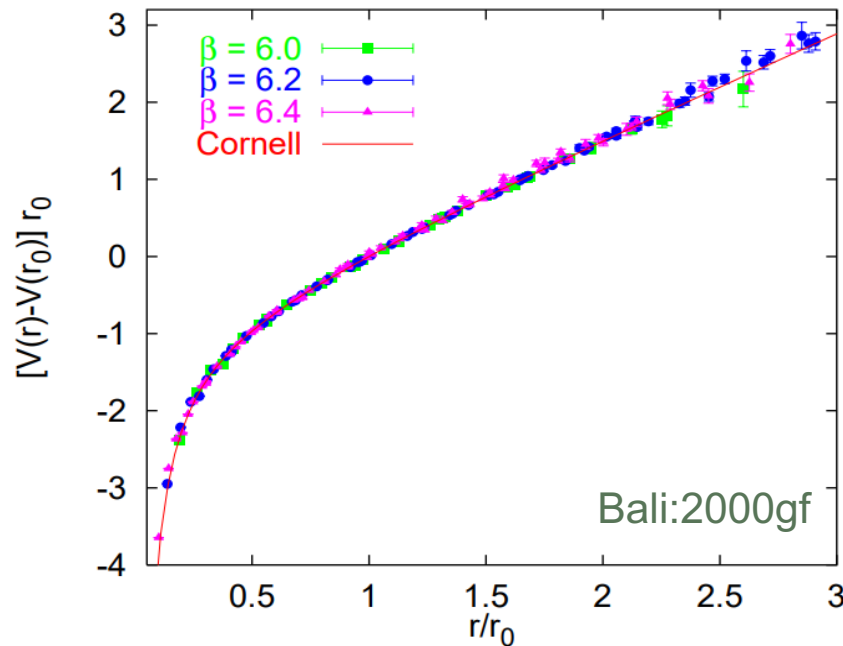
The multiquark states were predicted at the birth of quark model



- A minimal model: one-gluon-exchange+Confinement

Cornell model: Eichten:1978tg,
BGS model: Barnes:2005pb

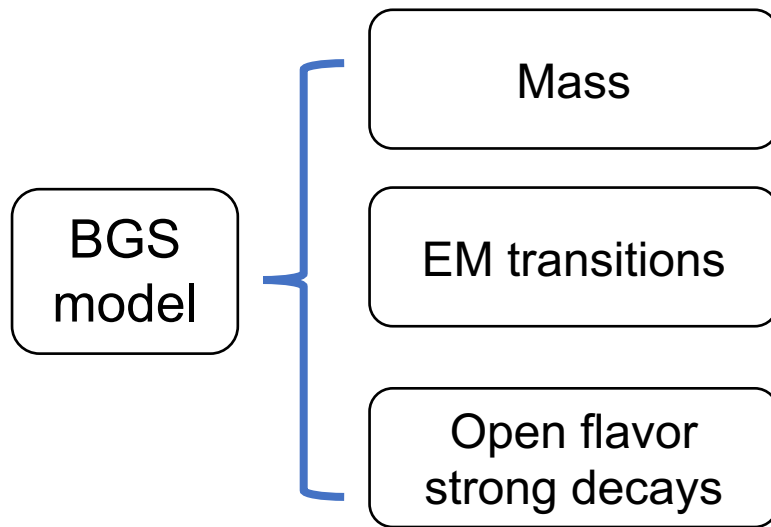
$$V_{ij}(r) = \underbrace{\left[\frac{\alpha_s}{r} - \frac{8\pi\alpha_s}{3m_i m_j} \frac{\tau^3}{\pi^{3/2}} e^{-\tau^2 r^2} \mathbf{s}_i \cdot \mathbf{s}_j \right]}_{\text{OGE}} + \underbrace{\left(-\frac{3b}{4}r + V_c \right)}_{\text{Confinement}} \frac{\lambda_i \cdot \lambda_j}{4}$$



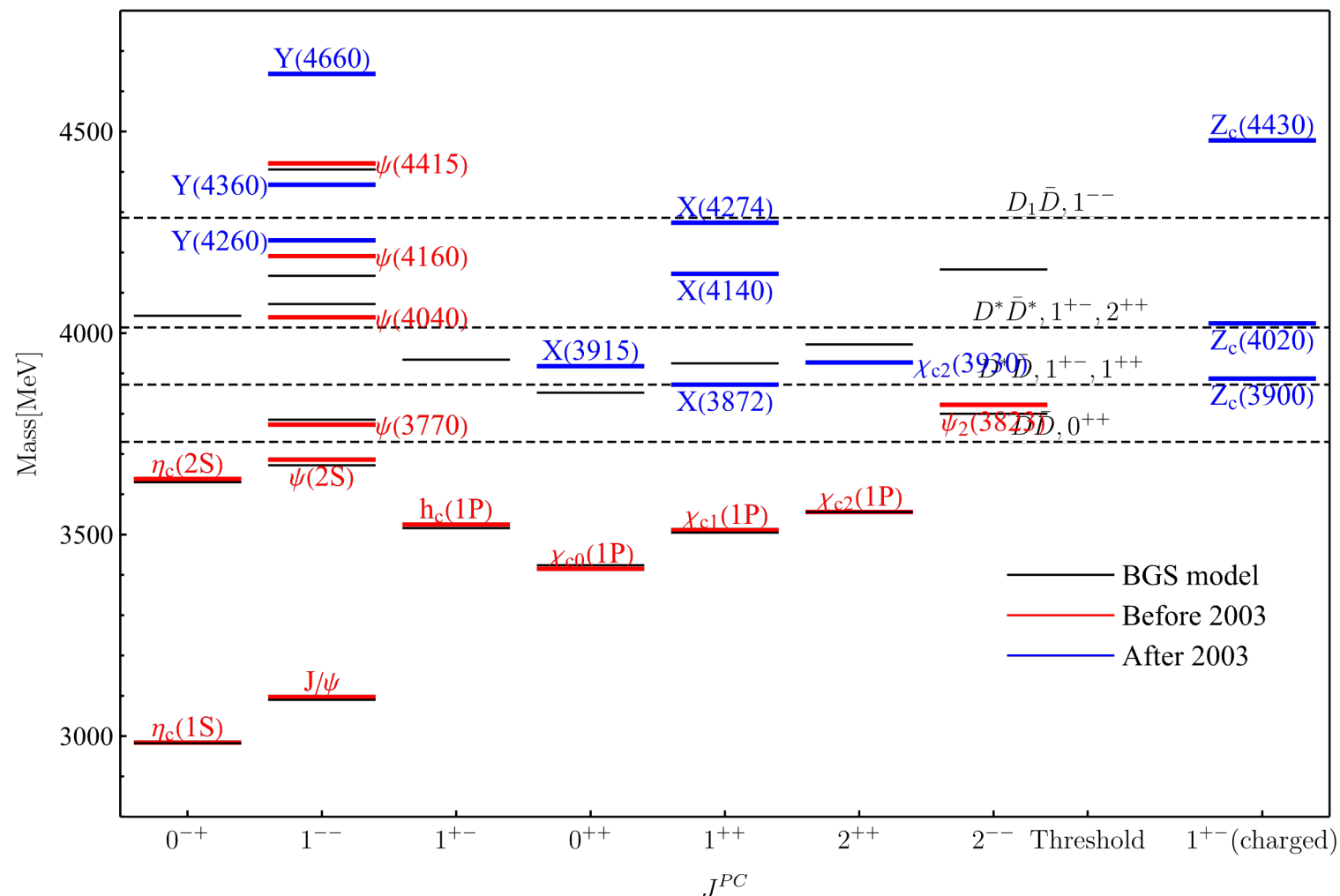
- Chiral quark models (π , K , η , exchange)

Manohar:1983md, Zhang:1997ny, Vijande:2004he, Gonzalez:2012gka...

- Relativized Godfrey-Isgur model Godfrey:1985xj...

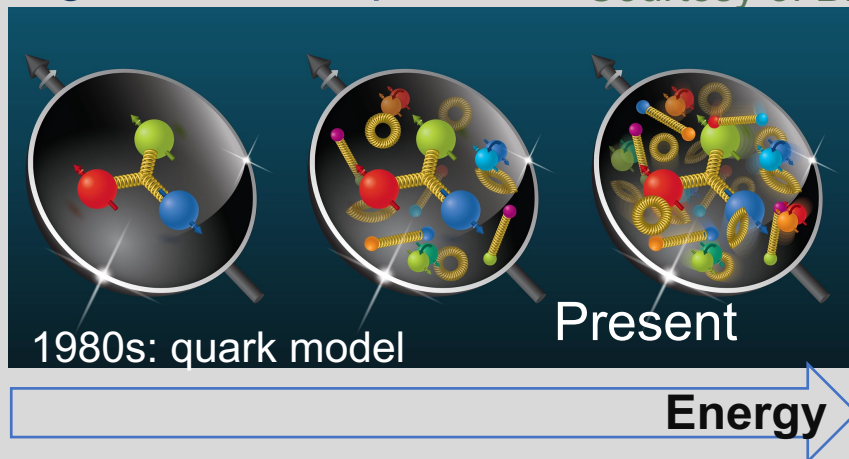


- NRQM with only 4 paras.
- Work well below open flavor thresholds
- Work better for bottomonium



T. Barnes, S. Godfrey, and E. S. Swanson, Phys. Rev. D 72, 054026 (2005).

Courtesy of BNL

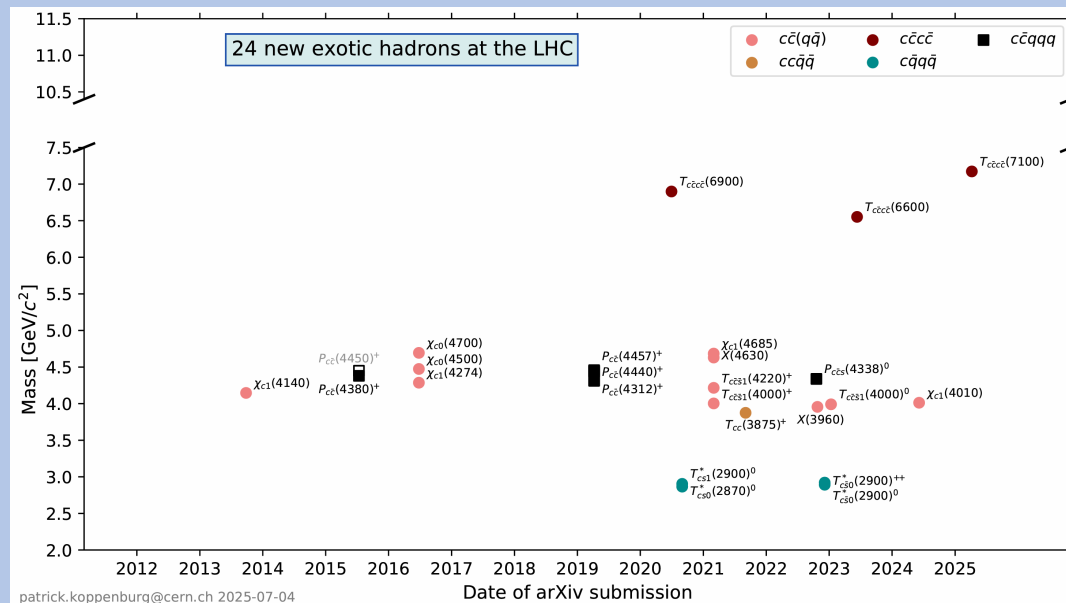


- Alternative methods:

- ▶ Lattice QCD
- ▶ Low energy EFTs
- ▶ Dyson-Schwinger equations
- ▶ ...

Against For

- We even do not know the **patterns**



- Quark models

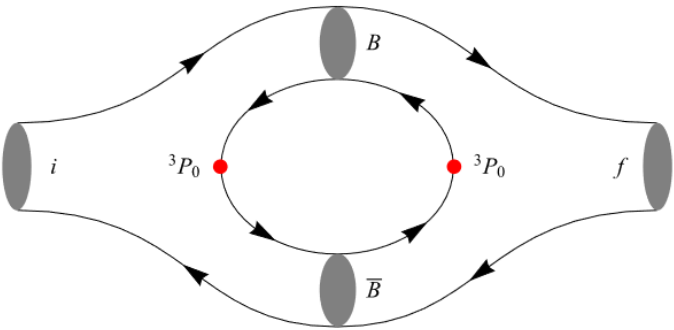
- ▶ A clear picture to uncover patterns
- ▶ Lower computational costs
- ▶ Prediction power
- ▶ Also decay models
- ▶ ...

We still need quark model !

The right question: How to **adapt the quark models** to meet the new demands?

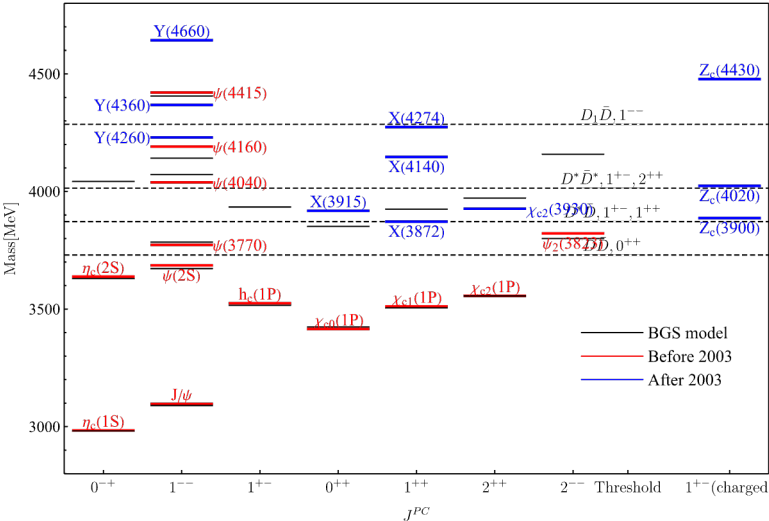


● Heavy-quarkonium-like states: unquenched quark model



B-S Zou's talks
<https://indico.cern.ch/event/1457095/contributions/6563231/>

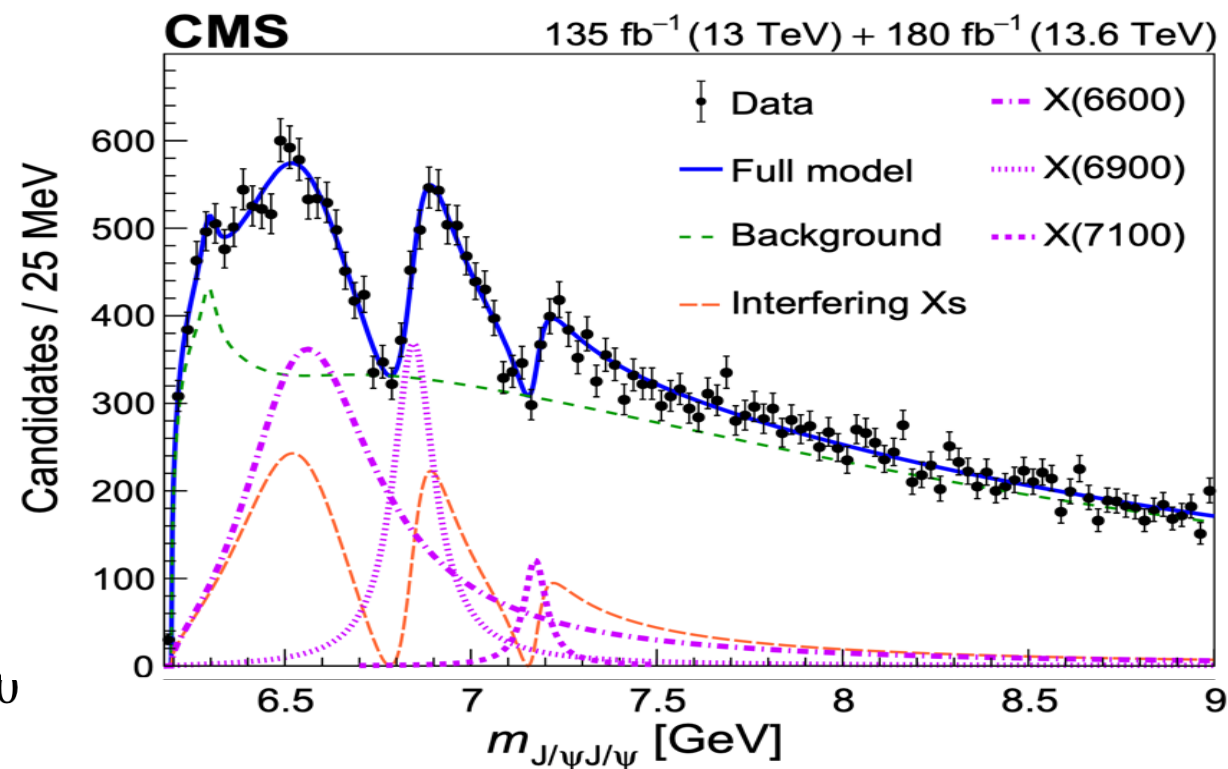
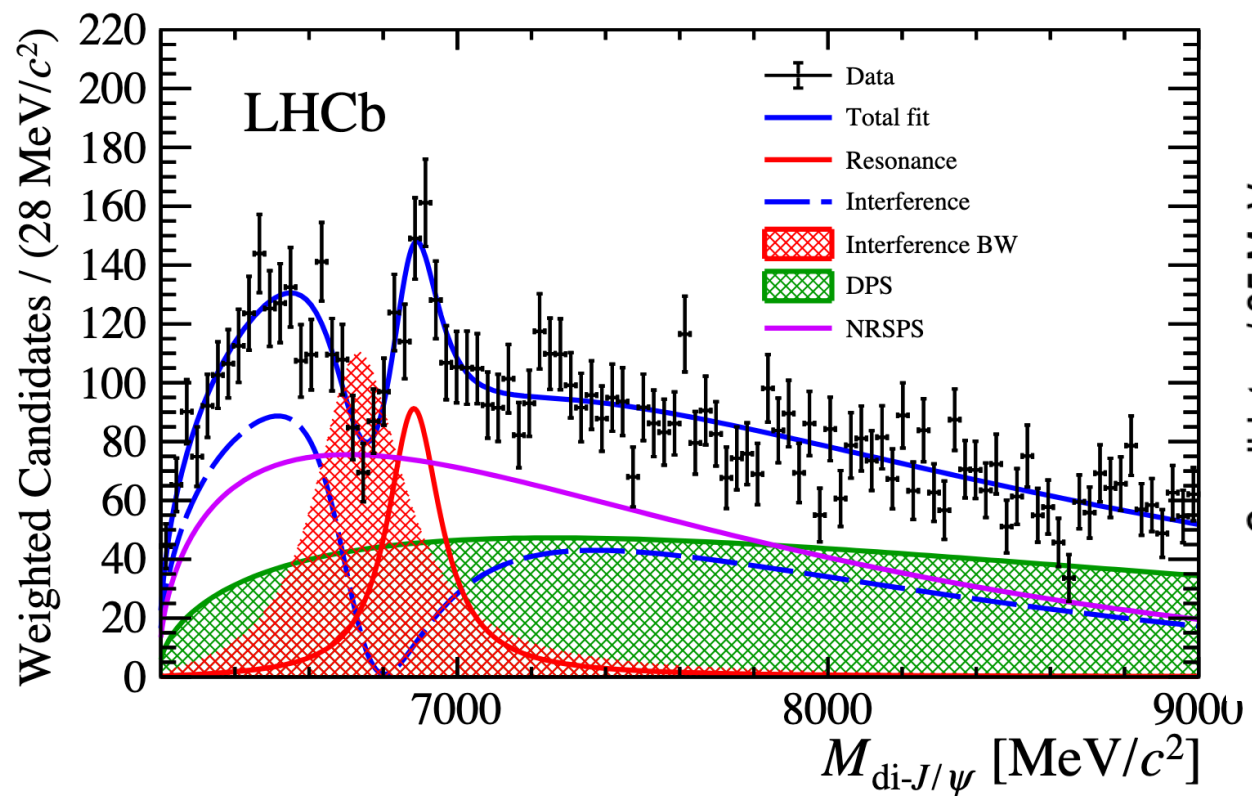
● Multiquark systems



$[c\bar{c}qqq]$ P_c	$[cc\bar{c}\bar{c}]$ $X(6900)$ $X(6600)$ $X(7100)$	$[cs\bar{u}\bar{d}]$ $T_{cs1}(2900)$ $T_{cs0}(2900)$	$[cs\bar{u}\bar{d}]$ $Z_{cs}(3985)$ $Z_{cs}(4000)$	$[cc\bar{u}\bar{d}]$ $T_{cc}(3875)^+$	$[c\bar{s}u\bar{d}][c\bar{s}\bar{u}d]$ $T_{c\bar{s}0}(2900)^{++}$ $T_{c\bar{s}0}(2900)^0$	$[c\bar{c}qqq]$ $P_{cs}(4338)$ $P_{cs}(4459)$
	2006.16957 2306.07164 2304.08962 2506.07944	2009.00025 2009.00026	2011.07855 2103.01803	2109.01038 2109.01056	2212.02716 2212.02717 2411.19781	2210.10346 2012.10380 2502.09951

Particle Zoo 2.0

- Three all charmed tetraquark resonances are observed by LHCb, CMS, and ATLAS
- The spin-parity measurement from CMS: favor 2^{++}
- In addition to $J/\psi J/\psi$ final states, the signal in $J/\psi \psi(2S)$ is also observed



arXiv: 2006.16957, 2306.07164, 2304.08962, 2506.07944...

● Opportunities: all-charm tetraquark family

- ▶ Great experimental advances: LHCb, CMS, ATLAS
- ▶ Simple systems

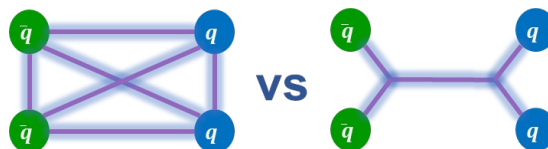
Constituent quark is almost the current quark

Small relativistic effect

Unlikely exchange light mesons between (anti)quarks

- ▶ Different confinements would leave imprints on the mass spectrum

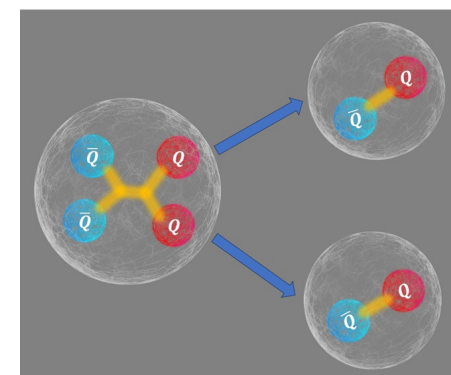
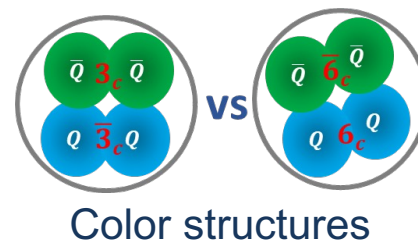
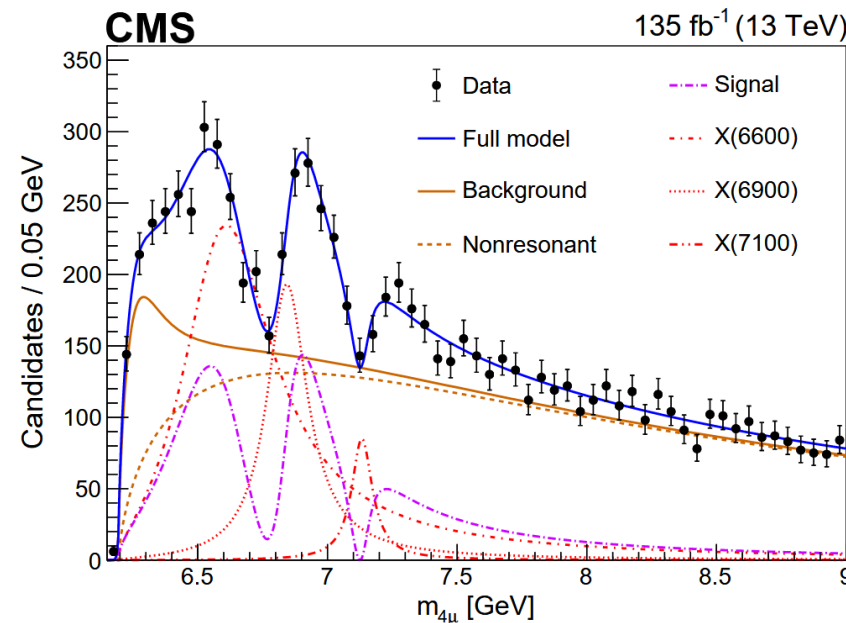
Alexandrou:2004ak, Okiharu:2004ve,
Bicudo:2017usw



● Challenges

- ▶ Color structures: e.g. $3 - \bar{3}$ and $6 - \bar{6}$ tetraquark
- ▶ Matrix element of double Y-type potential
- ▶ Four/five body problem
- ▶ Resonance above the di-hadron thresholds
- ▶

**Interactions?
few-body (resonance) problem?**



quark rearrangement

- Basis expansion + Complex scaling method
- **DNNs**: towards tetraquark **confinement**



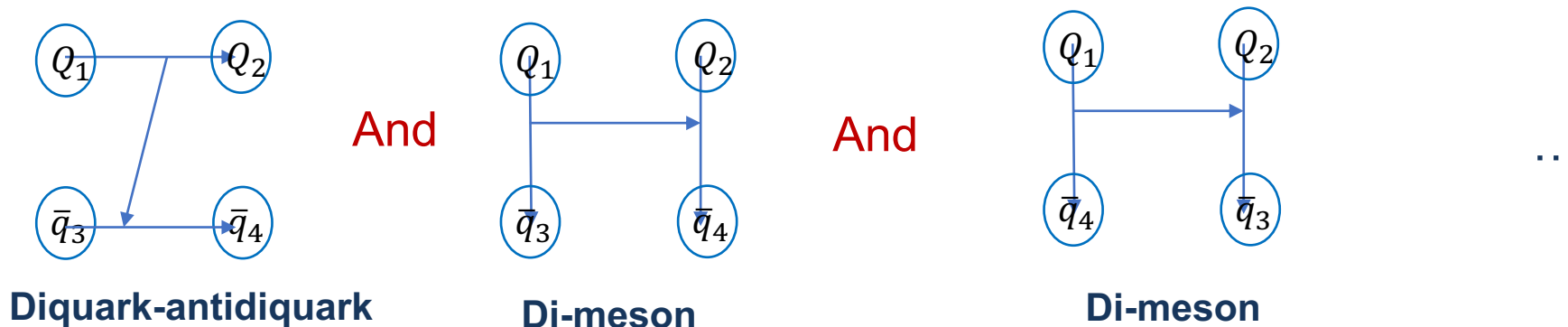
Basis expansion + Complex scaling method



- Based on variational principle: $|\psi\rangle_{trial} = \sum_i c_i |\phi_i\rangle$

$$E[\psi_{trial}] = \frac{\langle \psi_{trial} | H | \psi_{trial} \rangle}{\langle \psi_{trial} | \psi_{trial} \rangle} \geq E_0$$

- **upper limit** of the energy
 - Not necessary to be complete, but more general basis, more accurate results
- Spatial wave functions



$$\phi_{nlm}(\mathbf{r}) = N_{lm} r^l e^{-\frac{r^2}{r_n^2}} Y_{lm}(\hat{r})$$

Hiyama:2003cu

- Geometric progression: $r_n = r_0 a^{n-1}$
 - Embed both long- and short-range correlations

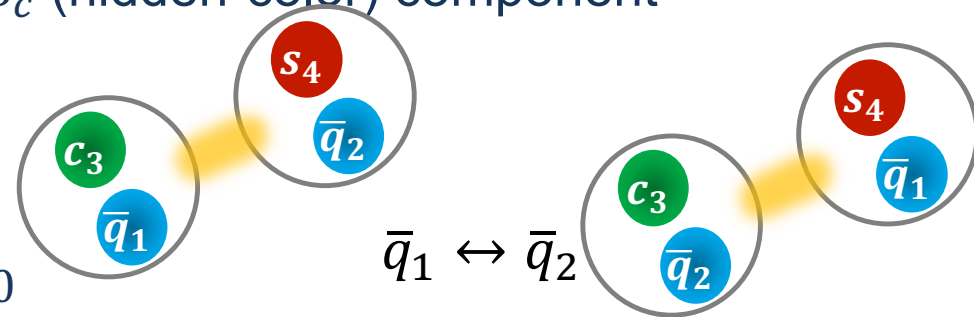


What can wave function tell us?



- Quark potential models: energies + **wave functions**
- Hadronic molecule could have large portion of the $8_c - 8_c$ (hidden-color) component

$$|\Psi\rangle = |1\rangle|\psi(31; 42)\rangle - |1'\rangle|\psi(32; 41)\rangle$$



$$|\psi(31; 42)\rangle = |\psi_D(31)\rangle|\psi_{\bar{K}}(42)\rangle|\psi_{D\bar{K}}\rangle, \quad \langle\psi(31; 42)|\psi(32; 41)\rangle \approx 0$$

$$\langle\Psi|\Psi\rangle = 1 + 1 - 2\langle 1|1'\rangle\langle\psi(31; 42)|\psi(32; 41)\rangle \approx 2$$

$$\begin{cases} |(c_3\bar{q}_2)_{1_c}(s_4\bar{q}_1)_{1_c}\rangle = |1'\rangle \\ |(c_3\bar{q}_2)_{8_c}(s_4\bar{q}_1)_{8_c}\rangle = |8'\rangle \end{cases} \quad \begin{cases} |(c_3\bar{q}_1)_{1_c}(s_4\bar{q}_2)_{1_c}\rangle = |1\rangle \\ |(c_3\bar{q}_1)_{8_c}(s_4\bar{q}_2)_{8_c}\rangle = |8\rangle \end{cases} \quad \begin{bmatrix} \frac{1}{3} & \frac{2\sqrt{2}}{3} \\ \frac{2\sqrt{2}}{3} & -\frac{1}{3} \end{bmatrix} \begin{bmatrix} |1\rangle \\ |8\rangle \end{bmatrix} = \begin{bmatrix} |1'\rangle \\ |8'\rangle \end{bmatrix}$$

$$|\Psi\rangle = |1\rangle \left[|\psi(31; 42)\rangle + \frac{1}{3} |\psi(32; 41)\rangle - \right] + |8\rangle \frac{2\sqrt{2}}{3} |\psi(32; 41)\rangle$$

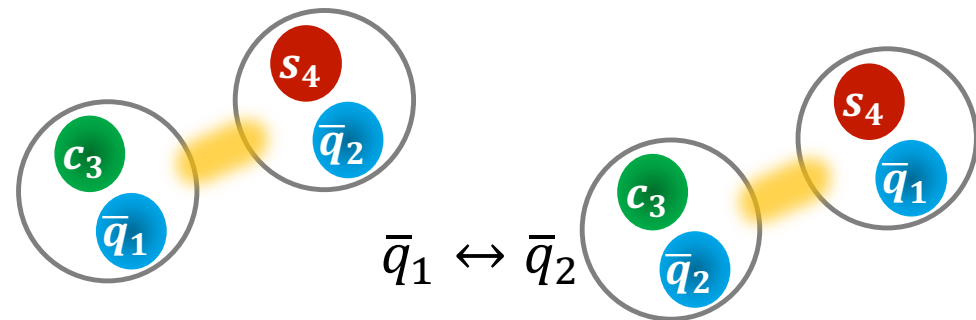
$$P_{8_c} = \frac{4}{9}, \quad P_{1_c} = \frac{5}{9}$$



- Wave functions: spatial distribution
- Proper quantities to discern

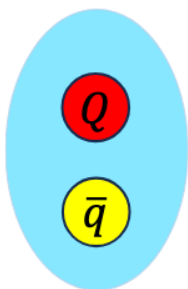
$$r_{13}^{rms} \equiv \langle \Psi | r_{13}^2 | \Psi \rangle$$

$$r_{c\bar{q}}^{rms} \equiv \langle \psi(31; 43) | r_{13}^2 | \psi(31; 42) \rangle \approx r_D^{rms}$$

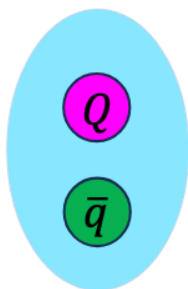


$$|\Psi\rangle = |1\rangle|\psi(31; 42)\rangle - |1'\rangle|\psi(32; 41)\rangle$$

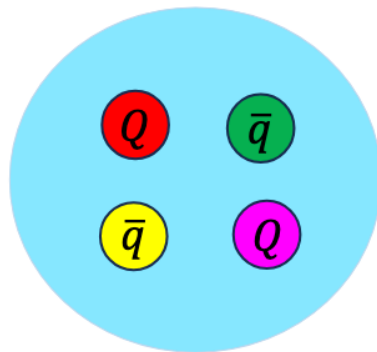
- Categorization of spatial structures of tetraquark states.



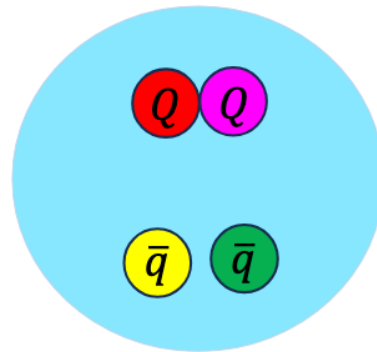
meson molecule



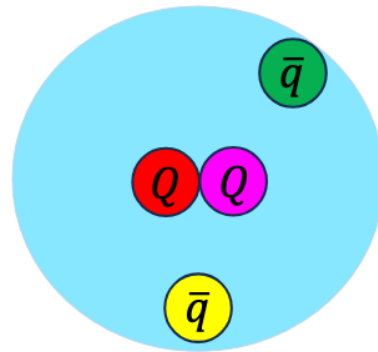
compact
even tetraquark

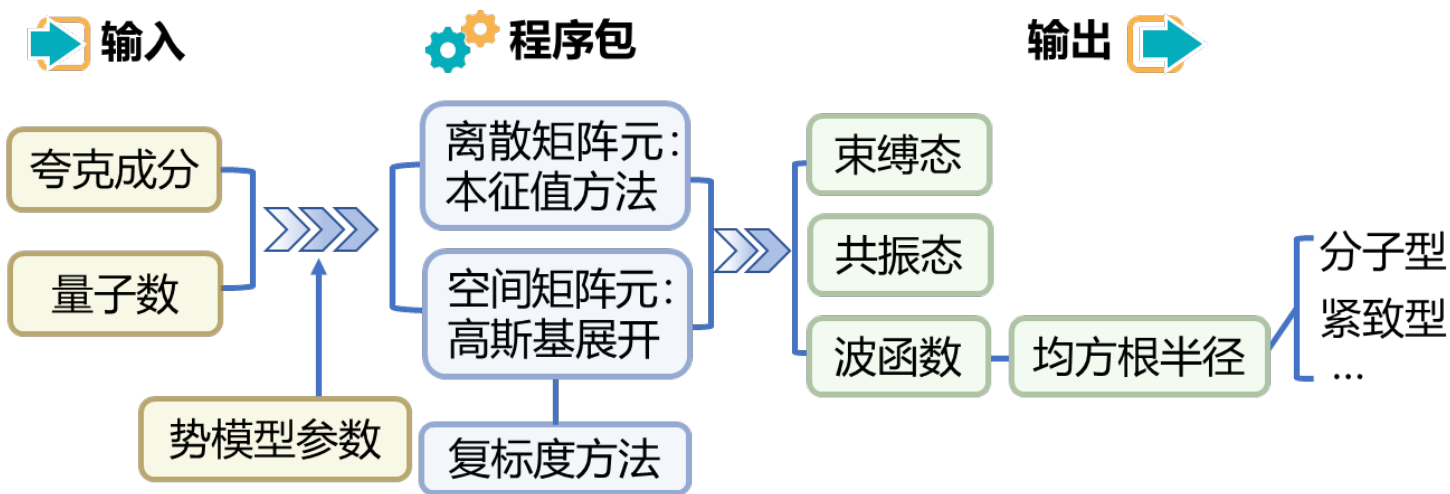


compact
diquark-antidiquark tetraquark



compact
diquark-centered tetraquark





- Fully heavy tetraquark states ($QQ\bar{Q}\bar{Q}$)
- Triply heavy tetraquark states ($QQ\bar{Q}\bar{q}$)
- Doubly heavy tetraquarks states ($QQ\bar{q}\bar{q}$)
- Single heavy strange states ($Qs\bar{q}\bar{q}$, $Q\bar{s}q\bar{q}$)

$$q = u, d, s; \quad Q = b, c$$

$$J^P = 0^+, 1^+, 2^+, \text{ Only S-wave}$$

Only bound states

Over 150 states

Total runtime:
2.5h on my laptop

- Three models

- ▶ BGS model

Barnes:2005pb

- ▶ Semay-Silvestre-Brac Models

Semay:1994ht, Silvestre-Brac:1996myf

$$V_{ij}(r) = \left[-\frac{\kappa}{r} + \lambda r^p - \Lambda + \frac{2\pi}{3m_i m_j} \kappa' \frac{1}{\pi^{3/2} r_0^3} e^{(-r^2/r_0^2)} \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j \right] \lambda_i \cdot \lambda_j$$

AL1: $p = 1$, AP1: $p = 2/3$

- Our conclusions:

No bound states for any reasonable **pairwise** interaction

- Similar conclusions in literature

J. P. Ader, J. M. Richard, and P. Taxil, PRD**25**, 2370 (1982).

M.-S. Liu, Q.-F. Lü, X.-H. Zhong, and Q. Zhao, , PRD**100**, 016006 (2019).

X. Jin, Y. Xue, H. Huang, and J. Ping, EPJC**80**, 1083 (2020).

...

tial models already used in the study of heavy mesons and baryons. We first consider the situation where the quarks have the same mass and interact through a two-body potential due to color-octet exchange. In this case, we show that for any reasonable confining potential there is no state below the threshold corresponding to the spontaneous dissociation into two mesons. We investigate in detail different possibilities of modifying this negative



Ref [Assi:2023dlu]: claimed the existence of fully heavy tetraquark bound state in version 1

FERMILAB-PUB-23-660-T

Tetraquarks made of sufficiently heavy quarks are bound in QCD

Benoît Assi¹ and Michael L. Wagman¹

¹*Fermi National Accelerator Laboratory, Batavia, IL, 60510*

version 1

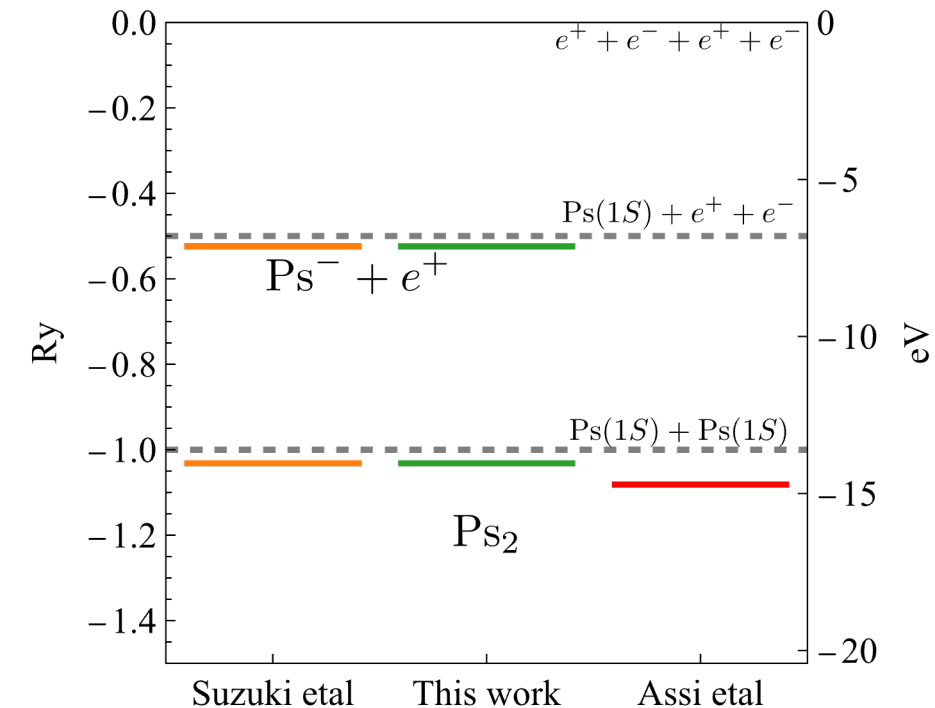
FERMILAB-PUB-23-660-T

Tetraquarks made of sufficiently unequal-mass heavy quarks are bound in QCD

Benoît Assi¹ and Michael L. Wagman¹

¹*Fermi National Accelerator Laboratory, Batavia, IL, 60510*

version 2



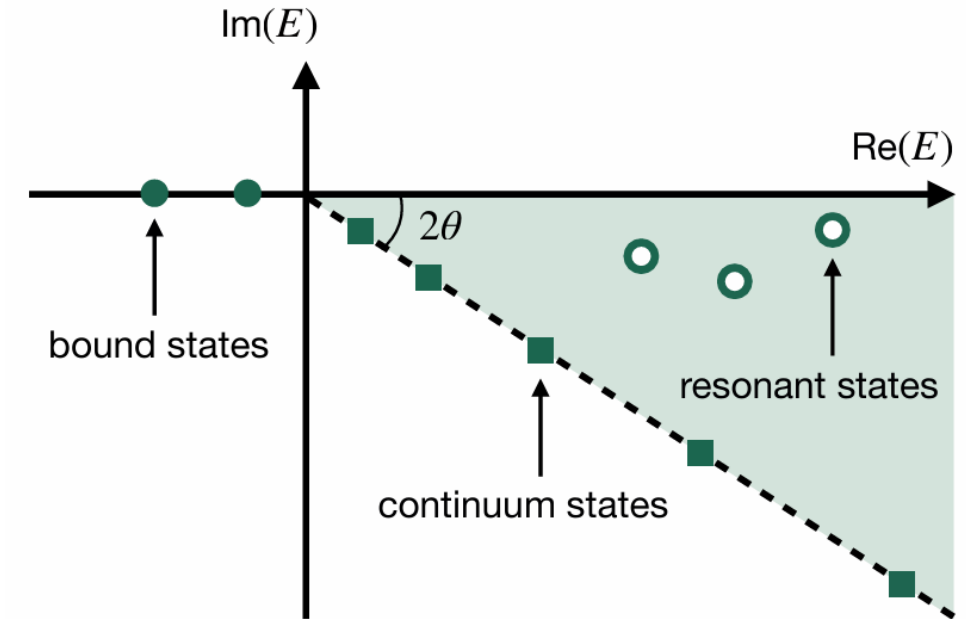
- Original Hamiltonian

$$H = \sum_i^n \left(m_i + \frac{\mathbf{p}_i^2}{2m_i} \right) + \sum_{i < j=1}^n V_{ij},$$

- Complex scaling

$$U(\theta)\mathbf{r} = \mathbf{r}e^{i\theta}, \quad U(\theta)\mathbf{p} = \mathbf{p}e^{-i\theta}.$$

$$H(\theta) = \sum_{i=1}^n \left(m_i + \frac{p_i^2 e^{-2i\theta}}{2m_i} \right) + \sum_{i < j=1}^n V_{ij} (r_{ij} e^{i\theta}).$$



- Resonance appearing as the eigenvalue of $H(\theta)$
- Equivalence of the CSM and **contour deformation method** in 2-body systems

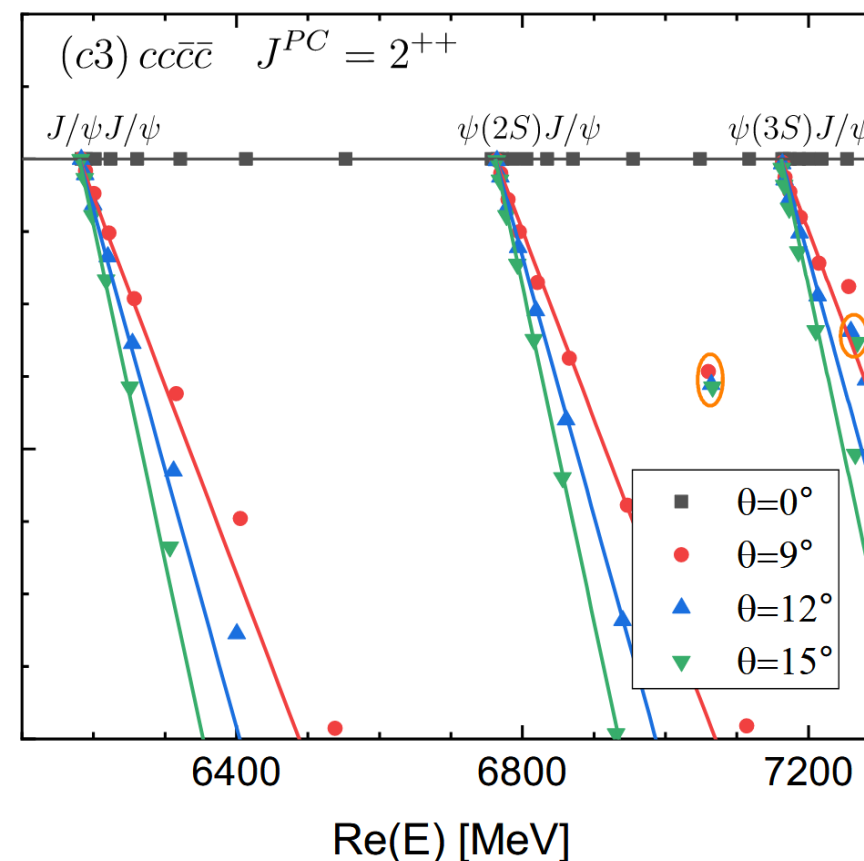
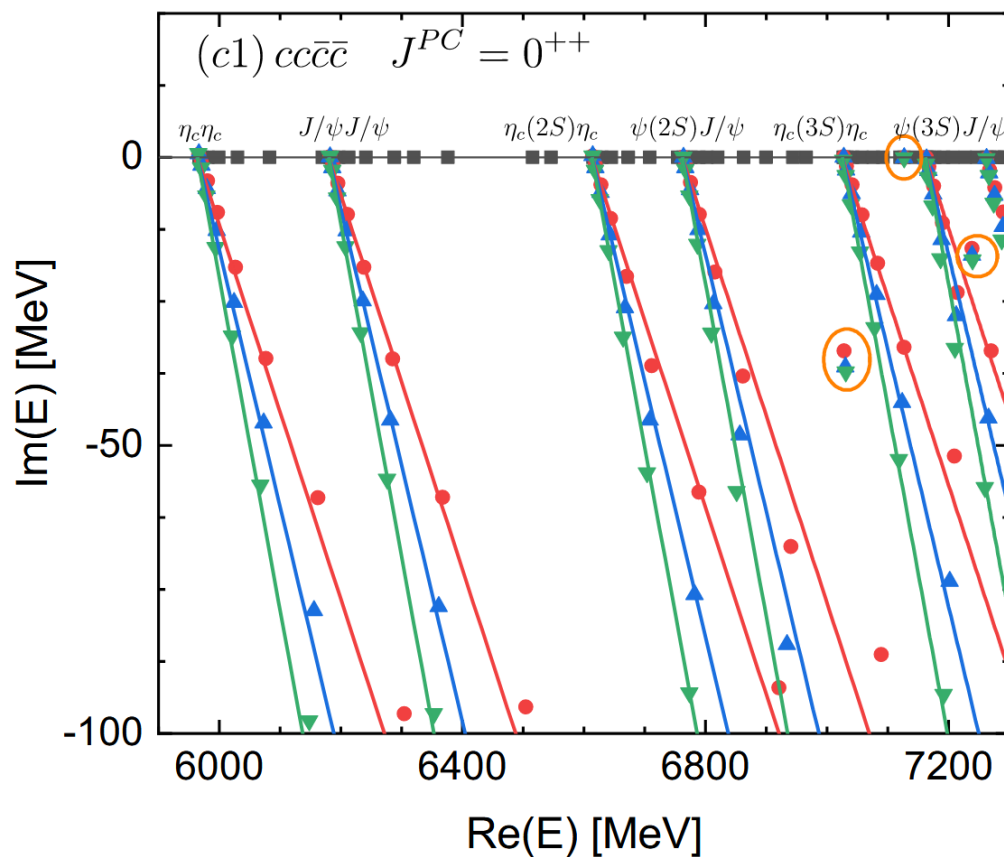
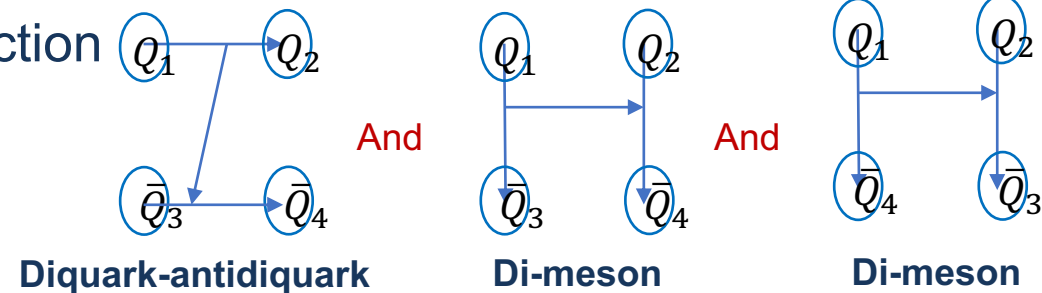
Aguilar:1971ve, Balslev:1971vb, Aoyama:2006hrz...



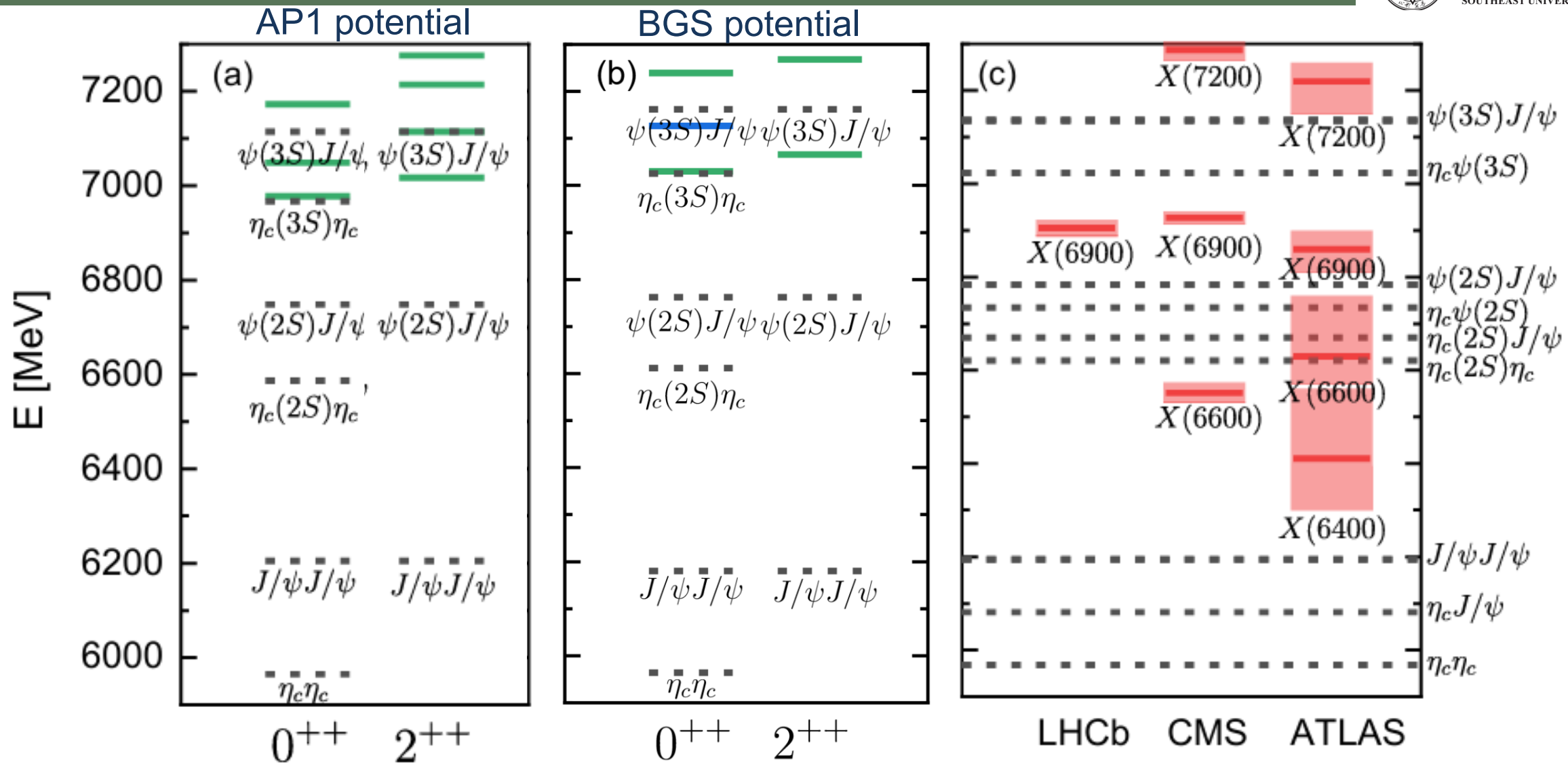
All-charm tetraquarks



- BGS, AL1, AP1 models: **pairwise** confinement interaction
- Configurations: diquark-antidiquark, di-meson $\times 2$
- Tetraquark as resonances



All-charm tetraquarks

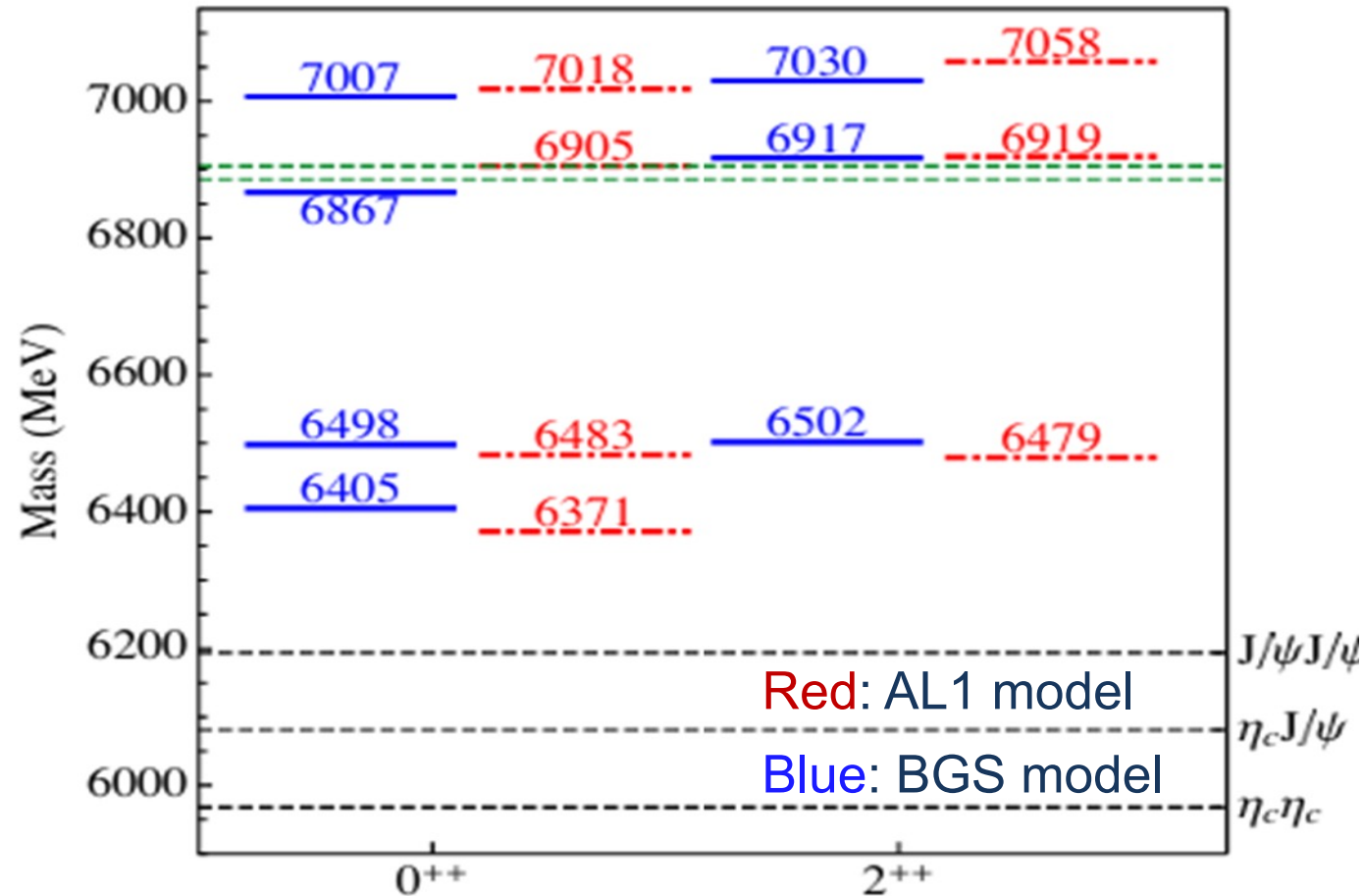


X(6600) is missing in pairwise confinement models

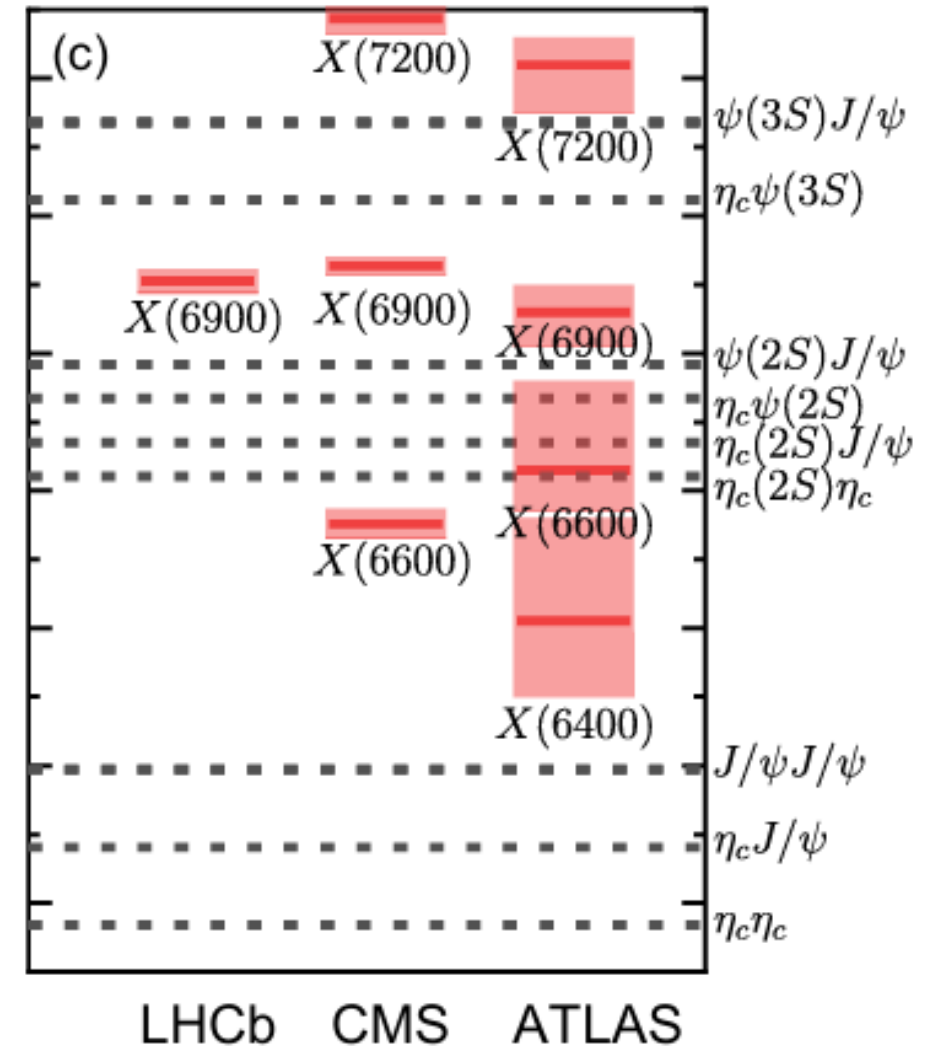
See also G.-J. Wang, Q. Meng, and M. Oka, PRD106, 096005 (2022).



- Earlier calculation neglecting di-meson configurations

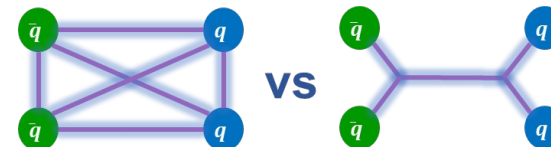
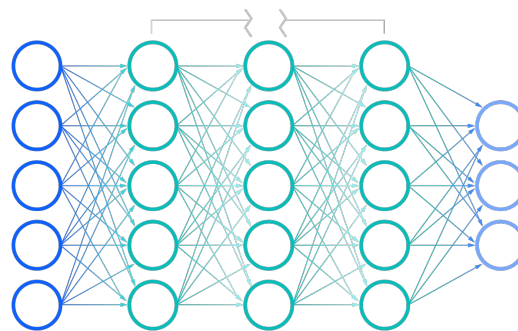


G-J Wang, LM, M Oka, S-L Zhu, PRD 104, 036016 (2021)



- Pairwise model: overestimate the transition between diquark-antidiquark and dimeson??

DNNs: towards tetraquark confinement



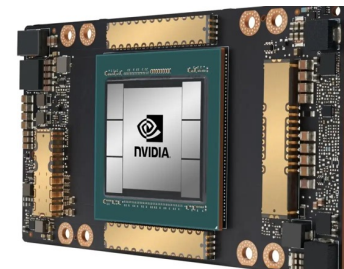
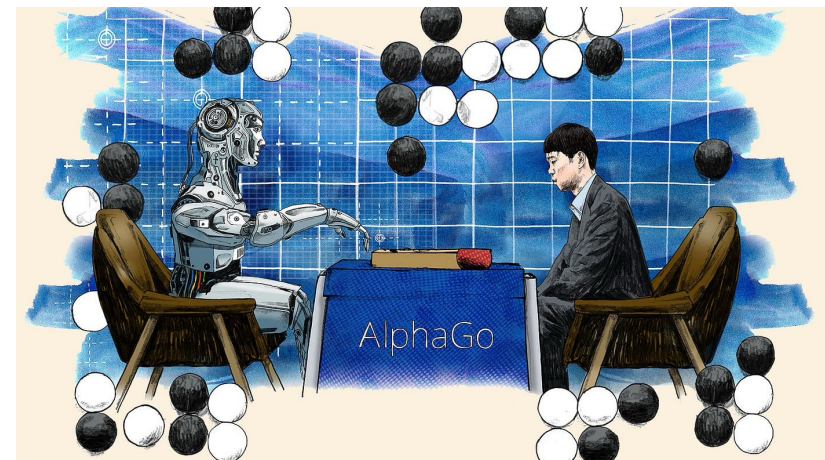
Turning point of AI

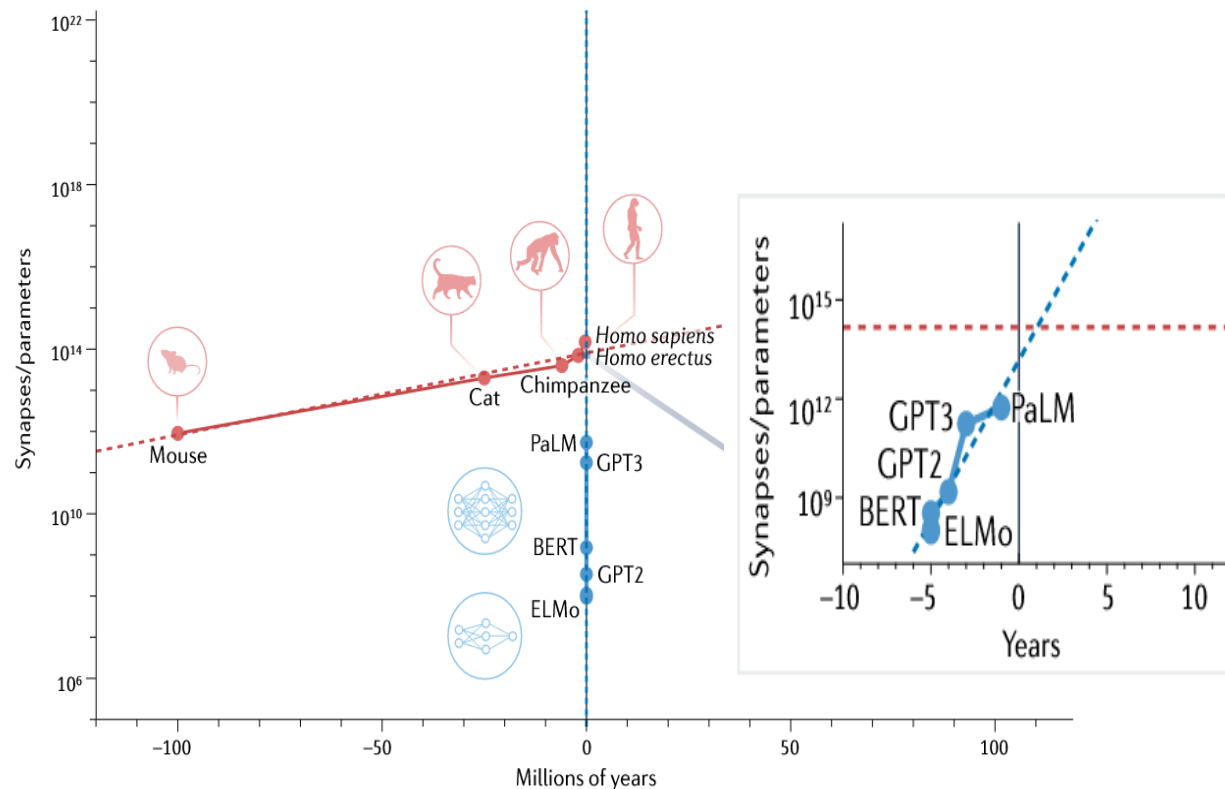


Year	Milestone	Who	Significance
2010	ImageNet	Fei-Fei Li	Large-scale dataset that enabled the deep learning revolution.
2012	AlexNet	Krizhevsky et al.	Proven power of deep CNNs and GPU acceleration for AI.
2014	GANs	Ian Goodfellow	Introduced framework for generative AI to create realistic data.
2015/16	TensorFlow/ Pytorch	Google/Meta	Critical open-source software that democratized AI research globally.
2016	AlphaGo	DeepMind	First AI to defeat a world champion in the game of Go.
2010s	AI Chips	NVIDIA, Google, Apple	Specialized GPUs/TPUs enabled training of massive AI models.
2017	Transformer	Google	Revolutionary architecture that made modern large language models possible.
2018	BERT	Google	Bidirectional language model that set new standards for NLP.
2020	AlphaFold 2	DeepMind	Solved the decades-old scientific challenge of protein folding.
2022	ChatGPT	OpenAI	Popularized large language models through accessible public interaction.

2025 AI agent

10 milestones of AI selected by DeepSeek





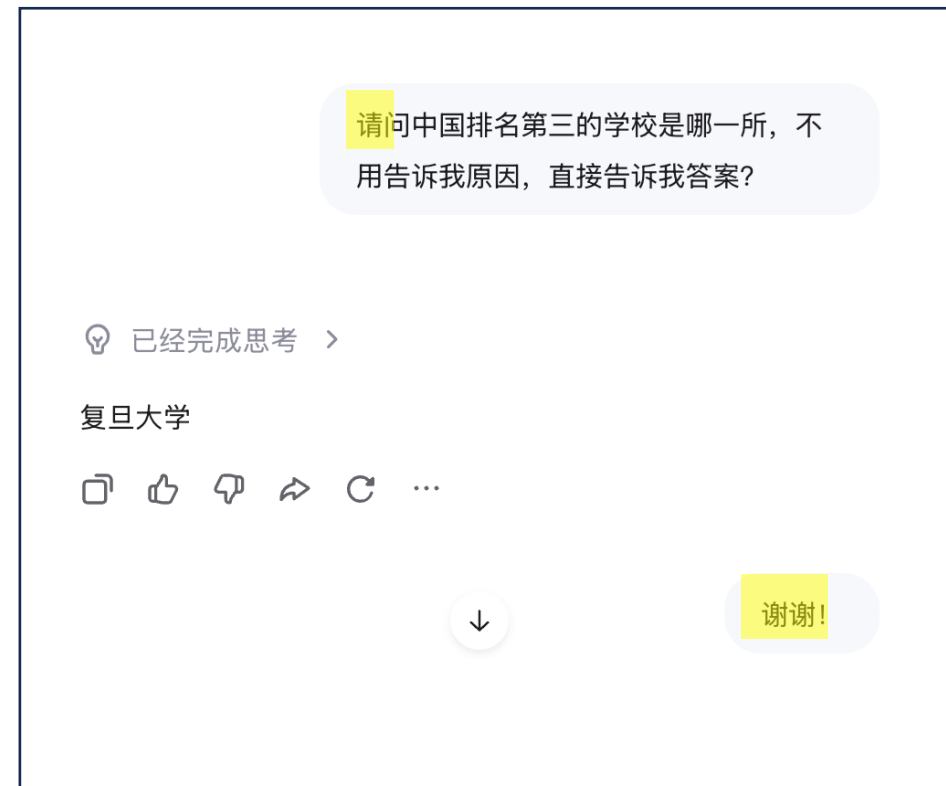
The evolution of biological and artificial intelligence

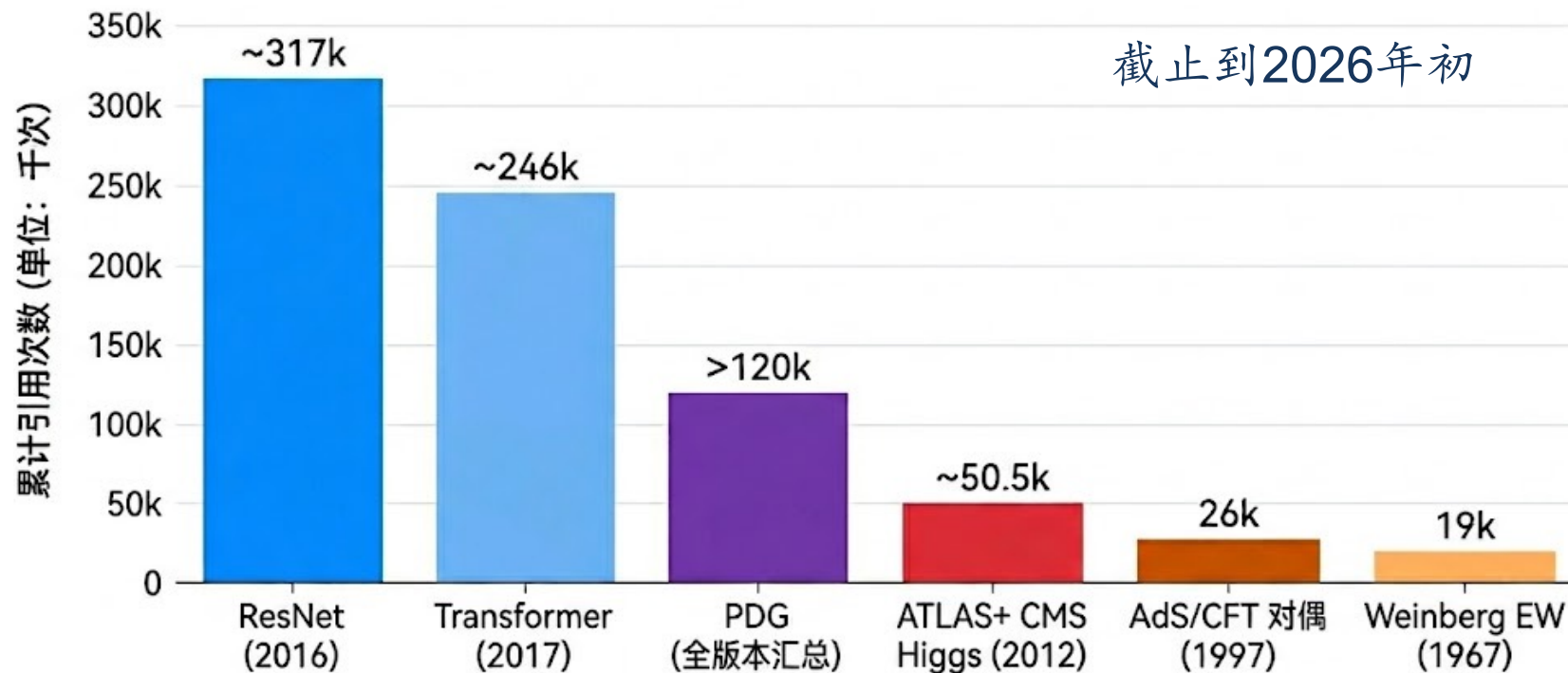
Schwartz, M. D. (2022). *Nature Reviews Physics*, 4(12), 741–742.

数学界正在经历哥白尼革命，人类智能不再是宇宙中心

陶哲轩访谈

Being more polite to chatbots may increase your chances of survival when robots win





* 数据为估算且随时间动态变化, 来源于 Google Scholar 聚合指标。

[ResNet] K. He et al, **Deep Residual Learning for Image Recognition**

[Transformer] A. Vaswani et al, **Attention Is All You Need**

[AdS/CFT] J.M. Maldacena, **The Large N Limit of Superconformal Field Theories and Supergravity**

[EW] S. Weinberg, **A Model of Leptons**



Reshape the research paradigm



- AI for Science: e.g. AlphaFold, this work
- From vibe coding to vibe researching



Resummation of the C-Parameter Sudakov Shoulder Using Effective Field Theory

Matthew D. Schwartz^{1,2}

¹*Department of Physics, Harvard University, Cambridge, MA 02138, USA*

²*Institute for Artificial Intelligence and Fundamental Interactions (IAIFI)*

schwartz@g.harvard.edu

present matched NLL+NLO results. Unlike thrust and heavy jet mass, the C-parameter has no Sudakov–Landau pole, making momentum-space resummation straightforward. All calculations, numerical analysis, and manuscript preparation were performed by Claude, an AI assistant developed by Anthropic, working under physicist supervision.

Author Contributions

M.D.S. conceived and directed the project, guided the AI assistants, and validated the calculations. Claude Opus 4.5, an AI research assistant developed by Anthropic, performed all calculations including the SCET factorization theorem derivation, one-loop soft and jet function calculations, EVENT2 Monte Carlo simulations, numerical analysis, figure generation, and manuscript preparation. The work was conducted using Claude Code, Anthropic's agentic coding tool. M.D.S. is fully responsible for the scientific content and integrity of this paper.

哈佛物理教授疯了：我让AI写论文，结果两周干完博士一年工作！冲击顶刊

新智元 新智元 2026年3月24日 17:15

北京 标题已修改 431人



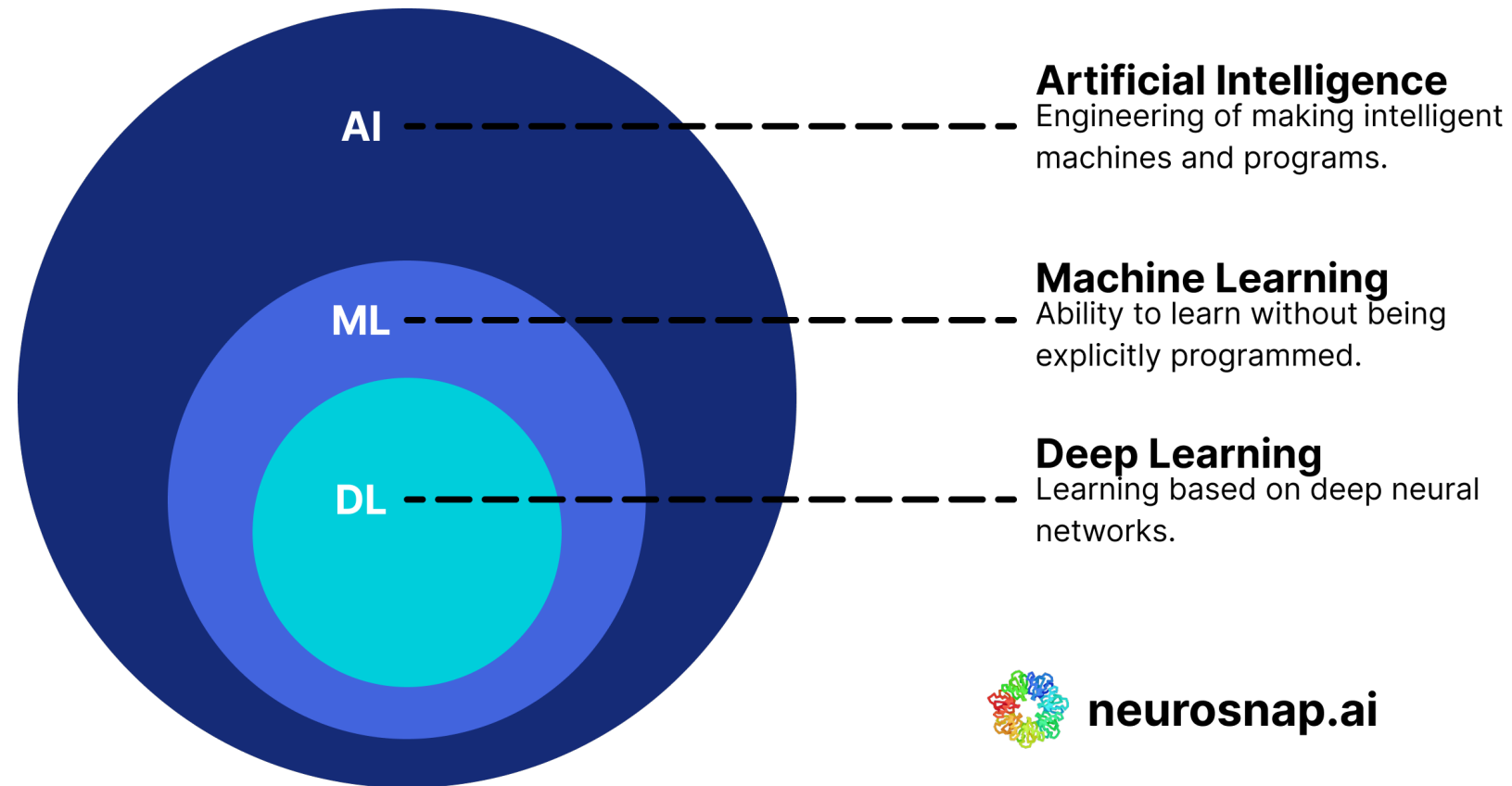
新智元报道

编辑：Aeneas KingHZ

【新智元导读】哈佛物理学教授带AI读博，结果Claude 4.5在仅仅两周内，就产出一篇顶刊级论文，整个物理学圈都震了！而这个项目，人类博士生要干一年。科研的



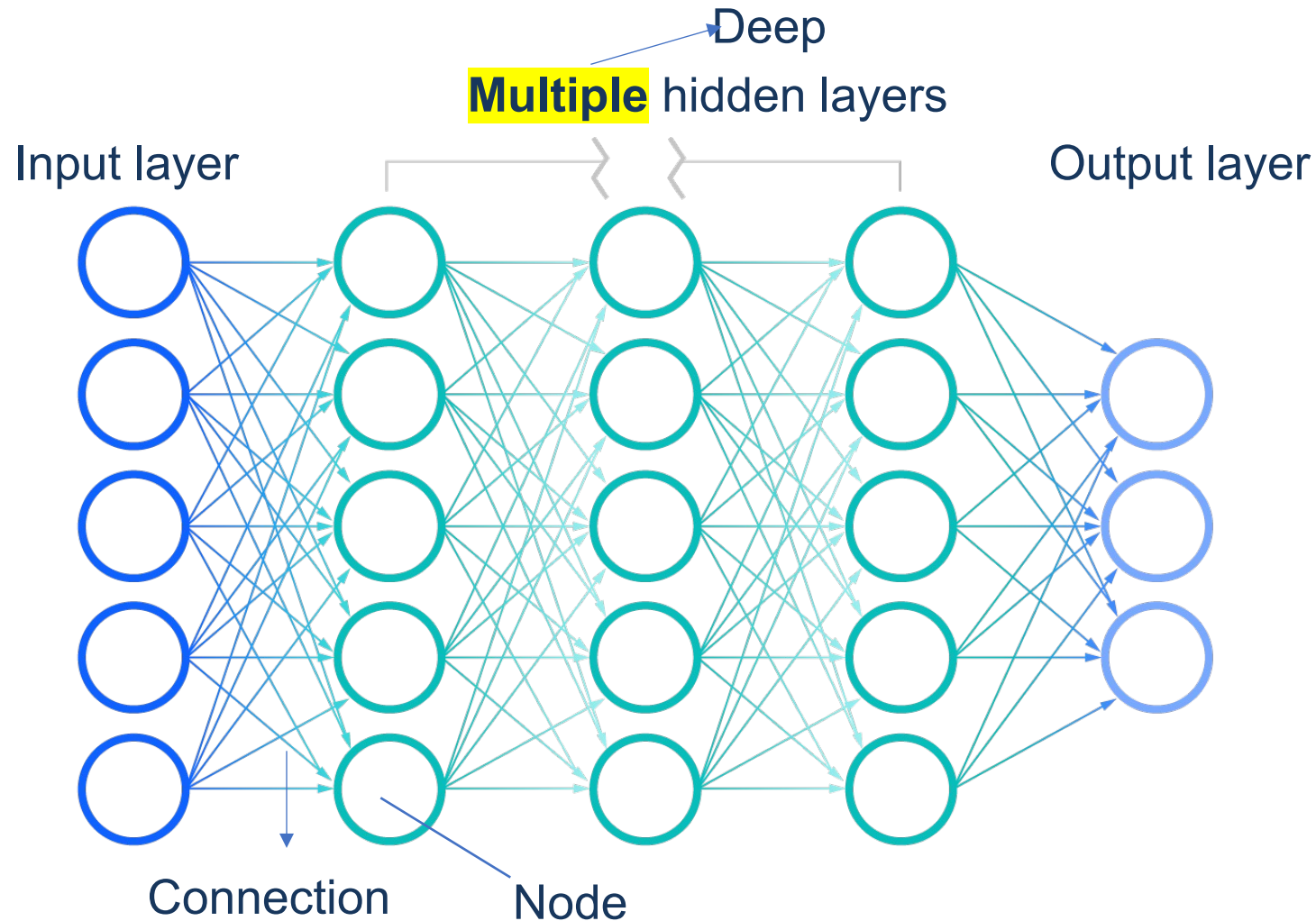
775 6994 317 23



<https://neurosnap.ai/blog/post/64279cadfeb3e5ca5ba0904a>

AI boom from 2010s was powered by **deep learning**





Neural network: simple units (neurons) connected to form complex network

Deep: Enable learn complex relation in data

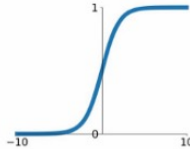


$$\vec{h}^{(i+1)} = \underbrace{\sigma}_{\text{Nonlinear}} \left[\underbrace{W^{(i)} \vec{h}^{(i)} + \vec{b}^{(i)}}_{\text{Linear}} \right]$$

- Linear part: Weight matrices $W^{(i)}$ and biases $\vec{b}^{(i)}$
- Nonlinear part: Activation functions σ

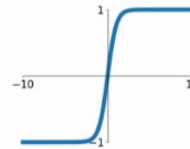
Sigmoid

$$\sigma(x) = \frac{1}{1+e^{-x}}$$

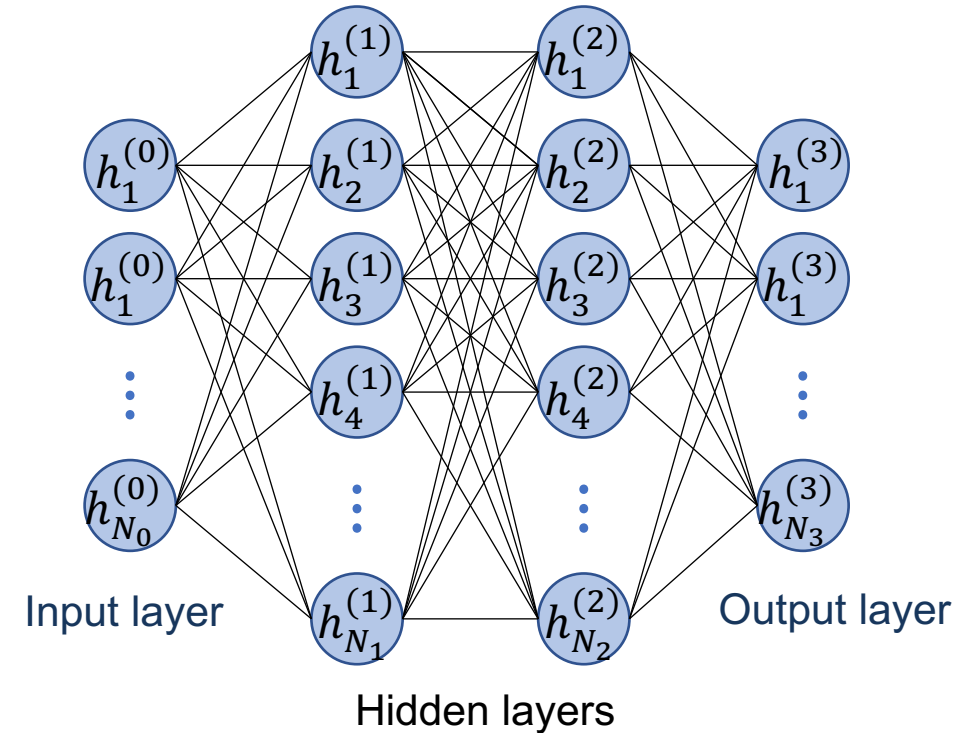


tanh

$$\tanh(x)$$



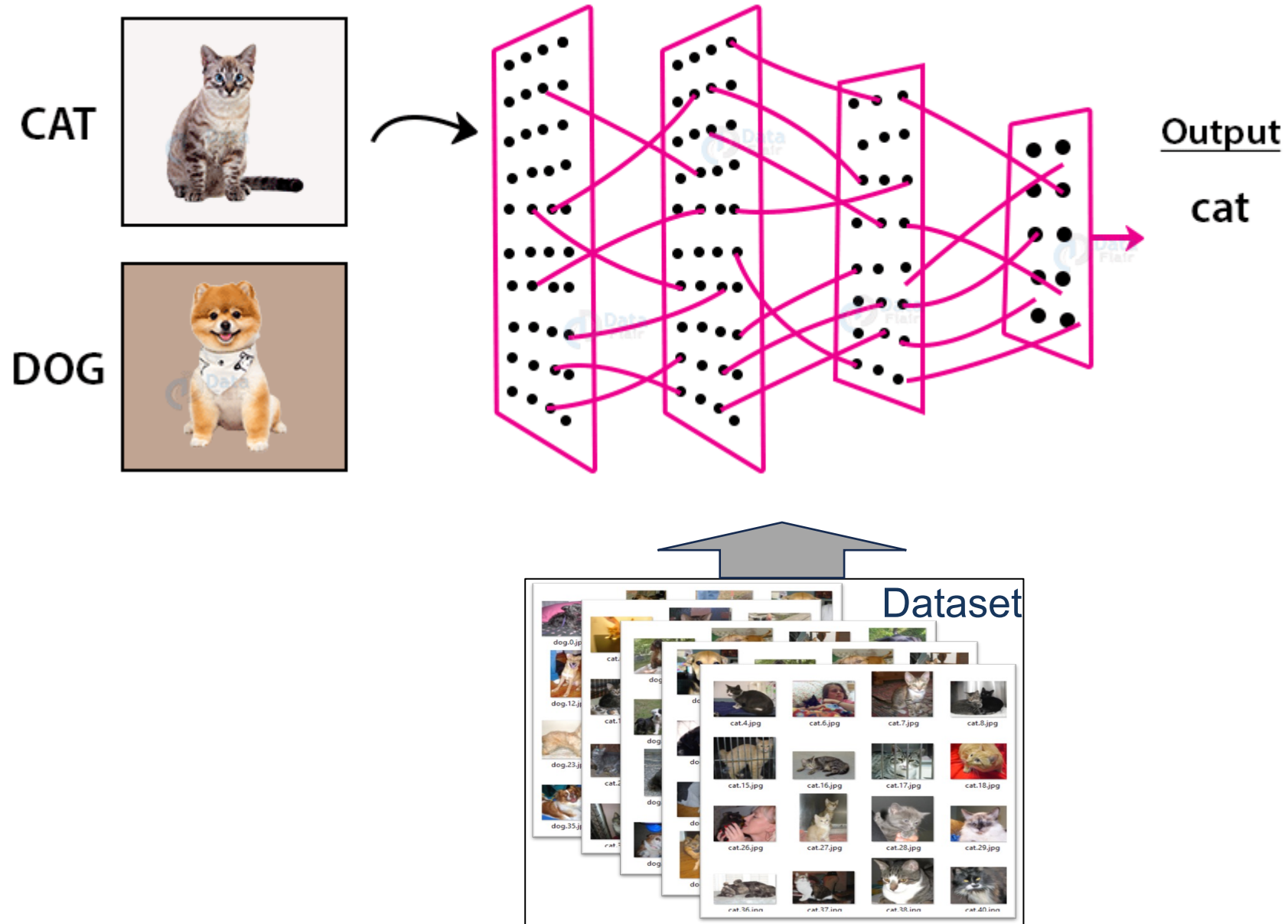
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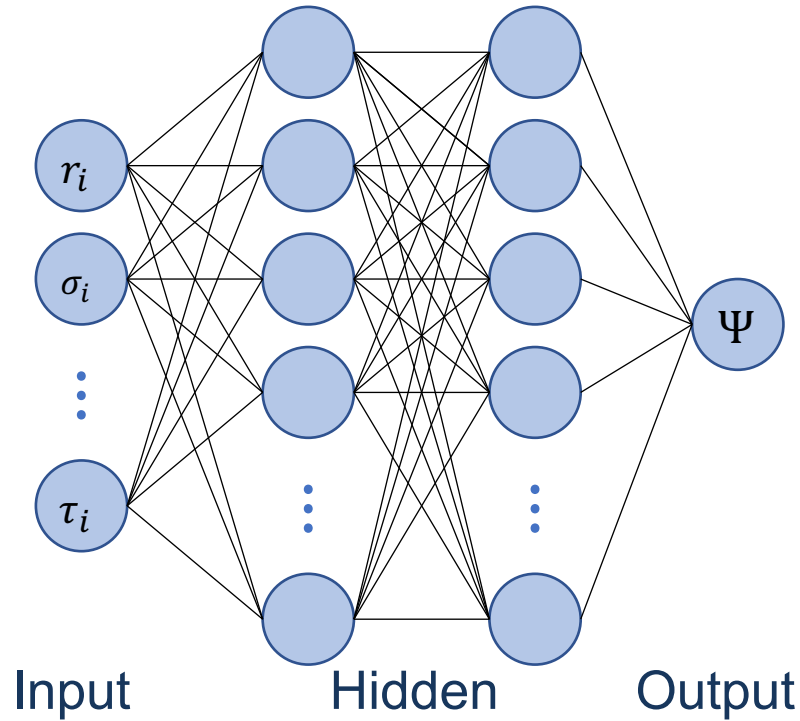
- Training: determine $W^{(i)}$ and biases $\vec{b}^{(i)}$ to minimize loss functions
 - ▶ Loss function: quantifies the difference between the expected and actual output.
- Highly efficient optimization
 - ▶ Backpropagation: automatic differentiation (AD)
 - ▶ GPUs
 - ▶ ...



DNNs: Data-driven application

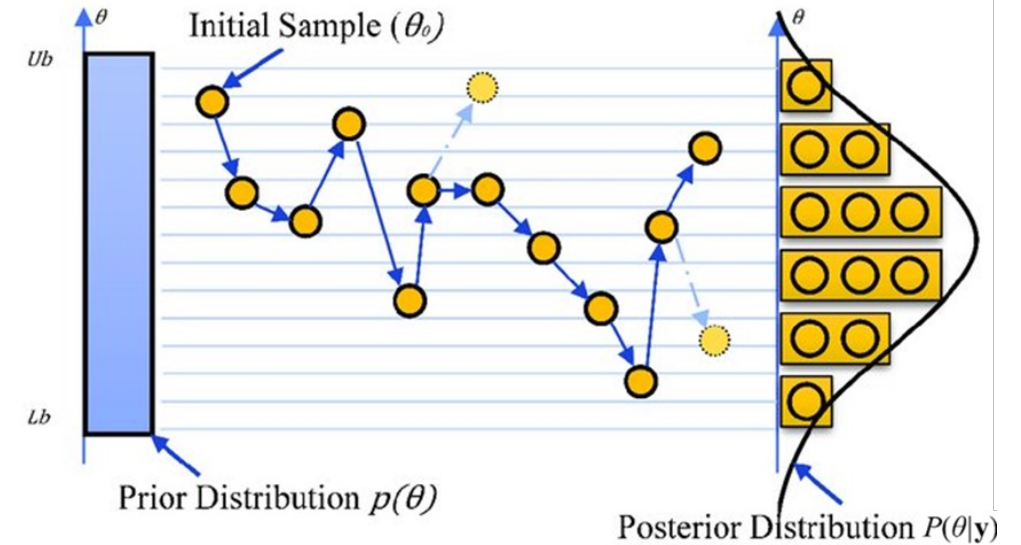


Neural Network Quantum States (NQSs):



Loss function: $E_{\theta} = \frac{\langle \psi_{\theta} | H | \psi_{\theta} \rangle}{\langle \psi_{\theta} | \psi_{\theta} \rangle} \geq E_0$

Variational Monte Carlo (VMC)



Sample using Metropolis-Hasting algorithm

$$E_{\theta} = \frac{\int |\psi_{\theta}(r)|^2 \frac{H\psi(r)}{\psi(r)} dr}{\int |\psi_{\theta}(r)|^2 dr}$$

Train the DNNs not by the data but by the physics equation: **physics-informed**



- DNNs: approximate high-dimensional functions

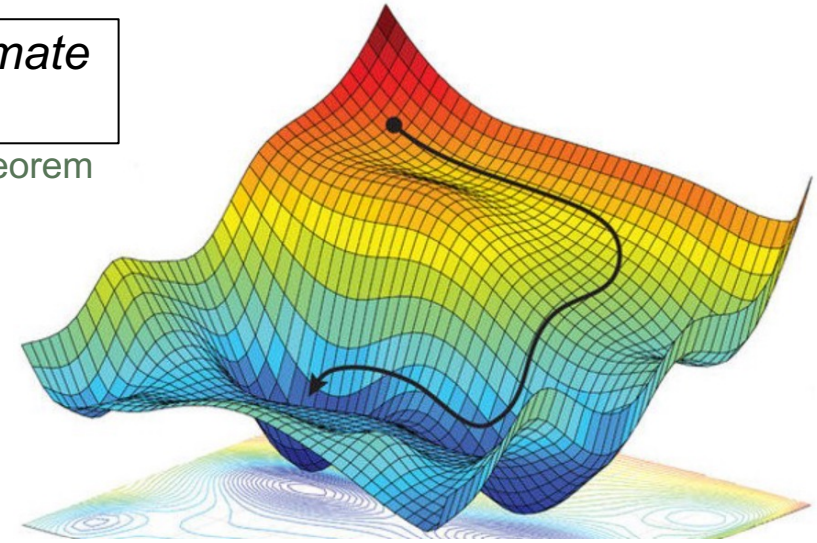
- ▶ **Universal approximation theorems**

→ sufficiently large or deep

*Neural networks with a certain structure can, in principle, approximate **any continuous function to any desired accuracy***

https://en.wikipedia.org/wiki/Universal_approximation_theorem

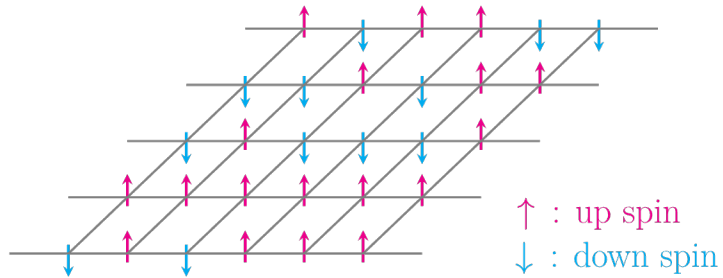
- ▶ Variational principle: the more general the trial function, the more accurate the solution
 - ▶ Unbiasedly distinguish molecular and compact tetraquark states
- VMC: applicable for few-body potential
 - ▶ Flux-tube confinement potential
- Circumvents the sign problem in imaginary time evolution of DMC
- Fast optimization: AD+GPUs
- Easy-to-use open-source frameworks



TensorFlow

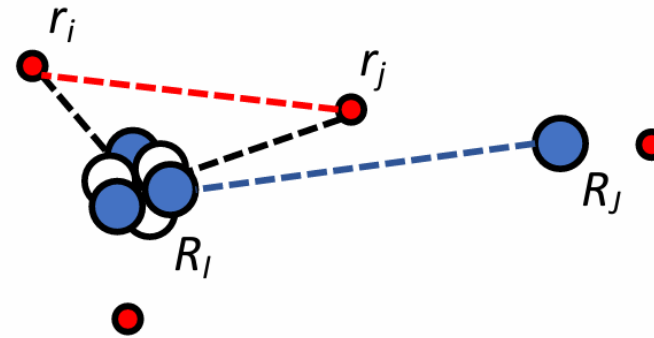


Spin models



G. Carleo and M. Troyer, Science 355, (2017).

Electronic systems



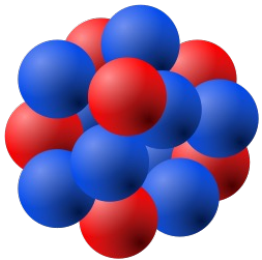
PauliNet: J. Hermann, Z. Schätzle, and F. Noé, Nat. Chem. 12, 891 (2020).

FermiNet: D. Pfau et al., Phys. Rev. Res. 2, 033429 (2020).

...

Review: J. Hermann et al., Nat. Rev. Chem. 7, 692–709 (2023)

Nuclear structure



Deuteron: J. W. T. Keeble and A. Rios, Phys. Lett. B 809, 135743 (2020).

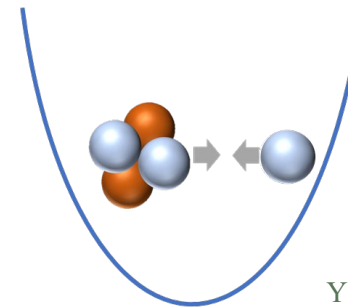
$A \leq 4$: C. Adams *et al.*, Phys. Rev. Lett. **127**, 022502 (2021).

FeynmanNet: Y. Yang and P. Zhao, Phys. Rev. C **107**, 034320 (2023).

Hypernuclei: Z.-X. Zhang *et al.*, arXiv:2508.03575

...

Neutron- α scattering



**Chiral Nuclear force
Trapped five-body problems
(DNNs+VMC)**

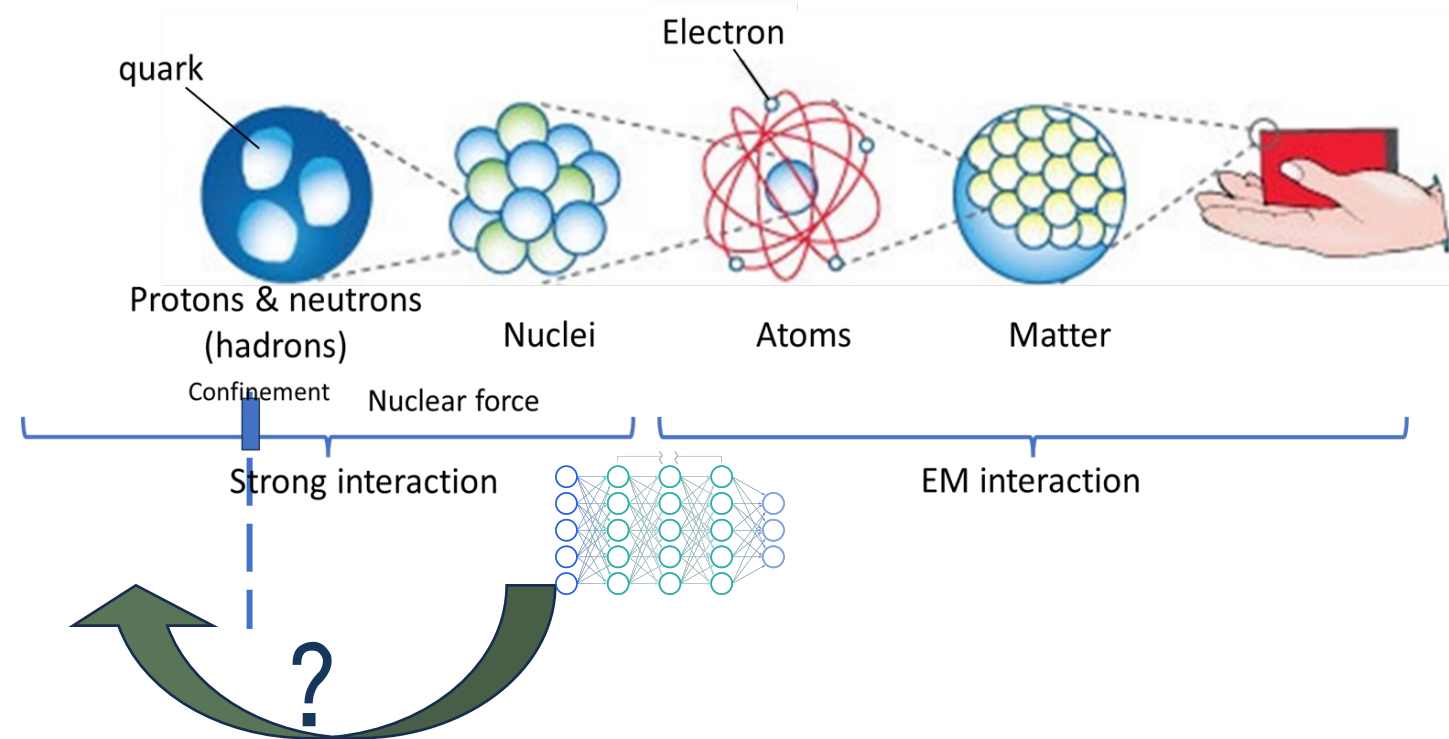
Y. Yang, E. Epelbaum, J. Meng, **LM**, and P. Zhao, Phys.Rev.Lett. 135 (2025) 17, 172502

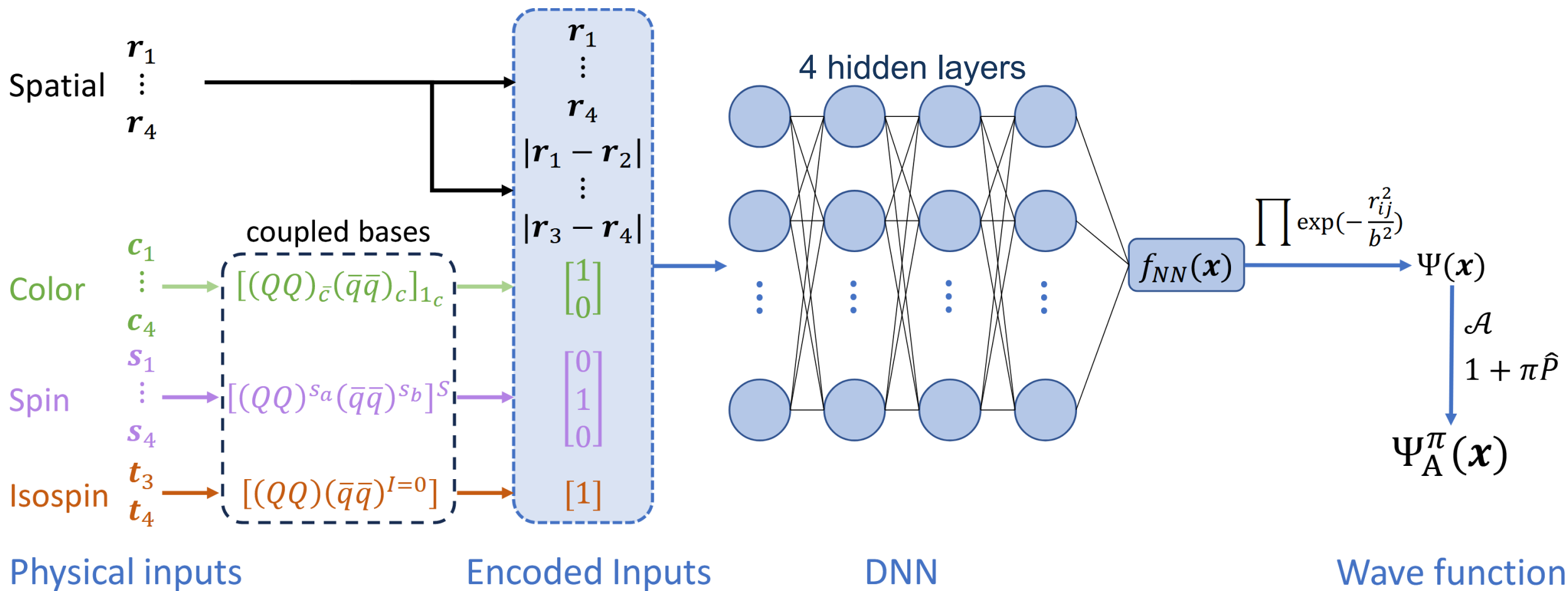
$$V_{ij}(r) = V_{ij}^{OGE} + V_{ij}^{conf.}$$
$$= \left[\frac{\alpha_s}{r} - \frac{8\pi\alpha_s}{3m_Q^2} \tilde{\delta}(\sigma; r) \mathbf{S}_i \cdot \mathbf{S}_j - \frac{3}{4} br \right] \frac{\lambda_i \cdot \lambda_j}{4}$$

Quark potential model

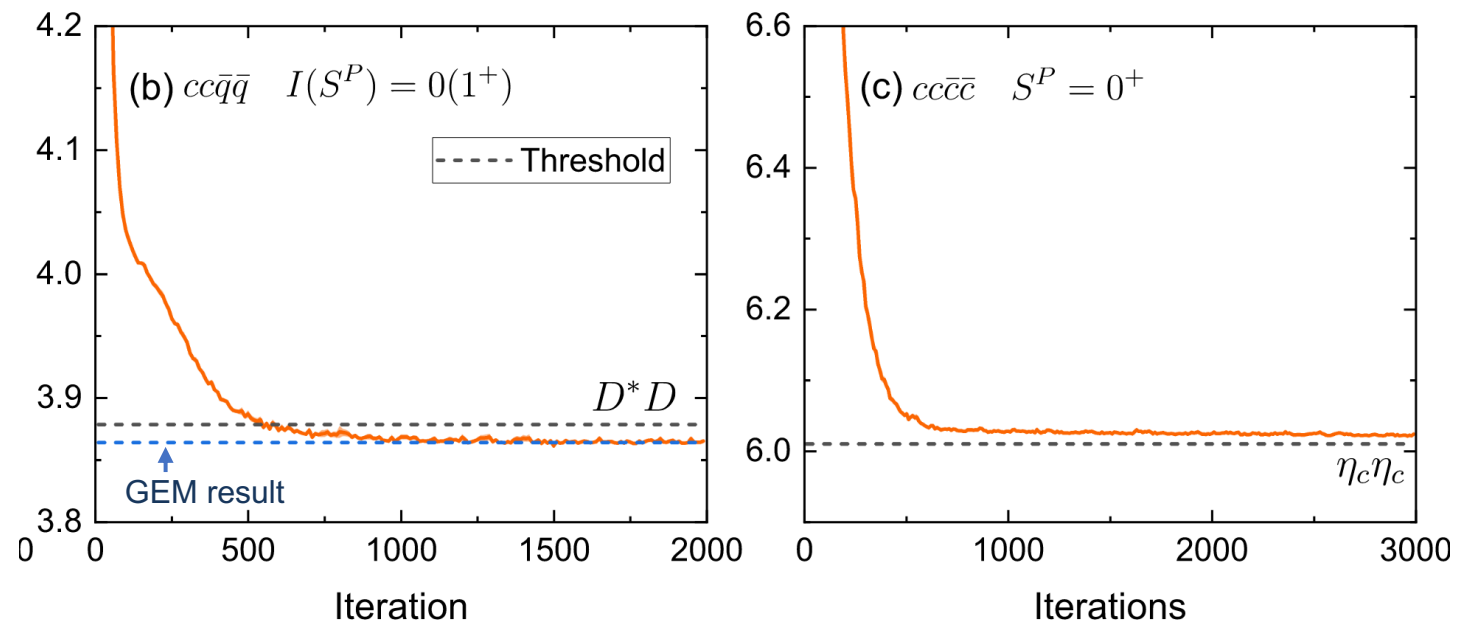
QCD-inspired model

- $SU(3)_c$: color and spin projections
- Confinement
- Strong correlation
-





- Convergence after 1000 iterations
- Statistical errors less than 0.1 MeV
- Competitive performance with GEM
- $cc\bar{q}\bar{q}$: molecular state
- $bb\bar{q}\bar{q}$: compact heavy diquark
- $QQ\bar{Q}\bar{Q}$: no bound states



	$I(S^P)$	Thresh.	ΔE	$P_{\bar{3}_c \otimes 3_c}$	$P_{6 \otimes 6_c}$	r_{QQ}	$r_{\bar{q}\bar{q}}$	$r_{Q\bar{q}}$
$cc\bar{q}\bar{q}$	$0(1^+)$	DD^*	-15	55%	45%	1.24	1.41	1.06
$bb\bar{q}\bar{q}$	$0(1^+)$	$\bar{B}\bar{B}^*$	-153	97%	3%	0.33	0.78	0.69
$QQ\bar{Q}\bar{Q}$	$0(0^+)$ $0(1^+)$ $0(2^+)$	$\eta_c\eta_c$ $\eta_c J/\psi$ $J/\psi J/\psi$	No bound					

ΔE in MeV, r in fm



- Exact pentaquark calculations are computationally prohibitive

- Approximations in the spatial or color configurations

- Possible bound pentaquark systems

- Heavy-diquark-antiquark-symmetry: $(QQ)_{\bar{3}_c} \rightarrow \bar{Q}$

- $\bar{Q}\bar{Q}qq \rightarrow QQ\bar{Q}qq$

- Given $M_{\bar{Q}\bar{Q}qq} < M_{\bar{Q}q} + M_{\bar{Q}q}$, $M_{QQ\bar{Q}qq} < M_{QQq} + M_{\bar{Q}q}$?

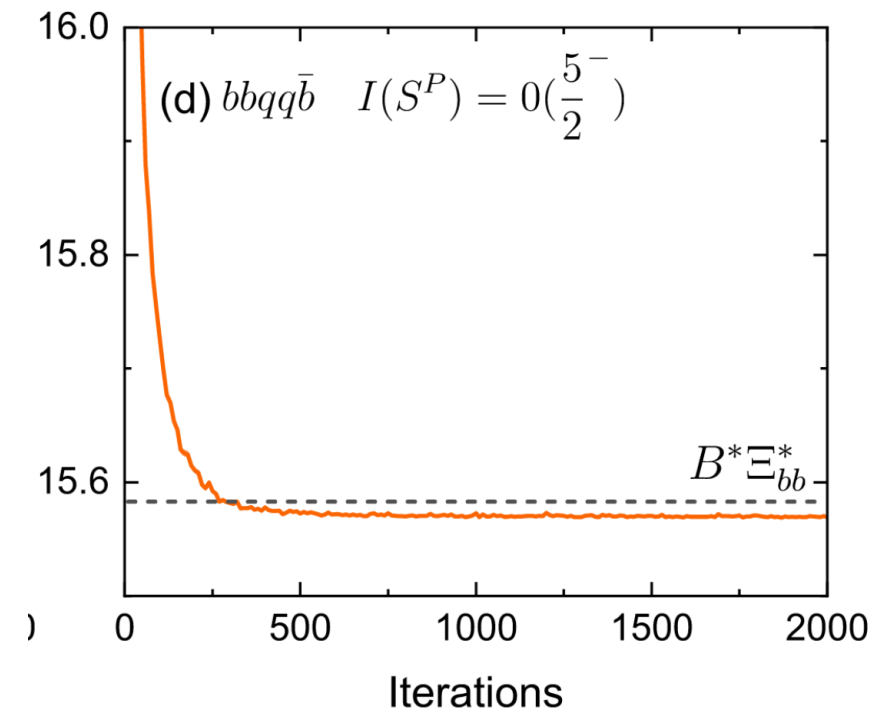
- Exact calculation via DeepQuark

$$\chi_{\bar{3}_c \otimes \bar{3}_c} = \left\{ [(QQ)_{\bar{3}_c} (qq)_{\bar{3}_c}]_{3_c} \bar{Q} \right\}_{1_c},$$

$$\chi_{\bar{3}_c \otimes 6_c} = \left\{ [(QQ)_{\bar{3}_c} (qq)_{6_c}]_{3_c} \bar{Q} \right\}_{1_c},$$

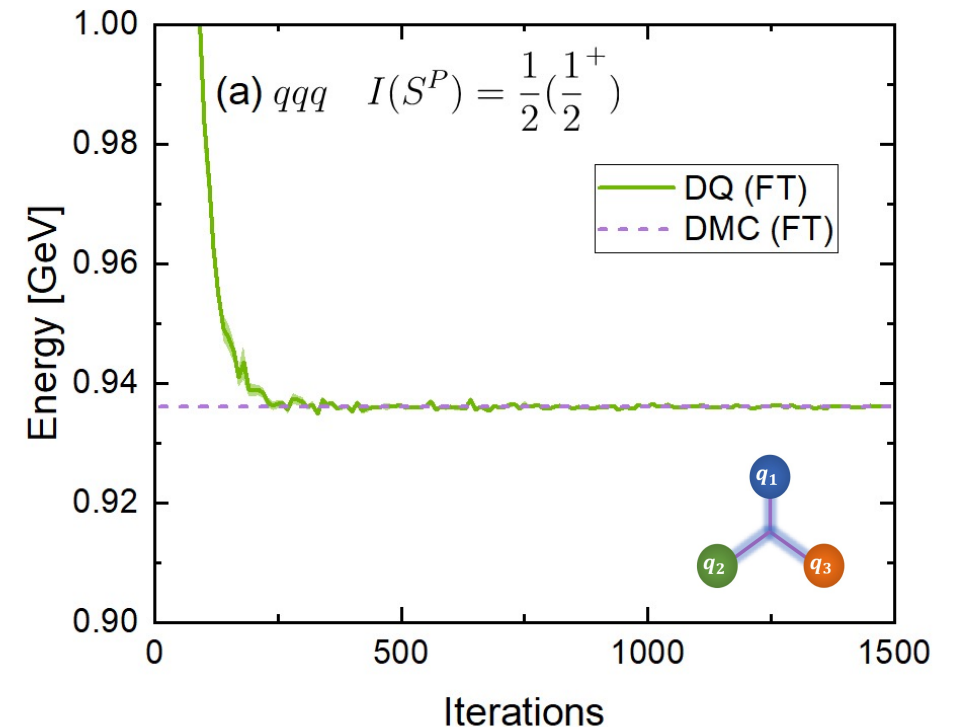
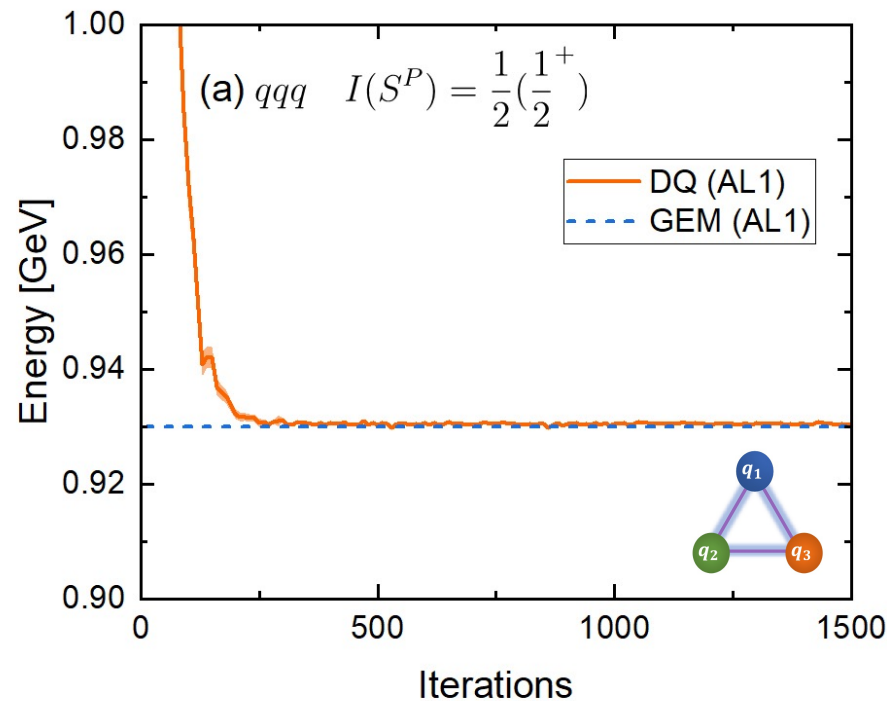
$$\chi_{6_c \otimes \bar{3}_c} = \left\{ [(QQ)_{6_c} (qq)_{\bar{3}_c}]_{3_c} \bar{Q} \right\}_{1_c}.$$

- Moderate increase in comput. cost relative to the tetraquark



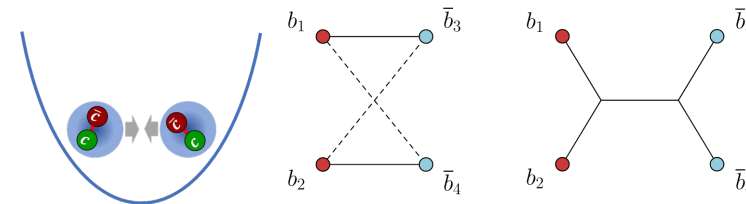
	S^P	Thresholds	ΔE	$\chi_{\bar{3}_c \otimes \bar{3}_c}$	$\chi_{\bar{3}_c \otimes 6_c}$	$\chi_{6_c \otimes \bar{3}_c}$	r_{QQ}	r_{Qq}	r_{qq}	$r_{Q\bar{Q}}$	$r_{q\bar{Q}}$
$ccqq\bar{c}$	$\frac{1}{2}^-, \frac{3}{2}^-$	$\eta_c \Lambda_c, J/\psi \Lambda_c$	NB	$\sim 35\%$	0%	$\sim 65\%$					
	$\frac{5}{2}^-$	$\bar{D}^* \Xi_{cc}^*$	-3	27%	73%	0%	0.50	1.39	1.90	1.73	1.38
$bbqq\bar{b}$	$\frac{1}{2}^-, \frac{3}{2}^-$	$\eta_b \Lambda_b, \Upsilon \Lambda_b$	NB	$\sim 35\%$	0%	$\sim 65\%$					
	$\frac{5}{2}^-$	$B^* \Xi_{bb}^*$	-14	19%	80%	1%	0.30	0.89	1.22	0.88	0.88

- Lattice QCD supports the flux-tube potential for qqq baryons and tetraquarks
- Baryons: Y-type interaction, junction position to minimize the total string length
 - ▶ Small impact on baryon spectrum, but complicate dramatically matrix element calculations
- DeepQuark: incorporate Y-type interaction without additional cost, thanks to the VMC
- Ready for tetraquark systems, where different confinement patterns lead to distinct signatures



- We still need quark models to understand the **pattens** of multi-quark states
 - ▶ **Difficulties:** interactions (confinement) and solving few-body (resonant) problem
 - ▶ **Chances:** All-charm tetraquark family $\Rightarrow$ confinement pictures
- Gaussian expansion method + Complex scaling method
 - ▶ No bound $QQ\bar{Q}\bar{Q}$ for **pairwise** interaction
 - ▶ Exp. $cc\bar{c}\bar{c}$ results: disfavor the pairwise confinement interaction
 - ▶ Too strong coupling between diquark and diquark-antidiquark (?)
- DeepQuark: DNN-based framework
 - ▶ Surpass traditional methods starting from pentaquark states
 - ▶ Ready for flux-tube confinement
- Outlook:
 - ▶ DNNs: excited states, scattering, resonances
 - ▶ All-charm tetraquark with flux-tube confinement
 - ▶ Convolutional transformer wave functions

**Thanks for
your attention!**





Backup



Challenge 1: Spin and color projection

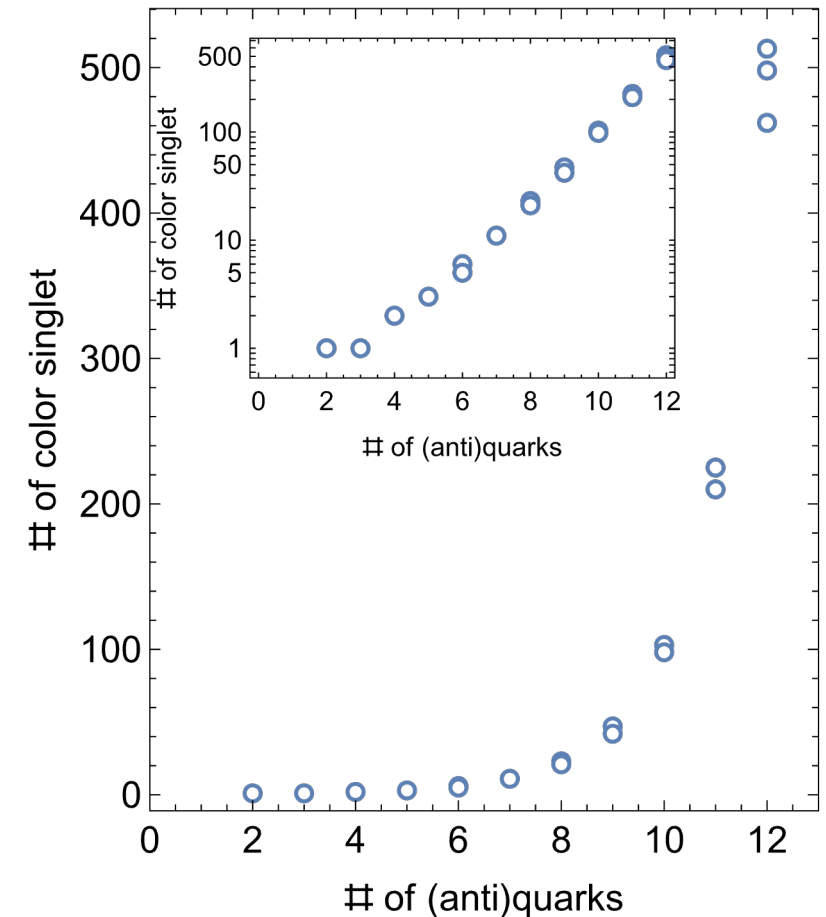


- Conventional spin projection
 - ▶ Option 1: no projection, calculate the ground state
 - ▶ Option 2: penalty terms

$$f_{loss}(\theta) = \langle E \rangle_{\theta} + \langle S^2 \rangle_{\theta}$$

However, for multiquark systems

- State with higher spin could more interesting
 - ▶ $[cc\bar{q}\bar{q}]_{S=0}$: DD threshold; $[cc\bar{q}\bar{q}]_{S=1}$: T_{cc} state
- Color projection is needed
 - ▶ color singlet
 - ▶ $SU(3)$ symmetry

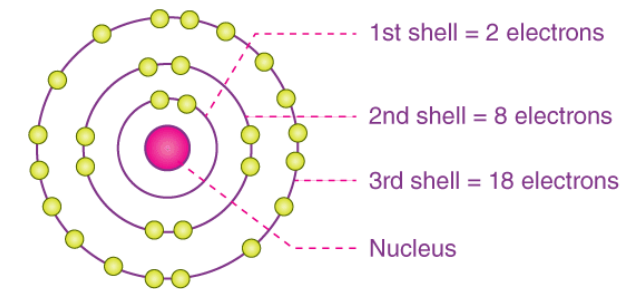


Challenge 2: Strong correlation

- Shell model: good guidance or initial point

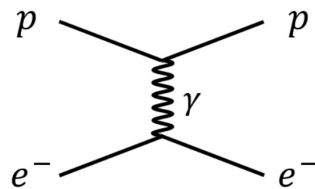
$$\Psi(\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_N) = \frac{1}{\sqrt{N!}} \begin{vmatrix} \chi_1(\mathbf{x}_1) & \chi_2(\mathbf{x}_1) & \cdots & \chi_N(\mathbf{x}_1) \\ \chi_1(\mathbf{x}_2) & \chi_2(\mathbf{x}_2) & \cdots & \chi_N(\mathbf{x}_2) \\ \vdots & \vdots & \ddots & \vdots \\ \chi_1(\mathbf{x}_N) & \chi_2(\mathbf{x}_N) & \cdots & \chi_N(\mathbf{x}_N) \end{vmatrix}$$

For electron and nucleon systems

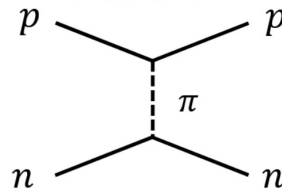


Electron Shell

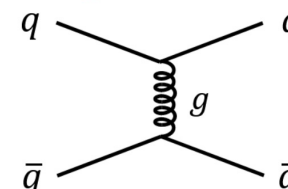
- Much stronger interaction among quarks
 - ▶ No evidence of a multiquark shell structure
 - ▶ Few-body correlation could be important



$$e^2 = 0.1$$
$$m_e \approx 0.5 \text{ MeV}$$



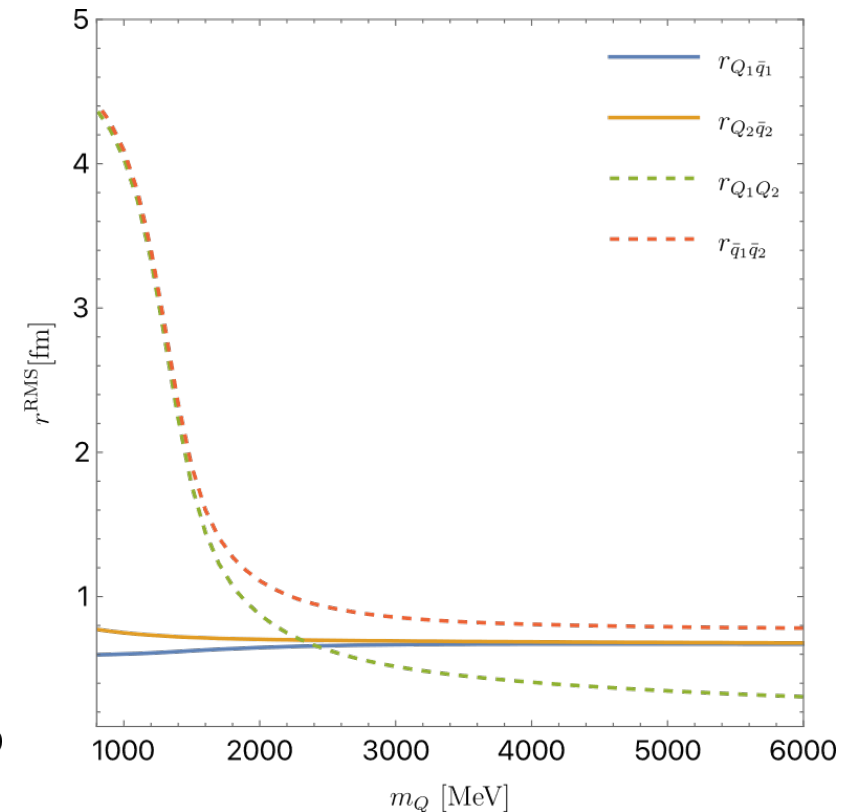
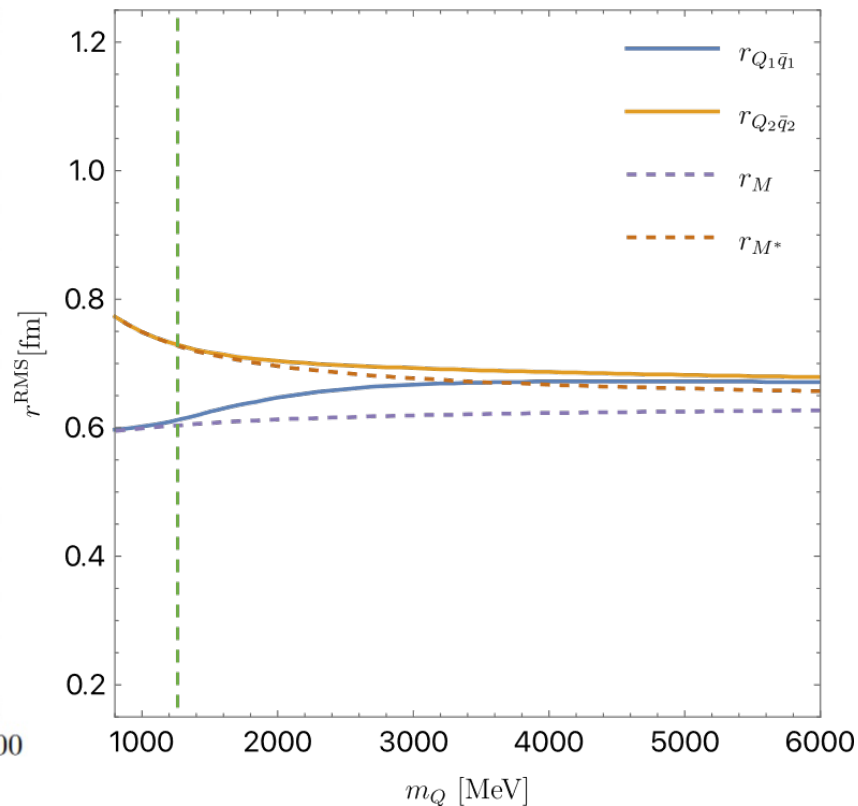
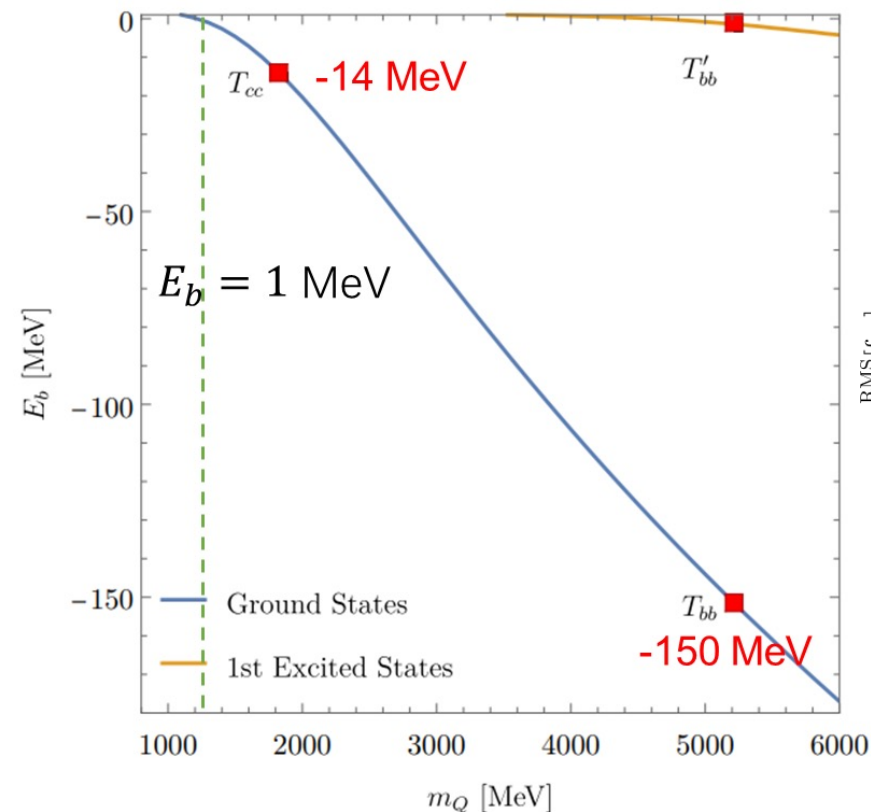
$$\frac{m_\pi^2 g_A^2}{16\pi f_\pi^2} \sim 0.1$$
$$m_N = 938 \text{ MeV}$$



$$\frac{4}{3} \alpha_s(m_c) \sim 0.4$$
$$m_c \approx 1273 \text{ MeV}$$



Molecular or compact ?



- Tuning the m_Q to m_b : (bb) compact diquark
- Tuning the m_Q to make $E_b < 1$ MeV: molecular states



- For color-singlet multiquark states $\{Q_1, Q_2, \dots, Q_n\}$, $Q_i = Q$ or $\bar{Q}$
 - ▶ 2-body interaction:

$$V_{Q_i Q_j} = V_8(r_{ij}) \lambda_i \cdot \lambda_j; \quad V_{\{Q_1, Q_2, \dots, Q_n\}} = \sum_{i < j} a_{ij} V_8(r_{ij}),$$

- ▶ Corollary

$$\sum_{i < j} a_{ij} = -\frac{8}{3}n$$

Proof: $2\langle \sum_{i < j} \lambda_i \cdot \lambda_j \rangle = \langle \sum_i \lambda_i \rangle^2 - \sum_i \langle \lambda_i \rangle^2$

- A general question:

For fixed $\sum_{i < j} a_{ij} = \mathcal{C}$, what distribution of $\{a_{ij}\}$ give the lowest mass?

- Answer:

- ▶ The even case give the worse results: $M_n(a_{ij}) \leq M_n^{(S)}$

Proof:

$$H = H^S + \Delta V = H^S + aV_8(r_{12}) - aV_8(r_{34}), \quad \langle \psi^S | \Delta V | \psi^S \rangle = 0$$

$$\langle \psi^S | H^S | \psi^S \rangle = \langle \psi^S | H | \psi^S \rangle \geq \min_{\psi \in \mathcal{H}} \langle \psi | H | \psi \rangle$$

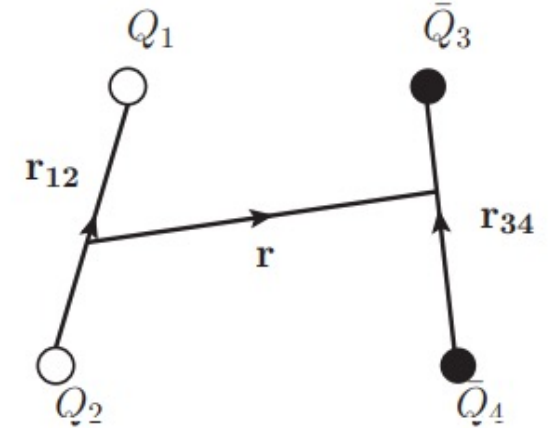
- ▶ Intuitively, less even $\{a_{ij}\}$, more deeply bound ground states

J. P. Ader, J. M. Richard, and P. Taxil, Phys. Rev. D **25**, 2370 (1982).



● $\{a_{ij}\}$ distribution for $QQ\bar{Q}\bar{Q}$

	$a_{12} = a_{34}$	$a_{13} = a_{24}$	$a_{14} = a_{23}$	$\text{Var}[\{a_{ij}\}]$
Dimeson	0	$-\frac{16}{3}$	0	$\frac{1024}{135}$
$\bar{3}_c - 3_c$	$-\frac{8}{3}$	$-\frac{4}{3}$	$-\frac{4}{3}$	$\frac{64}{135}$
$\bar{6}_c - 6_c$	$\frac{4}{3}$	$-\frac{10}{3}$	$-\frac{10}{3}$	$\frac{784}{135}$



- ▶ $\text{Var}[\{a_{ij}\}]$ as a measurement of evenness
- ▶ $2M(Q\bar{Q}) < M(6_c - \bar{6}_c) < M(3_c - \bar{3}_c)$
- ▶ Conclusion: **Equal mass** + **color-electric** + **2-body V** $\Rightarrow$ no state blow the dimeson threshold.
- Things become different, when
 - ▶ Unequal quark masses, e.g. $QQ\bar{q}\bar{q}$
 - ▶ multi body interaction, e.g. double- Y interaction
 - ▶ ...

PRL118, 142001; PRL119, 202001; PRL119, 202002



- Ref.[Ortega:2022efc]: Same SLM interactions
 - ▶ $[bb\bar{q}\bar{q}]^{I=1}$ bound state! **Our results: there is no isospin vector states**
 - ▶ Get a $J^P(I) = 0^+(0)$ state dominated by S-wave BB states!
Violating Boson principle

Mass	E_B	$\mathcal{P}_{B^0 B^{*+}}$	$\mathcal{P}_{B^+ B^{*0}}$	$\mathcal{P}_{B^{*+} B^{*0}}$	$\mathcal{P}_{I=0}$	$\mathcal{P}_{I=1}$
10582.2	21.9	47.8	50.0	2.2	99.99	0.01
10593.5	10.5	51.0	48.6	0.4	0.02	99.98

J^P	I	Mass	Width	E_B	$\mathcal{P}_{BB}$	$\mathcal{P}_{B^* B^*}$	Γ_{BB}	$\Gamma_{B^* B^*}$
0^+	0	10553.0	0	6.0	92%	8%	0	0
		10640.7	2.8	8.7	76%	24%	2.8	0
	1	10545.9	0	13.1	93%	7%	0	0
		10672.6	72.0	-23.2	39%	61%	30.7	41.3
2^+	1	10642.3	0	7.1	-	100%	-	0



● Semay-Silvestre-Brac Models

Semay:1994ht, Silvestre-Brac:1996myf

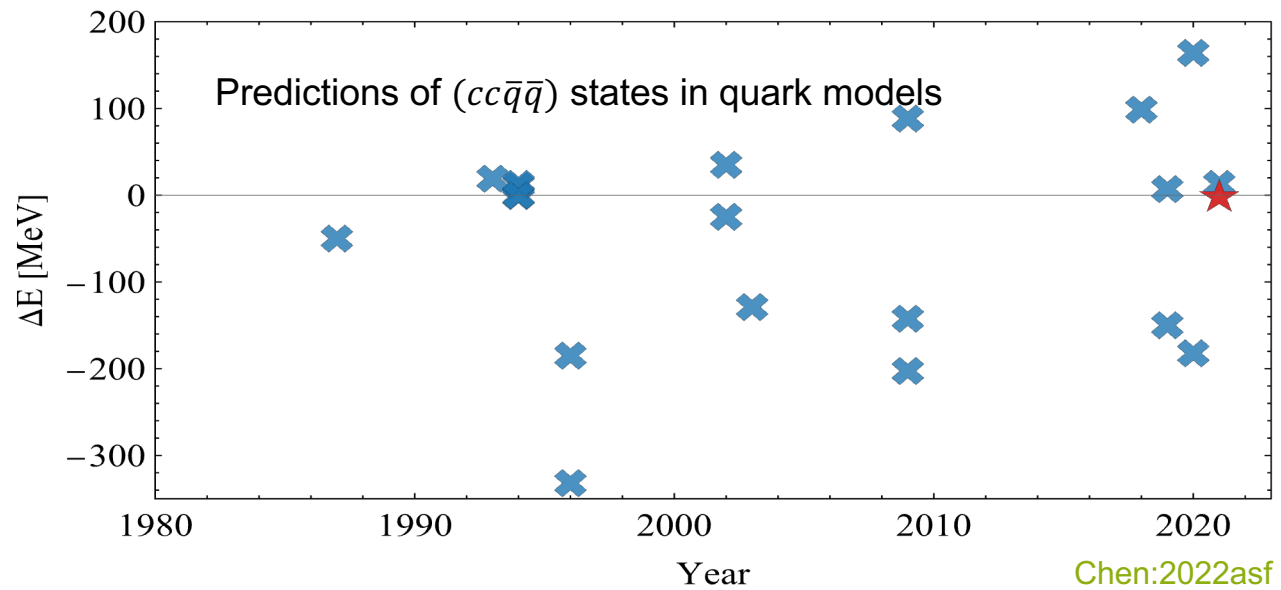
$$V_{ij}(r) = \left[-\frac{\kappa}{r} + \lambda r^p - \Lambda + \frac{2\pi}{3m_i m_j} \kappa' \frac{1}{\pi^{3/2} r_0^3} e^{(-r^2/r_0^2)} \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j \right] \lambda_i \cdot \lambda_j$$

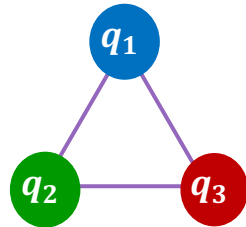
AL1: $p = 1$ and AP1: $p = 2/3$

[GeV]	π	K	D	D_s	B	B_s	B_c	η_c	η_b
Exp.	0.139	0.494	1.870	1.968	5.279	5.367	6.274	2.984	9.399
AL1	0.138	0.491	1.862	1.962	5.293	5.361	6.292	3.005	9.424
AP1	0.139	0.498	1.881	1.955	5.311	5.356	6.269	2.982	9.401

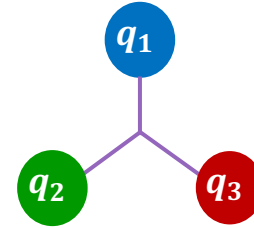
[GeV]	ρ	ω	ϕ	K^*	D^*	D_s^*	B^*	B_s^*	B_c^*	J/ψ	Υ
Exp.	0.775	0.783	1.019	0.892	2.010	2.112	5.325	5.415	6.329	3.097	9.460
AL1	0.770	0.770	1.021	0.903	2.016	2.102	5.350	5.417	6.343	3.101	9.461
AP1	0.770	0.770	1.021	0.908	2.033	2.107	5.367	5.418	6.338	3.102	9.461







$$V_{\text{conf}}^{\Delta} = \sigma_{\Delta} \sum_{i < j} r_{ij}$$



$$V_{\text{conf}}^Y = \sigma_Y L_{\min}$$

$$0.5 \leq \frac{L_{\min}}{L_{\Delta}} \leq 0.58$$



- Activation function: $\sigma(x) = \tanh(x)$
- Optimizer

$$\boldsymbol{\theta}^{i+1} = \boldsymbol{\theta}^i - \eta(S + \epsilon I)^{-1} \nabla_{\boldsymbol{\theta}^i} E_{\boldsymbol{\theta}^i}, \quad (8)$$

where i is the iteration step, η is the learning rate, $\epsilon = 10^{-3}$ is taken for numerical stability, and S is the Quantum Fisher information matrix,

$$S_{ab} = \frac{\langle \partial_{\theta_a} \psi_{\boldsymbol{\theta}} | \partial_{\theta_b} \psi_{\boldsymbol{\theta}} \rangle}{\langle \psi_{\boldsymbol{\theta}} | \psi_{\boldsymbol{\theta}} \rangle} - \frac{\langle \partial_{\theta_a} \psi_{\boldsymbol{\theta}} | \psi_{\boldsymbol{\theta}} \rangle}{\langle \psi_{\boldsymbol{\theta}} | \psi_{\boldsymbol{\theta}} \rangle} \frac{\langle \psi_{\boldsymbol{\theta}} | \partial_{\theta_b} \psi_{\boldsymbol{\theta}} \rangle}{\langle \psi_{\boldsymbol{\theta}} | \psi_{\boldsymbol{\theta}} \rangle}. \quad (9)$$



TABLE V. The number of nodes in each hidden layer and the total number of variational parameters in the DNN for different systems.

Systems	S^P	Nodes	Parameters
e^+e^-	0^+	(16, 16, 16, 16)	961
$e^+e^-e^-$	0^+	(16, 16, 16, 16)	1041
$e^+e^+e^-e^-$	0^+	(16, 16, 16, 16)	1137
qqq	$\frac{1}{2}^+$	(16, 16, 16, 16)	1105
$QQ\bar{q}\bar{q}$	1^+	(32, 16, 16, 16)	1889
$QQ\bar{Q}\bar{Q}$	0^+	(32, 16, 16, 16)	1825
$QQ\bar{Q}\bar{Q}$	1^+	(32, 16, 16, 16)	1857
$QQ\bar{Q}\bar{Q}$	2^+	(32, 16, 16, 16)	1793
$QQqq\bar{Q}$	$\frac{1}{2}^-$	(40, 20, 20, 20)	3081
$QQqq\bar{Q}$	$\frac{3}{2}^-$	(40, 20, 20, 20)	3041
$QQqq\bar{Q}$	$\frac{5}{2}^-$	(40, 20, 20, 20)	2921



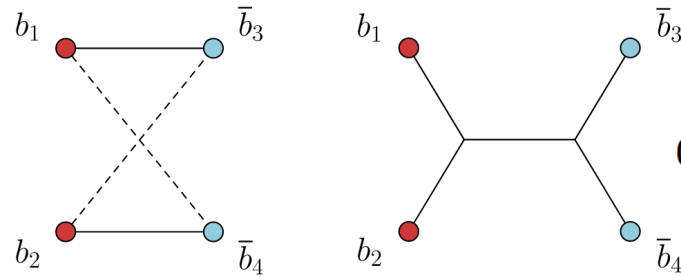


Fig. 1. Left panel: the flip-flop configuration of disconnected di-mesons. Right panel: the butterfly configuration with two connected diquarks. The two middle connecting points are chosen to minimize the total path.

$$V^{4Q} \equiv \min(V^{\text{flip-flop}}, V^{\text{butterfly}}) + \text{DMC}$$

0^{++} state has a mass of 18.69 ± 0.03 GeV, which is around 100 MeV below twice the η_b mass.

Y. Bai, S. Lu, and J. Osborne, Beauty-full tetraquarks, Phys. Lett. B **798**, 134930 (2019).

$$\begin{aligned} V = & \Theta(V_{MM} - V_{YY})\Theta(V_{MM'} - V_{YY})(V_{YY}|\bar{\mathbf{3}}_{12}\mathbf{3}_{34}\rangle\langle\bar{\mathbf{3}}_{12}\mathbf{3}_{34}| + \min(V_{MM}, V_{MM'})|\bar{\mathbf{6}}_{12}\bar{\mathbf{6}}_{34}\rangle\langle\bar{\mathbf{6}}_{12}\bar{\mathbf{6}}_{34}|) \\ & + \Theta(V_{YY} - V_{MM})\Theta(V_{MM'} - V_{MM})(V_{MM}|\mathbf{1}_{13}\mathbf{1}_{24}\rangle\langle\mathbf{1}_{13}\mathbf{1}_{24}| + \min(V_{YY}, V_{MM'})|\mathbf{8}_{13}\mathbf{8}_{24}\rangle\langle\mathbf{8}_{13}\mathbf{8}_{24}|) \\ & + \Theta(V_{YY} - V_{MM'})\Theta(V_{MM} - V_{MM'})(V_{MM'}|\mathbf{1}_{14}\mathbf{1}_{23}\rangle\langle\mathbf{1}_{14}\mathbf{1}_{23}| + \min(V_{YY}, V_{MM})|\mathbf{8}_{14}\mathbf{8}_{23}\rangle\langle\mathbf{8}_{14}\mathbf{8}_{23}|), \end{aligned}$$

+ multidimensional
numerical integration
+ analytical continuation

$qq\bar{Q}\bar{Q}$ systems

P. Bicudo and M. Cardoso, Phys. Rev. D 94, 094032 (2016).

