



東南大學
SOUTHEAST UNIVERSITY

兰州大学-核学院-报告

Unveiling the All-Charmed Tetraquark Spectrum with Deep Neural Networks

Lu Meng (孟璐)

Southeast University

Nov. 28th, 2025



Shi-Lin Zhu Wei-Lin Wu Yao Ma Yan-Ke Chen

Based on: [PRD108, 114016 \(2023\)](#), [PRD110, 034030 \(2024\)](#),
[PRD109, 054034 \(2024\)](#), [arXiv:2506.20555 ...](#)



Phys.Lett. 8 (1964) 214-215

Volume 8, number 3 PHYSICS LETTERS 1 February 1964

A SCHEMATIC MODEL OF BARYONS AND MESONS *

M. GELL-MANN

California Institute of Technology, Pasadena, California

Received 4 January 1964

...
A simpler and more elegant scheme can be constructed if we allow non-integral values for the charges. We can dispense entirely with the basic baryon b if we assign to the triplet t the following properties: spin $\frac{1}{2}$, $z = -\frac{1}{3}$, and baryon number $\frac{1}{3}$. We then refer to the members $u^{\frac{2}{3}}$, $d^{-\frac{1}{3}}$, and $s^{-\frac{1}{3}}$ of the triplet as "quarks" q and the members of the anti-triplet as anti-quarks \bar{q} . Baryons can now be constructed from quarks by using the combinations (qqq) , $(qqq\bar{q})$, etc., while mesons are made out of $(q\bar{q})$, $(qq\bar{q}\bar{q})$, etc. It is assumed that the lowest baryon configuration (qqq) gives just the represen-



8419/TH.412
21 February 1964

AN SU_3 MODEL FOR STRONG INTERACTION SYMMETRY AND ITS BREAKING

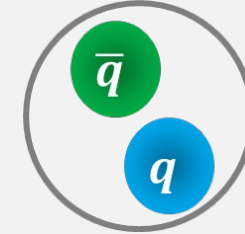
II *)

G. Zweig
CERN---Geneva

*) Version I is CERN preprint 8182/TH.401, Jan. 17, 1964.

6) In general, we would expect that baryons are built not only from the product of three aces, AAA , but also from $\bar{A}AAAA$, $\bar{A}AAAAA$, etc., where \bar{A} denotes an anti-ace. Similarly, mesons could be formed from $\bar{A}A$, $A\bar{A}A$ etc. For the low mass mesons and baryons we will assume the simplest possibilities, $\bar{A}A$ and AAA , that is, "deuces and treys".

Meson

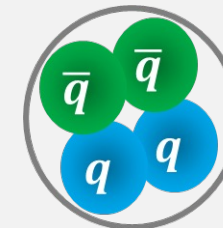


Baryon

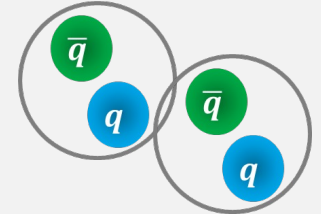


Conventional hadrons

Compact type



Molecular type



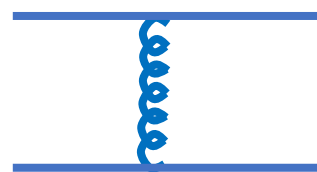
Multiquark states

The multiquark states were predicted at the birth of quark model



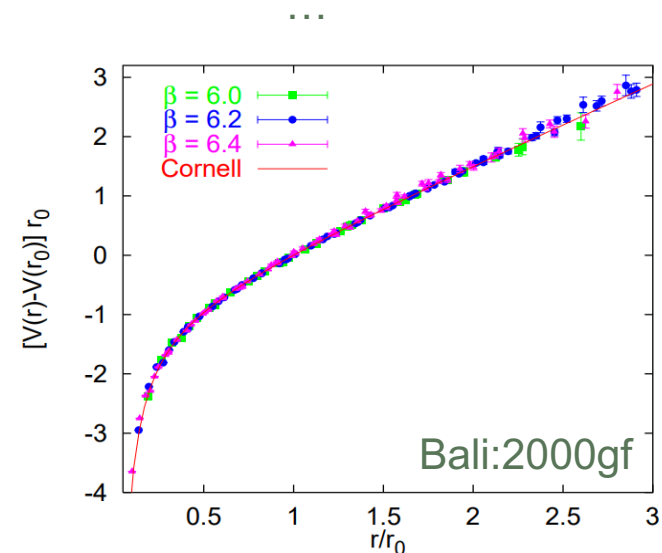
- A minimal model: one-gluon-exchange+Confinement

$$V_{ij}(r) = \left[\underbrace{\frac{\alpha_s}{r} - \frac{8\pi\alpha_s}{3m_i m_j} \frac{\tau^3}{\pi^{3/2}} e^{-\tau^2 r^2} \mathbf{s}_i \cdot \mathbf{s}_j}_{\text{OGE}} + \underbrace{\left(-\frac{3b}{4}r + V_c\right)}_{\text{Confinement}} \right] \frac{\lambda_i \cdot \lambda_j}{4}$$



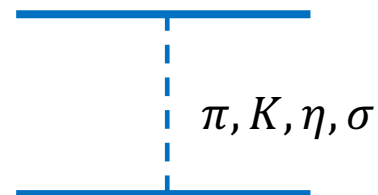
One-gluon-exchange

Cornell model: Eichten:1978tg,
BGS model: Barnes:2005pb



- Chiral quark models

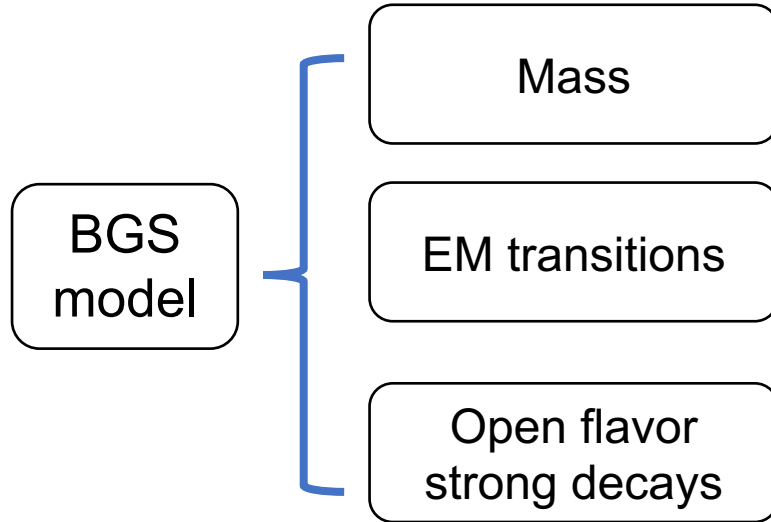
Manohar:1983md, Zhang:1997ny, Vijande:2004he, Gonzalez:2012gka...



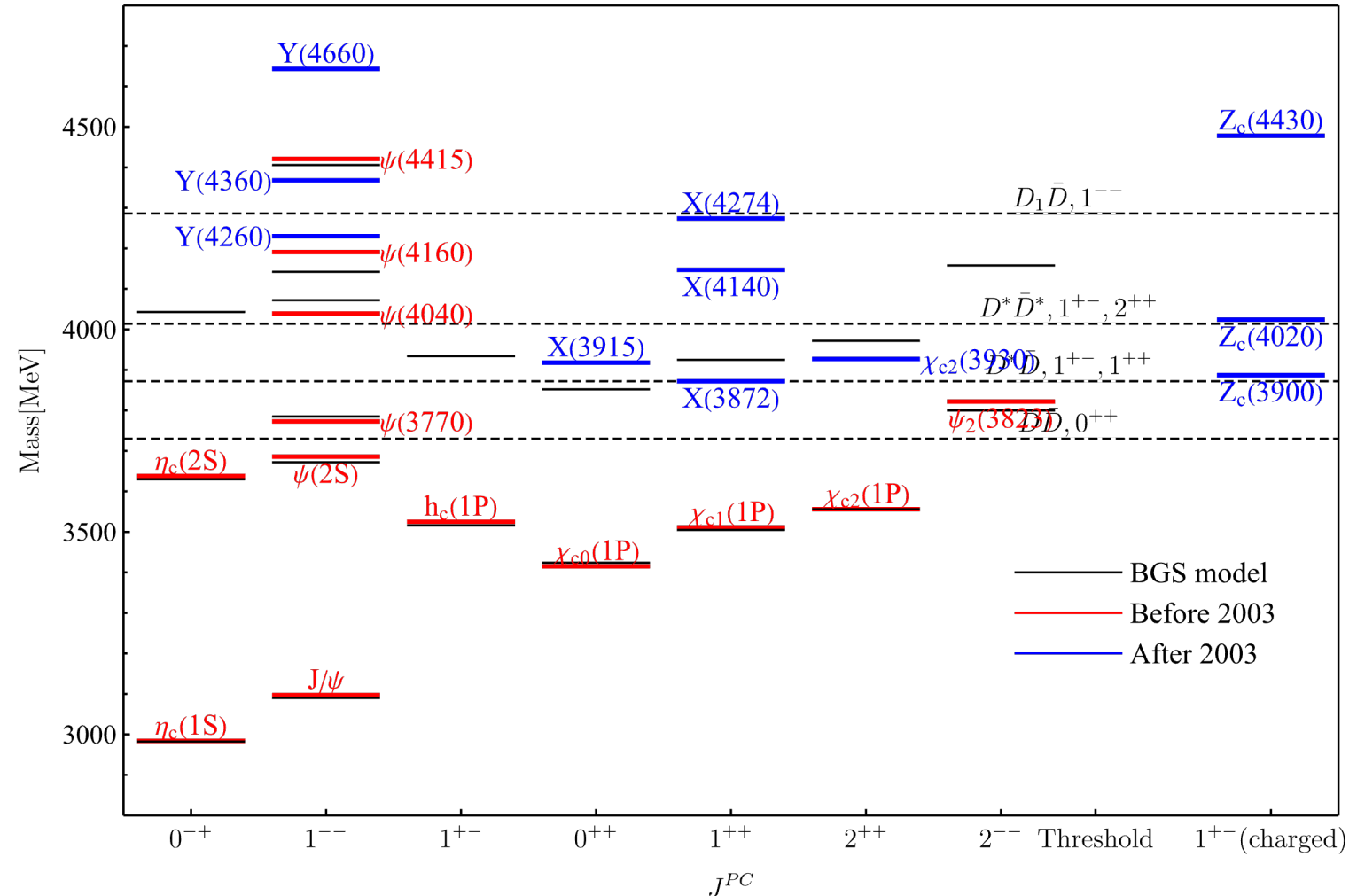
- Relativized Godfrey-Isgur model

Godfrey:1985xj...





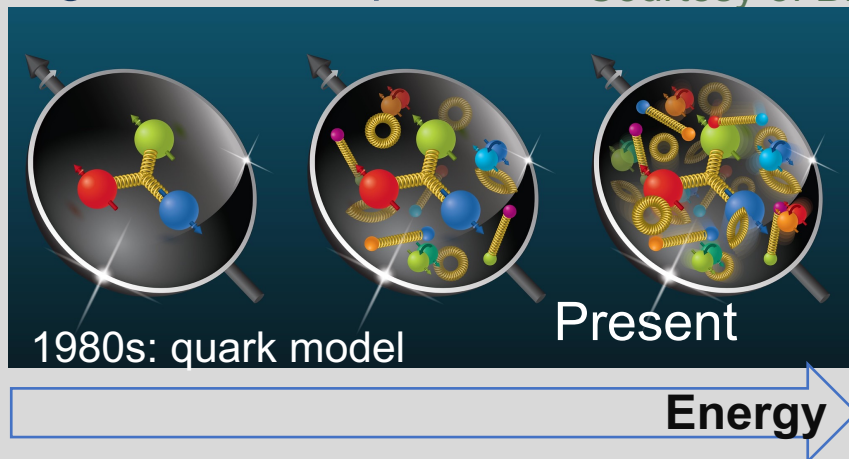
- NRQM with only 4 paras.
- Work well below open flavor thresholds
- Work better for bottomonium



T. Barnes, S. Godfrey, and E. S. Swanson, Phys. Rev. D 72, 054026 (2005).

● Evolving view of the proton

Courtesy of BNL

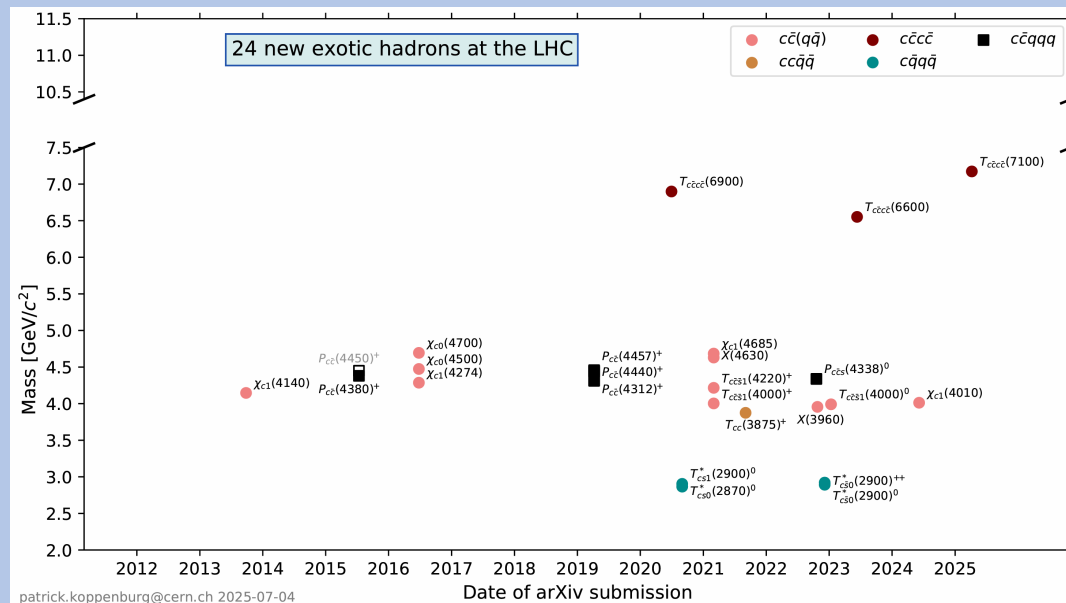


● Alternative methods:

- ▶ Lattice QCD
- ▶ Low energy EFTs
- ▶ Dyson-Schwinger equations
- ▶ ...

Against For

● We even do not know the patterns



● Quark models

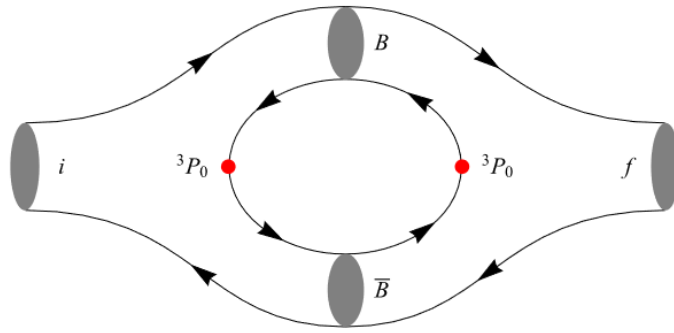
- ▶ A clear picture to uncover patterns
- ▶ Lower computational costs
- ▶ Prediction power
- ▶ Also decay models
- ▶ ...

We still need quark model !

The right question: How to **adapt the quark models to meet the new demands?**

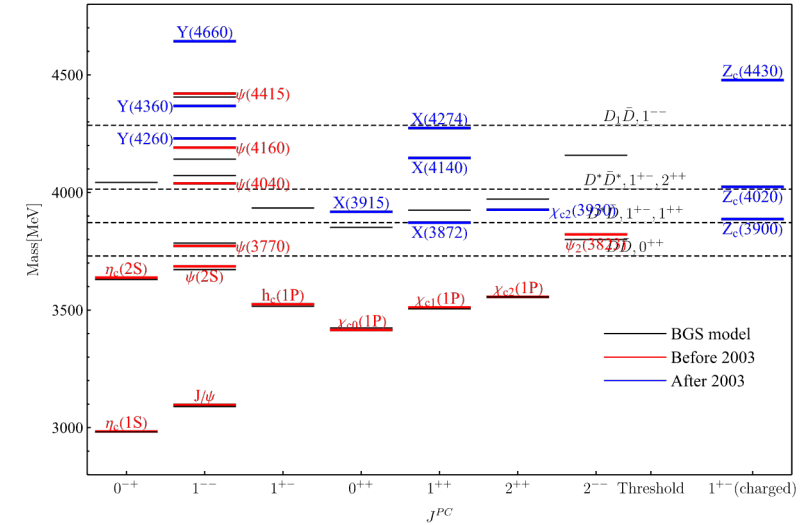


- Heavy-quarkonium-like states: unquenched quark model



B-S Zou's talks

<https://indico.cern.ch/event/1457095/contributions/6563231/>



- Multiquark systems

$[c\bar{c}qqq]$ P_c	$[cc\bar{c}\bar{c}]$	$[cs\bar{u}\bar{d}]$	$[cs\bar{u}\bar{d}]$	$[cc\bar{u}\bar{d}]$	$[c\bar{s}u\bar{d}][c\bar{s}\bar{u}d]$	$[c\bar{c}qqq]$
	X(6900)	$T_{cs1}(2900)$	$Z_{cs}(3985)$	$T_{cc}(3875)^+$	$T_{c\bar{s}0}(2900)^{++}$	$P_{cs}(4338)$
	X(6600)	$T_{cs0}(2900)$	$Z_{cs}(4000)$		$T_{c\bar{s}0}(2900)^0$	$P_{cs}(4459)$
	X(7100)					
	2006.16957	2009.00025	2011.07855	2109.01038	2212.02716	2210.10346
	2306.07164	2009.00026	2103.01803	2109.01056	2212.02717	2012.10380
	2304.08962				2411.19781	2502.09951
	2506.07944					

Particle Zoo 2.0

- **Opportunities: all-charm tetraquark family**

- ▶ Great experimental advances: LHCb, CMS, ATLAS
- ▶ Simple systems

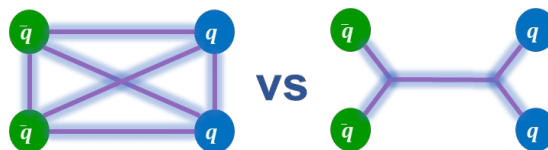
Constituent quark is almost the current quark

Small relativistic effect

Unlikely exchange light mesons between (anti)quarks

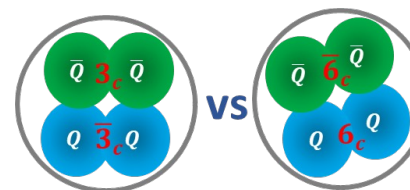
- ▶ Different confinements would leave imprints on the mass spectrum

Alexandrou:2004ak, Okiharu:2004ve,
Bicudo:2017usw

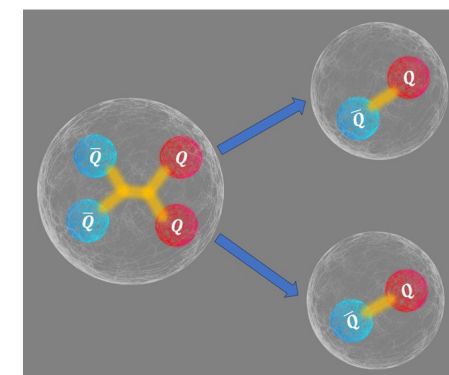


- **Challenges**

- ▶ Color structures: e.g. $3 - \bar{3}$ and $6 - \bar{6}$ tetraquark
- ▶ Matrix element of double Y-type potential
- ▶ Four/five body problem
- ▶ Resonance above the di-hadron thresholds
- ▶

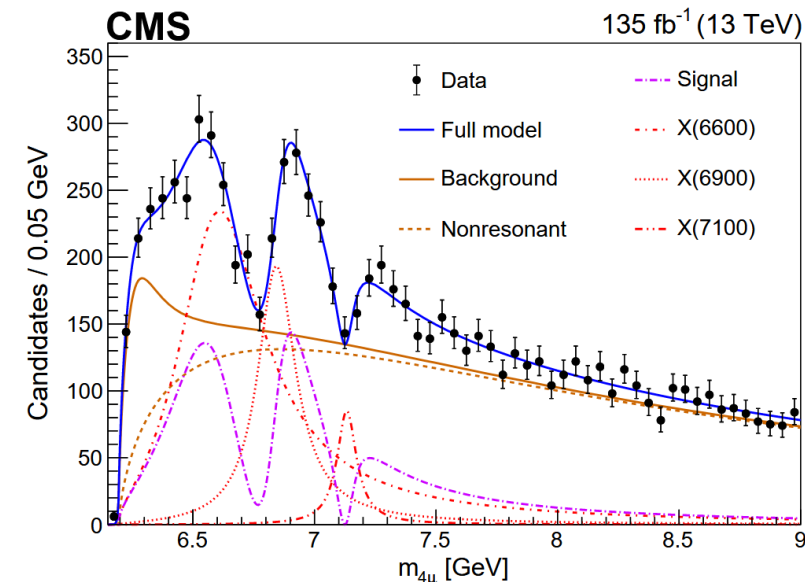


Color structures



quark rearrangement

**Interactions?
few-body (resonance) problem?**



- **Bound state**
- **Resonance:** complex scaling method
- **DNNs:** towards tetraquark **confinement**





Bound state



- For color-singlet multiquark states $\{Q_1, Q_2, \dots, Q_n\}$, $Q_i = Q$ or \bar{Q}
 - ▶ 2-body interaction:

$$V_{Q_i Q_j} = V_8(r_{ij}) \lambda_i \cdot \lambda_j; \quad V_{\{Q_1, Q_2, \dots, Q_n\}} = \sum_{i < j} a_{ij} V_8(r_{ij}),$$

- ▶ Corollary

$$\sum_{i < j} a_{ij} = -\frac{8}{3}n$$

Proof: $2\langle \sum_{i < j} \lambda_i \cdot \lambda_j \rangle = \langle \sum_i \lambda_i \rangle^2 - \sum_i \langle \lambda_i \rangle^2$

- A general question:

For fixed $\sum_{i < j} a_{ij} = C$, what distribution of $\{a_{ij}\}$ give the lowest mass?

- Answer:

- ▶ The even case give the worse results: $M_n(a_{ij}) \leq M_n^{(S)}$

Proof:

$$H = H^S + \Delta V = H^S + aV_8(r_{12}) - aV_8(r_{34}), \quad \langle \psi^S | \Delta V | \psi^S \rangle = 0$$

$$\langle \psi^S | H^S | \psi^S \rangle = \langle \psi^S | H | \psi^S \rangle \geq \min_{\psi \in \mathcal{H}} \langle \psi | H | \psi \rangle$$

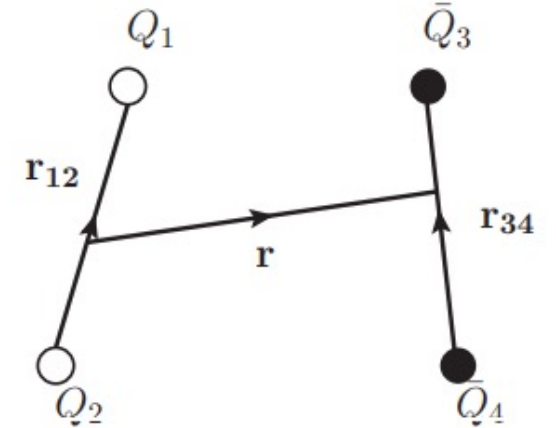
- ▶ Intuitively, less even $\{a_{ij}\}$, more deeply bound ground states

J. P. Ader, J. M. Richard, and P. Taxil, Phys. Rev. D **25**, 2370 (1982).



● $\{a_{ij}\}$ distribution for $QQ\bar{Q}\bar{Q}$

	$a_{12} = a_{34}$	$a_{13} = a_{24}$	$a_{14} = a_{23}$	$\text{Var}[\{a_{ij}\}]$
Dimeson	0	$-\frac{16}{3}$	0	$\frac{1024}{135}$
$\bar{3}_c - 3_c$	$-\frac{8}{3}$	$-\frac{4}{3}$	$-\frac{4}{3}$	$\frac{64}{135}$
$\bar{6}_c - 6_c$	$\frac{4}{3}$	$-\frac{10}{3}$	$-\frac{10}{3}$	$\frac{784}{135}$



- ▶ $\text{Var}[\{a_{ij}\}]$ as a measurement of evenness
- ▶ $2M(Q\bar{Q}) < M(6_c - \bar{6}_c) < M(3_c - \bar{3}_c)$
- ▶ Conclusion: **Equal mass + color-electric + 2-body V** \Rightarrow no state blow the dimeson threshold.
- Things become different, when
 - ▶ Unequal quark masses, e.g. $QQ\bar{q}\bar{q}$
 - ▶ multi body interaction, e.g. double- Y interaction
 - ▶ ...

PRL118, 142001; PRL119, 202001; PRL119, 202002



● Semay-Silvestre-Brac Models

Semay:1994ht, Silvestre-Brac:1996myf

$$V_{ij}(r) = \left[-\frac{\kappa}{r} + \lambda r^p - \Lambda + \frac{2\pi}{3m_i m_j} \kappa' \frac{1}{\pi^{3/2} r_0^3} e^{(-r^2/r_0^2)} \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j \right] \lambda_i \cdot \lambda_j$$

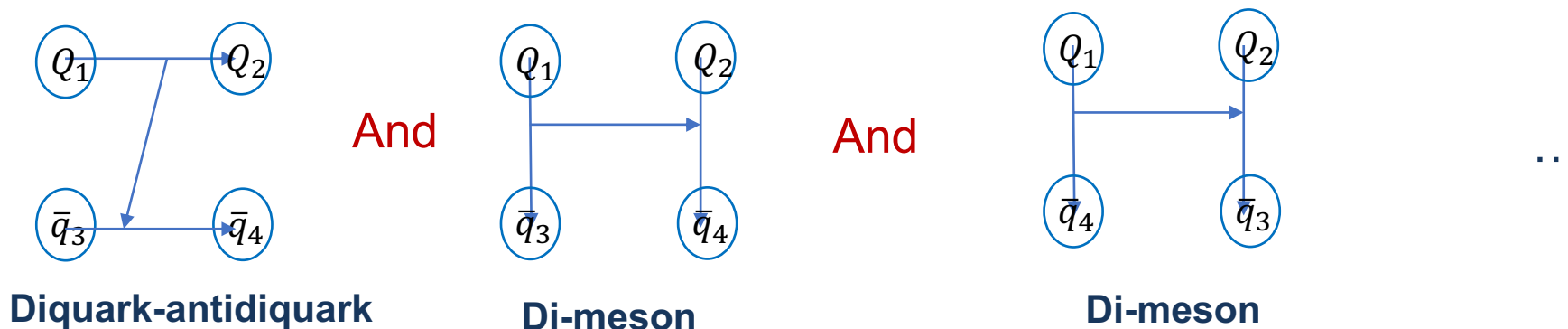
AL1: $p = 1$ and AP1: $p = 2/3$

[GeV]	π	K	D	D_s	B	B_s	B_c	η_c	η_b
Exp.	0.139	0.494	1.870	1.968	5.279	5.367	6.274	2.984	9.399
AL1	0.138	0.491	1.862	1.962	5.293	5.361	6.292	3.005	9.424
AP1	0.139	0.498	1.881	1.955	5.311	5.356	6.269	2.982	9.401

[GeV]	ρ	ω	ϕ	K^*	D^*	D_s^*	B^*	B_s^*	B_c^*	J/ψ	Υ
Exp.	0.775	0.783	1.019	0.892	2.010	2.112	5.325	5.415	6.329	3.097	9.460
AL1	0.770	0.770	1.021	0.903	2.016	2.102	5.350	5.417	6.343	3.101	9.461
AP1	0.770	0.770	1.021	0.908	2.033	2.107	5.367	5.418	6.338	3.102	9.461



- Spatial wave functions



$$\phi_{nlm}(\mathbf{r}) = N_{lm} r^l e^{-\frac{r^2}{r_n^2}} Y_{lm}(\hat{r})$$

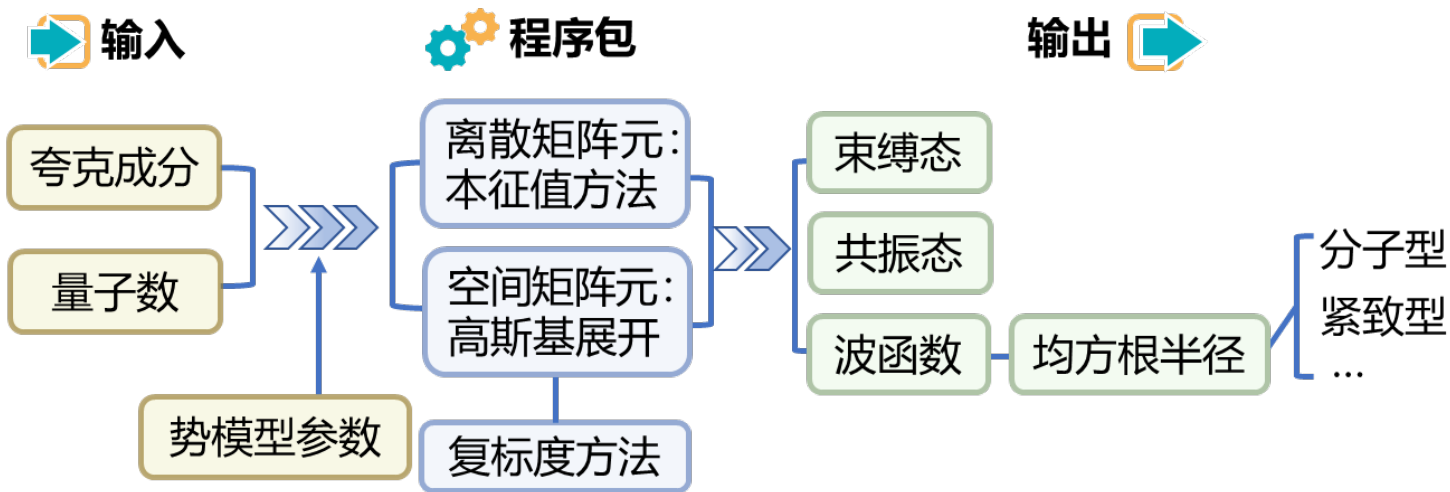
- ▶ Geometric progression: $r_n = r_0 a^{n-1}$
- ▶ Embed both long- and short-range correlations
- Antisymmetrization (e.g. $Q_1 = Q_2$ and $q_3 = q_4$):

$$\psi = \mathcal{A}[\psi_{color} \otimes \psi_{spin} \otimes \psi_{spatial} \otimes \psi_{flavor}], \quad \mathcal{A} = (1 - P_{12})(1 - P_{34})$$

- Based on variational principle: **upper limit** of the energy

Hiyama:2003cu





- Fully heavy tetraquark states ($QQ\bar{Q}\bar{Q}$)
- Triply heavy tetraquark states ($QQ\bar{Q}\bar{q}$)
- Doubly heavy tetraquarks states ($QQ\bar{q}\bar{q}$)
- Single heavy strange states ($Qs\bar{q}\bar{q}$, $Q\bar{s}q\bar{q}$)

$$q = u, d, s; \quad Q = b, c$$

$$J^P = 0^+, 1^+, 2^+, \text{ Only S-wave}$$

Only bound states

Over 150 states

Total runtime:
2.5h on my laptop



- Our conclusions:

No bound state for any reasonable pairwise interaction

- Ref [Ader:1981db]

J. P. Ader, J. M. Richard, and P. Taxil, PRD**25**, 2370 (1982).
 M.-S. Liu, Q.-F. Lü, X.-H. Zhong, and Q. Zhao, PRD**100**, 016006 (2019).
 X. Jin, Y. Xue, H. Huang, and J. Ping, EPJC**80**, 1083 (2020).

...

tial models already used in the study of heavy mesons and baryons. We first consider the situation where the quarks have the same mass and interact through a two-body potential due to color-octet exchange. In this case, we show that for any reasonable confining potential there is no state below the threshold corresponding to the spontaneous dissociation into two mesons. We investigate in detail different possibilities of modifying this negative

- Ref [Assi:2023dlu]: claimed the existence of fully heavy tetraquark bound state in version 1

FERMILAB-PUB-23-660-T

Tetraquarks made of sufficiently heavy quarks are bound in QCD

Benoît Assi¹ and Michael L. Wagman¹ version 1

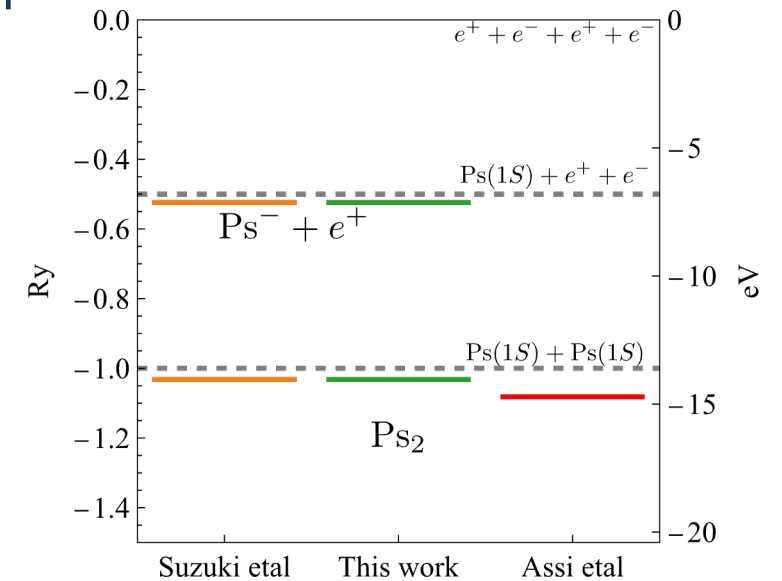
¹Fermi National Accelerator Laboratory, Batavia, IL, 60510

FERMILAB-PUB-23-660-T

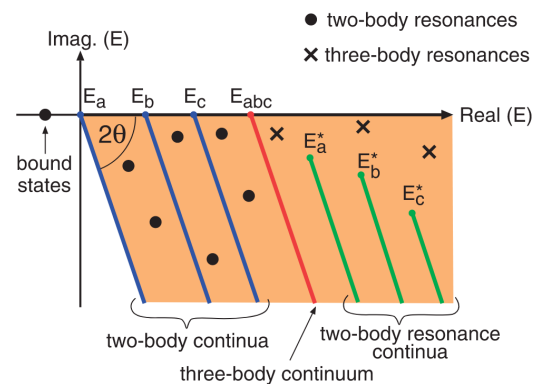
Tetraquarks made of sufficiently unequal-mass heavy quarks are bound in QCD

Benoît Assi¹ and Michael L. Wagman¹ version 2

¹Fermi National Accelerator Laboratory, Batavia, IL, 60510



Resonance: complex scaling method



Benchmark calculations for 4 nucleon resonant states are still missing.

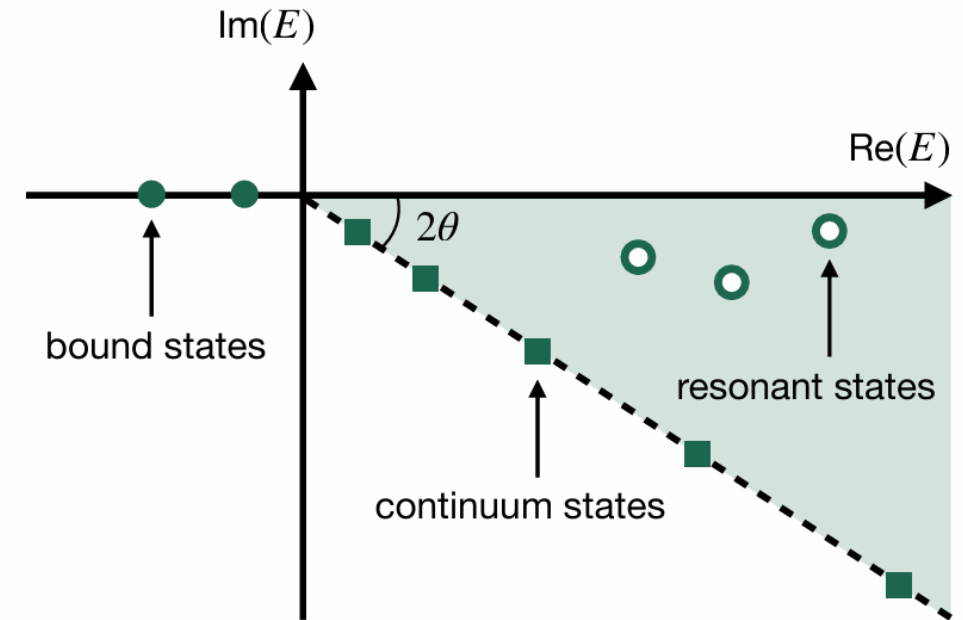
- Original Hamiltonian

$$H = \sum_i^n \left(m_i + \frac{\mathbf{p}_i^2}{2m_i} \right) + \sum_{i<j=1}^n V_{ij},$$

- Complex scaling

$$U(\theta)\mathbf{r} = \mathbf{r}e^{i\theta}, \quad U(\theta)\mathbf{p} = \mathbf{p}e^{-i\theta}.$$

$$H(\theta) = \sum_{i=1}^n \left(m_i + \frac{p_i^2 e^{-2i\theta}}{2m_i} \right) + \sum_{i<j=1}^n V_{ij} (r_{ij} e^{i\theta}).$$



- Resonance appearing as the eigenvalue of $H(\theta)$

- Equivalence of the CSM and **contour deformation method** in 2-body systems

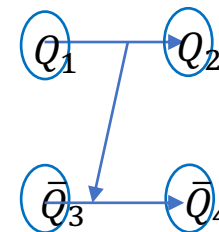
Aguilar:1971ve, Balslev:1971vb, Aoyama:2006hrz...



All-charm tetraquarks

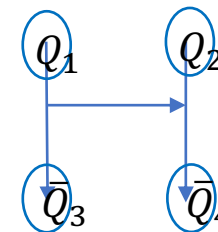


- BGS, AL1, AP1 models: **pairwise** confinement interaction
- Configurations: diquark-antidiquark, di-meson \times 2
- Tetraquark as resonances



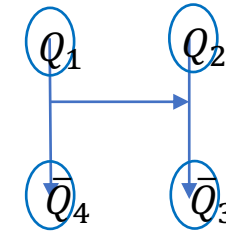
Diquark-antidiquark

And

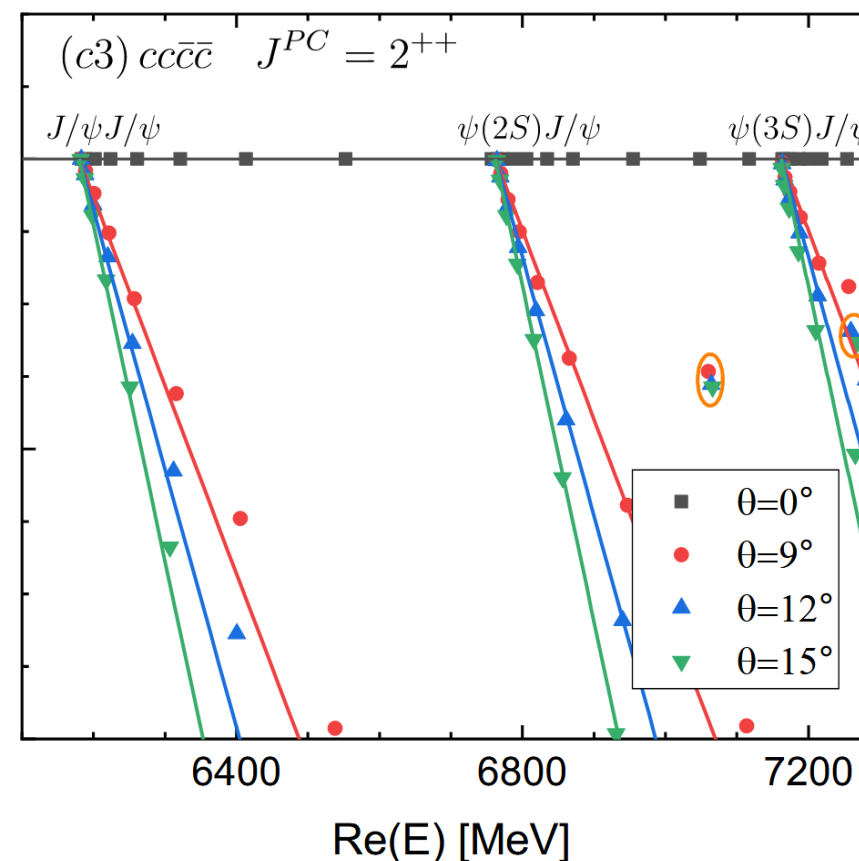
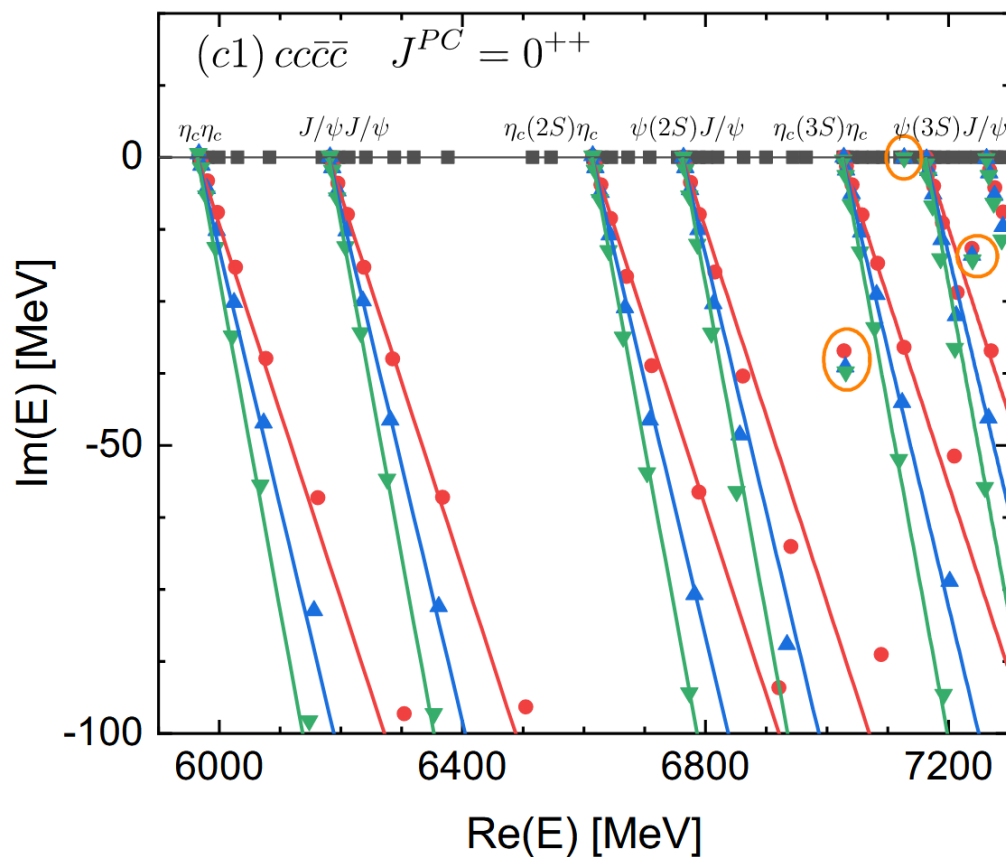


Di-meson

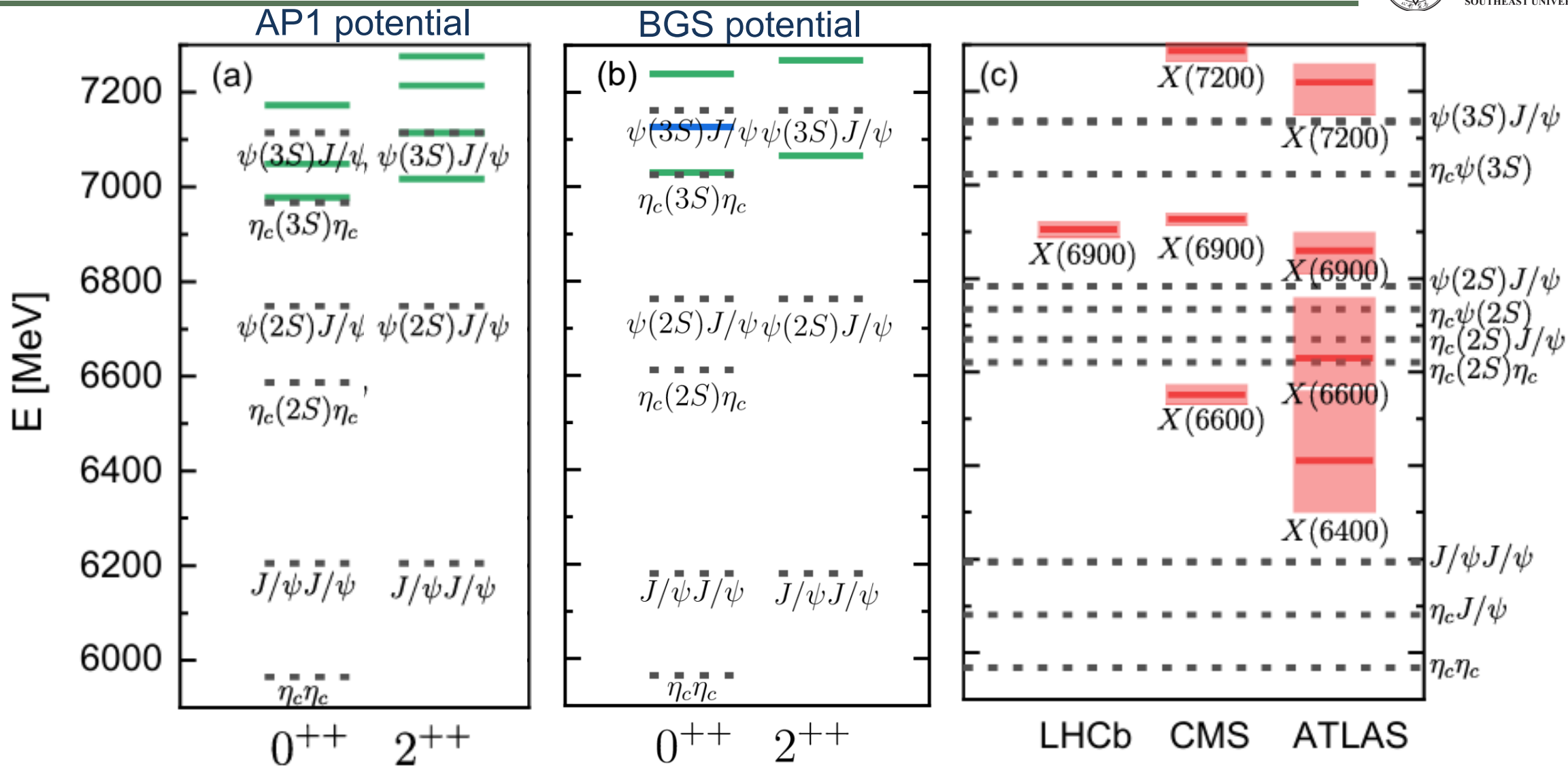
And



Di-meson



All-charm tetraquarks

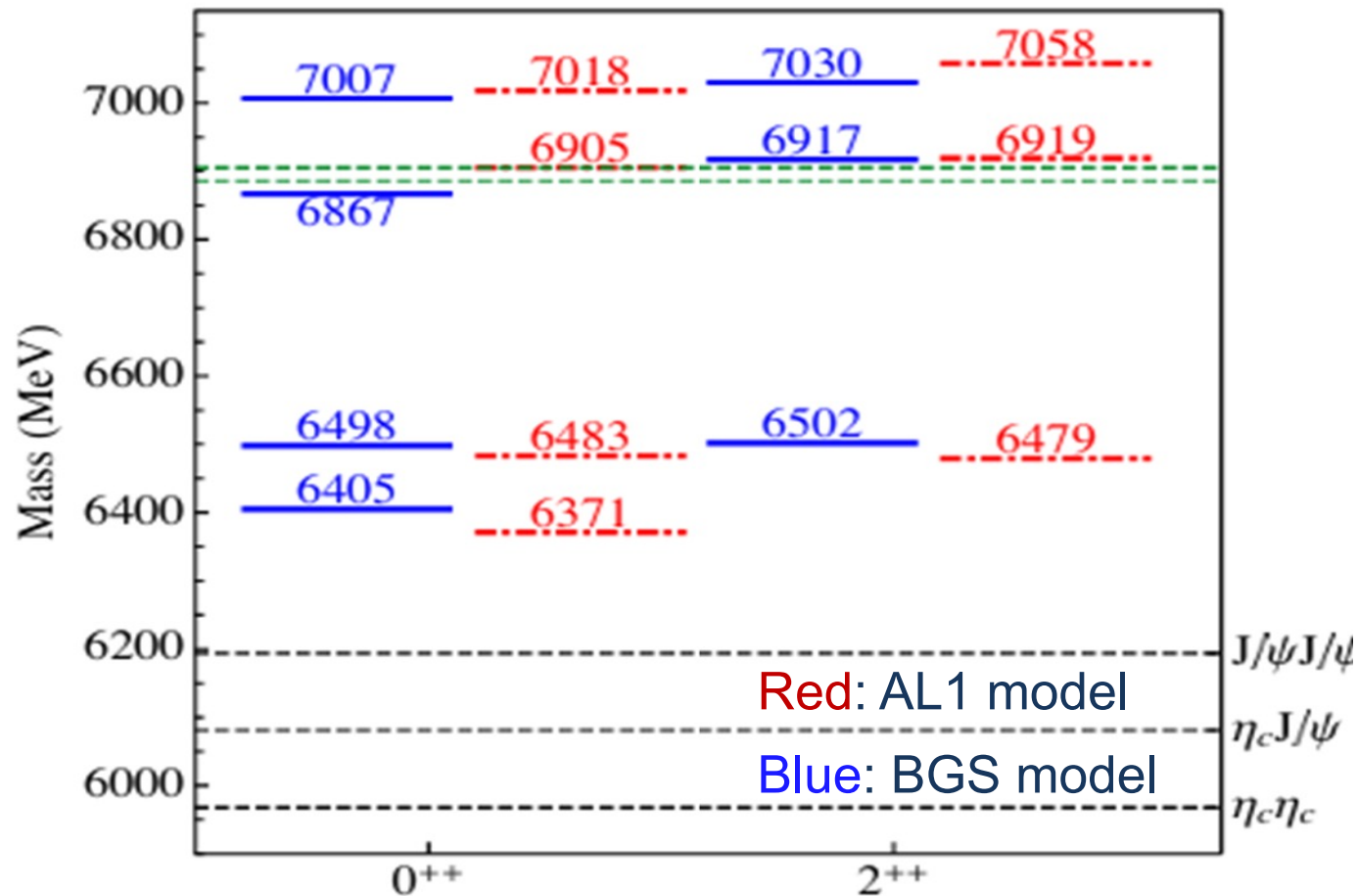


X(6600) is missing in pairwise confinement models

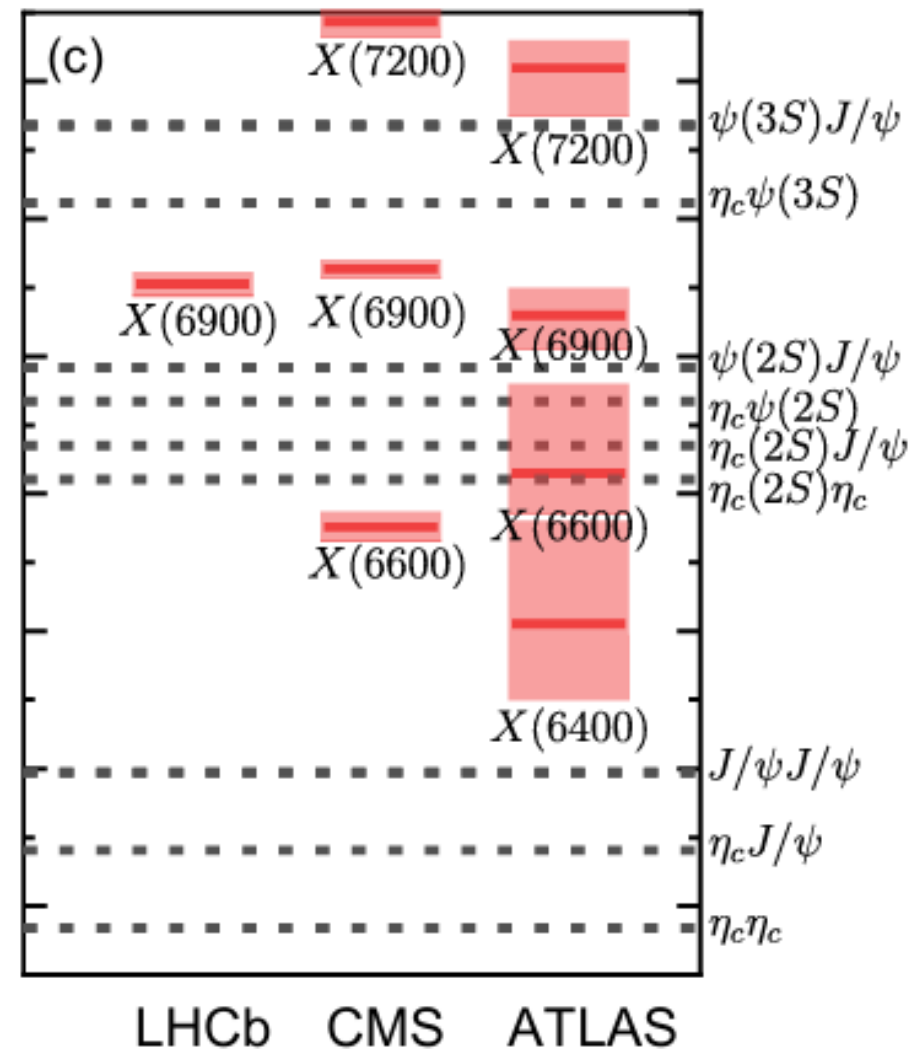
See also G.-J. Wang, Q. Meng, and M. Oka, PRD106, 096005 (2022).



- Earlier calculation neglecting di-meson configurations



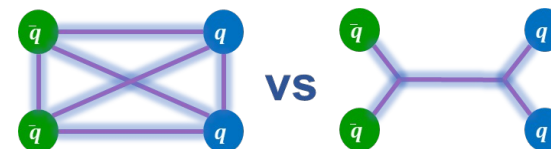
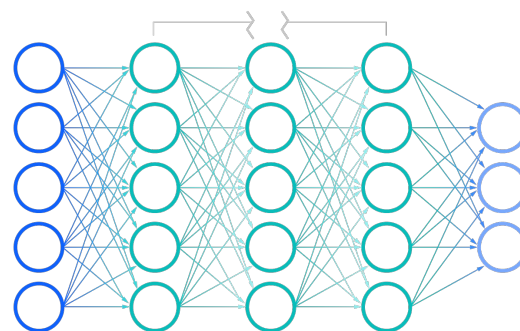
G-J Wang, LM, M Oka, S-L Zhu, PRD 104, 036016 (2021)



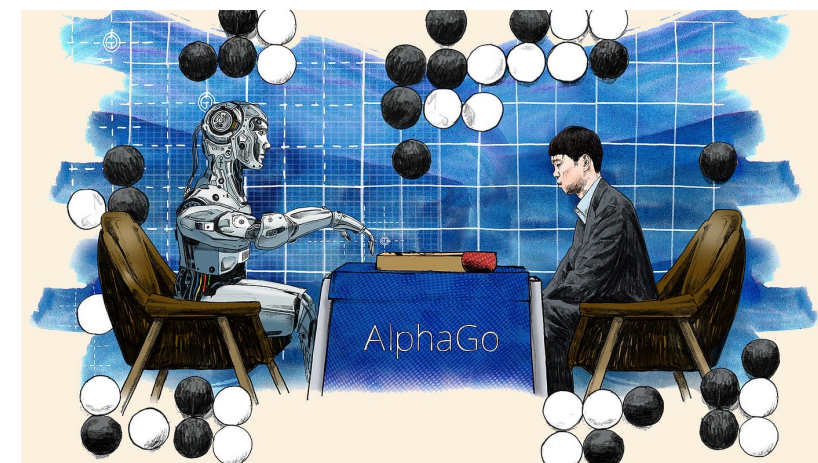
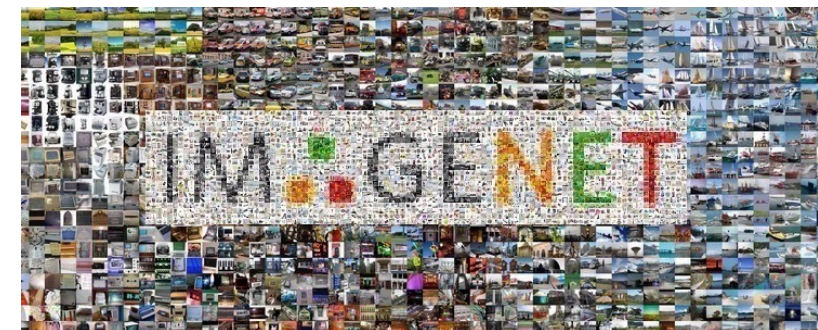
- Pairwise model: overestimate the transition between diquark-antidiquark and dimeson??



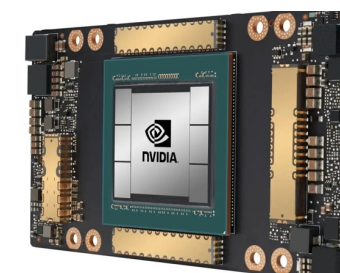
DNNs: towards tetraquark confinement

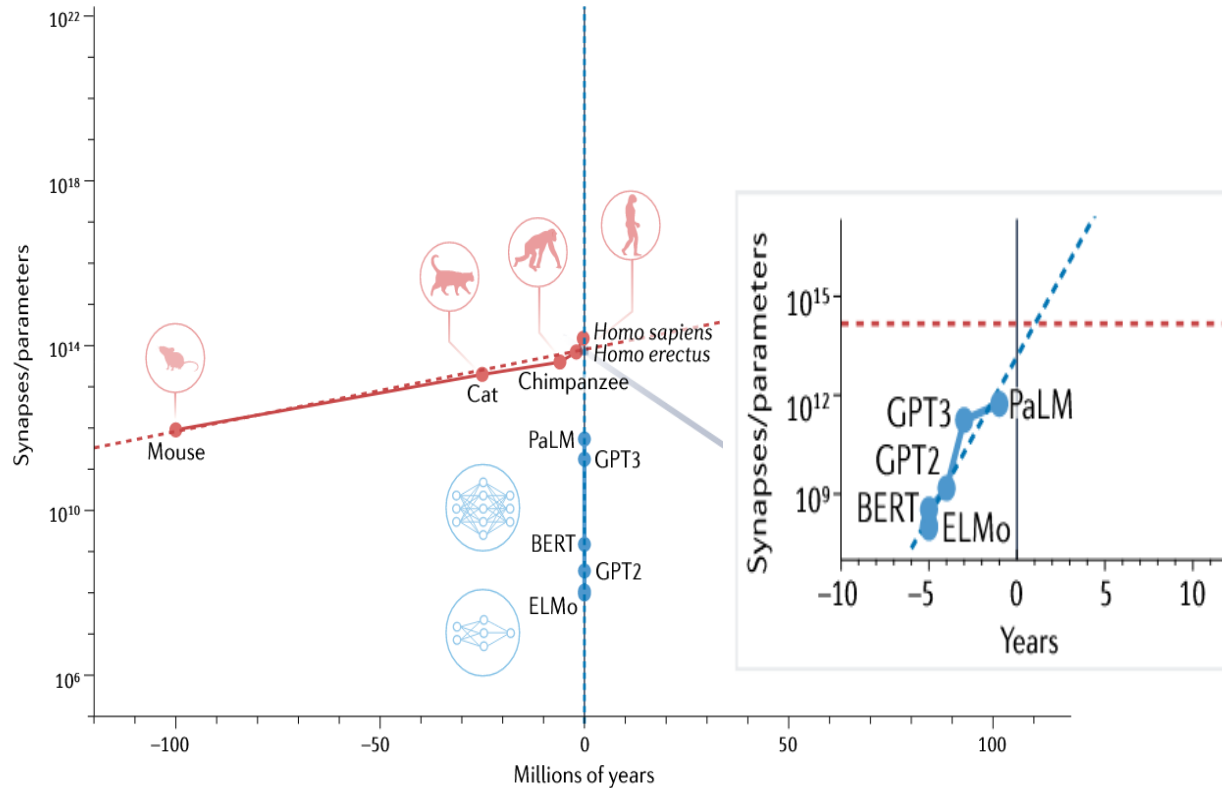


Year	Milestone	Who	Significance
2010	ImageNet	Fei-Fei Li	Large-scale dataset that enabled the deep learning revolution.
2012	AlexNet	Krizhevsky et al.	Proven power of deep CNNs and GPU acceleration for AI.
2014	GANs	Ian Goodfellow	Introduced framework for generative AI to create realistic data.
2015/16	TensorFlow/ Pytorch	Google/Meta	Critical open-source software that democratized AI research globally.
2016	AlphaGo	DeepMind	First AI to defeat a world champion in the game of Go.
2010s	AI Chips	NVIDIA, Google, Apple	Specialized GPUs/TPUs enabled training of massive AI models.
2017	Transformer	Google	Revolutionary architecture that made modern large language models possible.
2018	BERT	Google	Bidirectional language model that set new standards for NLP.
2020	AlphaFold 2	DeepMind	Solved the decades-old scientific challenge of protein folding.
2022	ChatGPT	OpenAI	Popularized large language models through accessible public interaction.



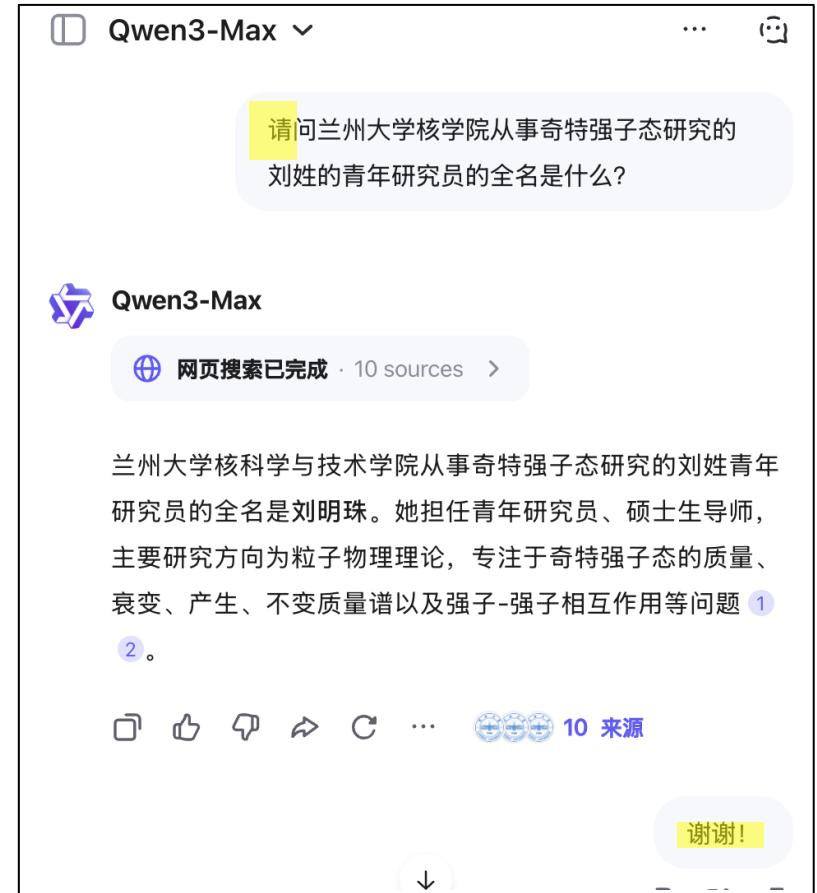
10 milestones of AI selected by DeepSeek





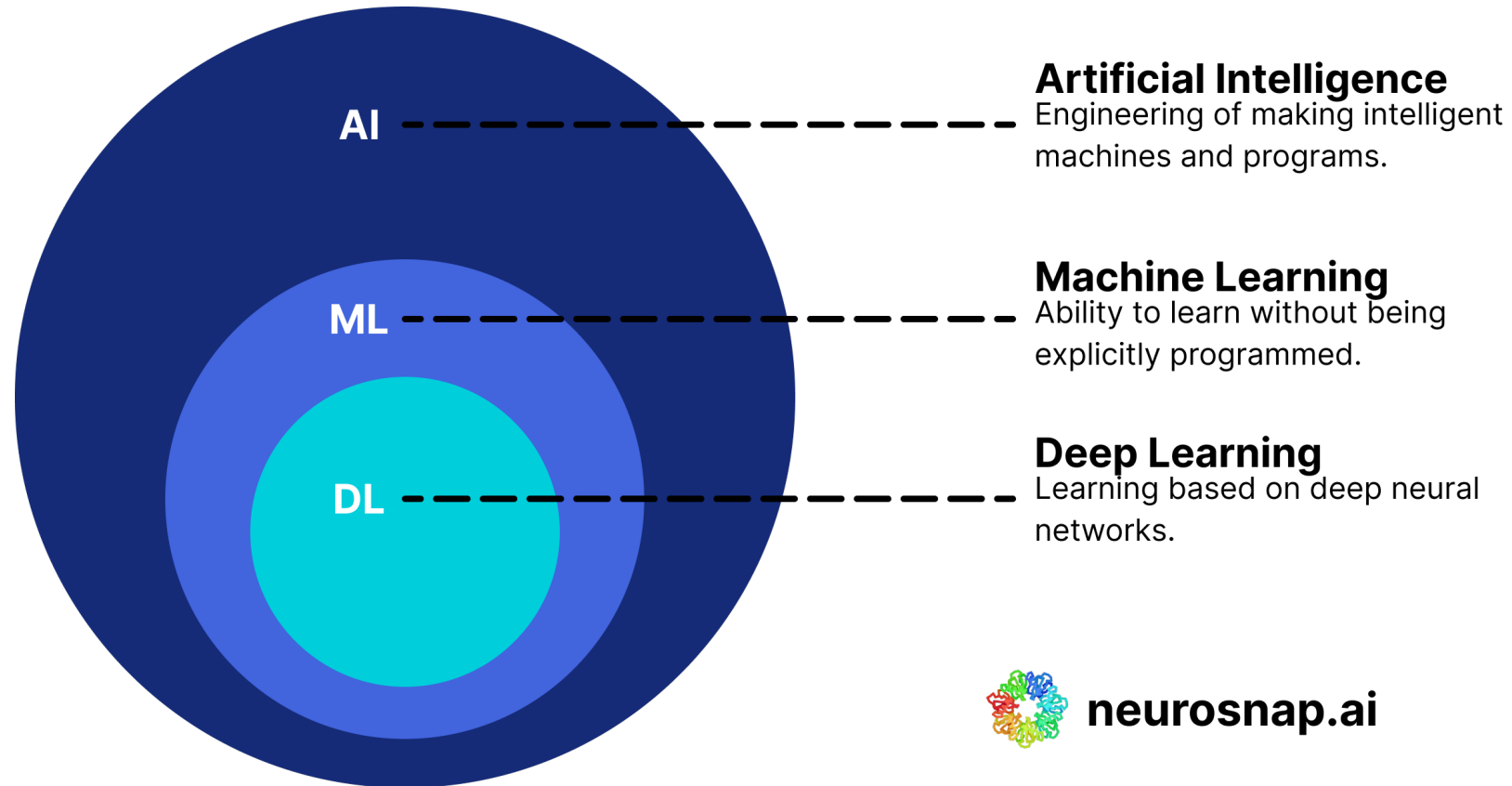
The evolution of biological and artificial intelligence

Schwartz, M. D. (2022). *Nature Reviews Physics*, 4(12), 741–742.



Being more polite to chatbots may increase your chances of survival when robots win



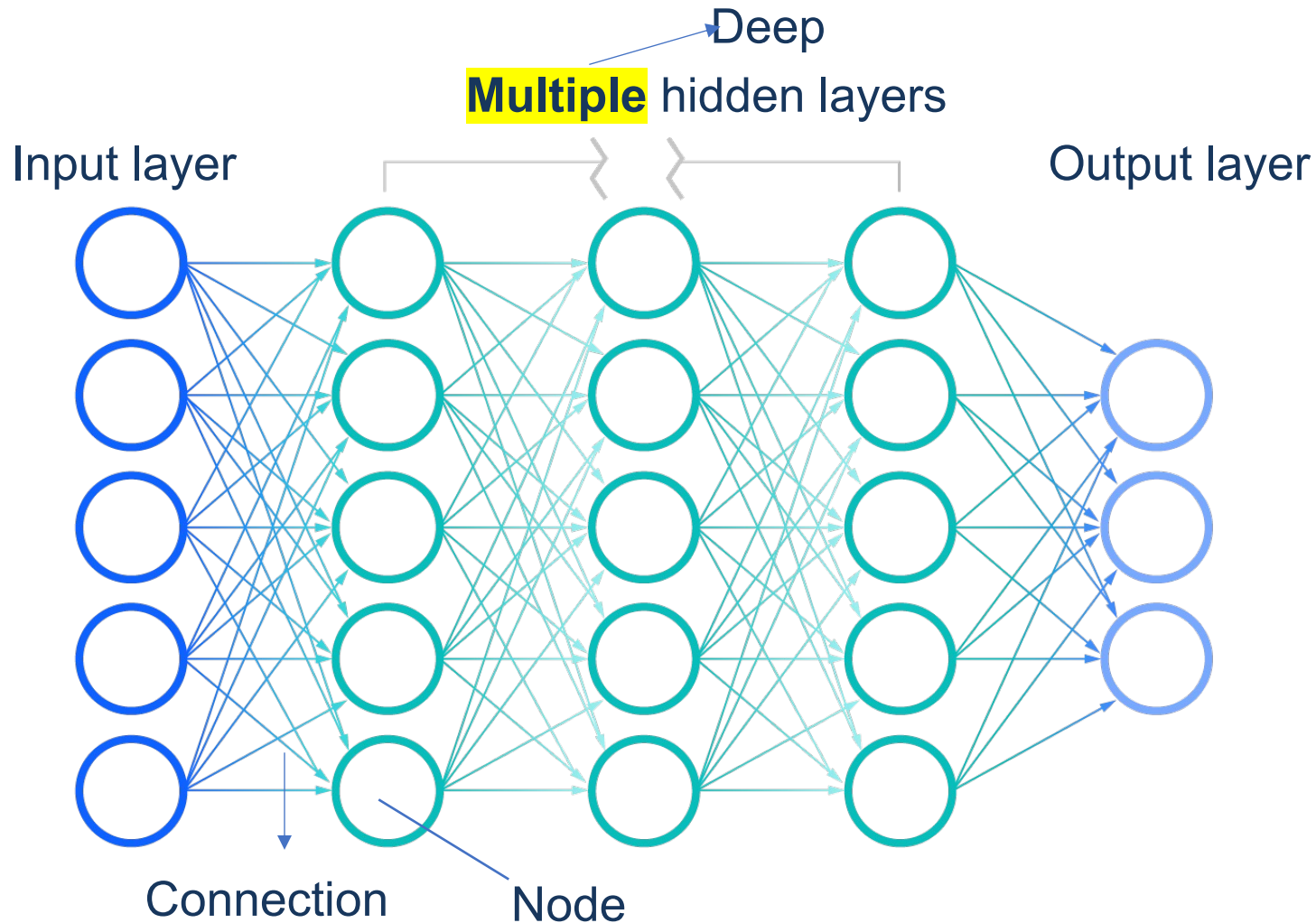


neurosnap.ai

<https://neurosnap.ai/blog/post/64279cadfeb3e5ca5ba0904a>

AI boom from 2010s was powered by **deep learning**





Neural network: simple units (neurons) connected to form complex network

Deep: Enable learn complex relation in data



$$\vec{h}^{(i+1)} = \sigma [W^{(i)} \vec{h}^{(i)} + \vec{b}^{(i)}]$$

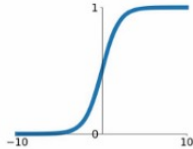
Nonlinear

Linear

- Linear part: Weight matrices $W^{(i)}$ and biases $\vec{b}^{(i)}$
- Nonlinear part: Activation functions σ

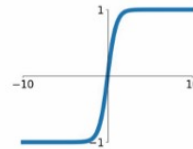
Sigmoid

$$\sigma(x) = \frac{1}{1+e^{-x}}$$

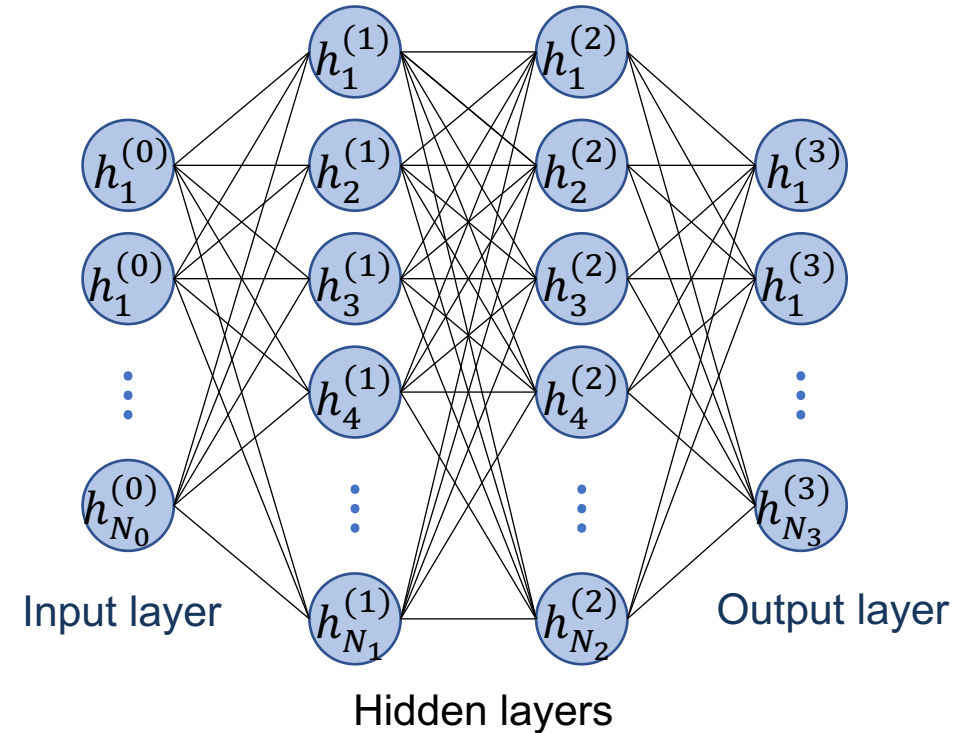


tanh

$$\tanh(x)$$



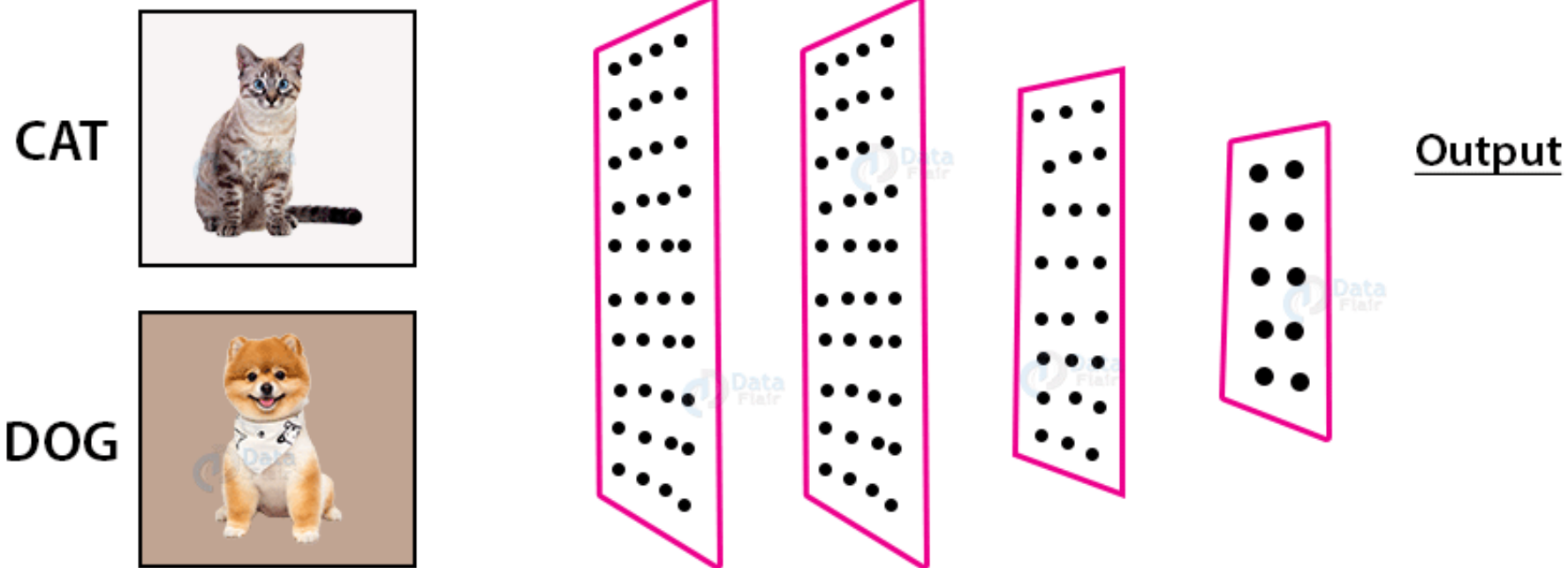
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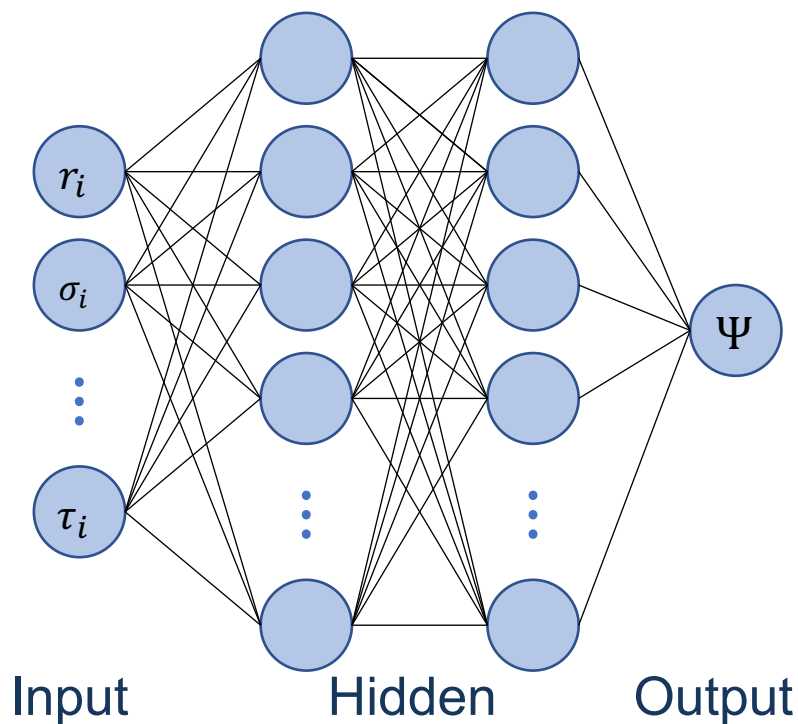
- Training: determine $W^{(i)}$ and biases $\vec{b}^{(i)}$ to minimize loss functions
 - ▶ Loss function: quantifies the difference between the expected and actual output.
- Highly efficient optimization
 - ▶ Backpropagation: automatic differentiation (AD)
 - ▶ GPUs
 - ▶ ...



DNNs: Data-driven application

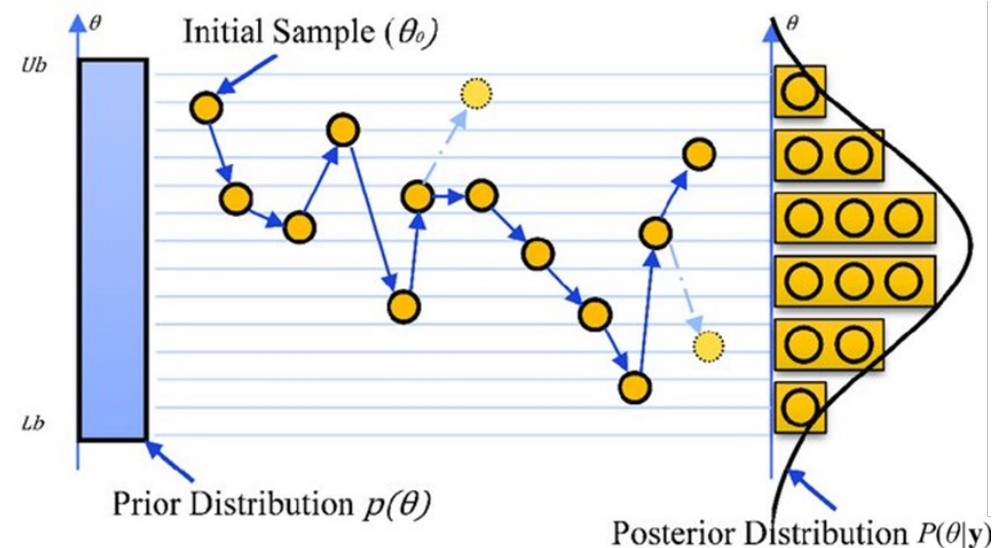


Neural Network Quantum States (NQSs):



Loss function: $E_{\theta} = \frac{\langle \psi_{\theta} | H | \psi_{\theta} \rangle}{\langle \psi_{\theta} | \psi_{\theta} \rangle} \geq E_0$

Variational Monte Carlo (VMC)



Sample using Metropolis-Hasting algorithm

$$E_{\theta} = \frac{\int |\psi_{\theta}(r)|^2 \frac{H\psi(r)}{\psi(r)} dr}{\int |\psi_{\theta}(r)|^2 dr}$$

Train the DNNs not by the data but by the physics equation: **physics-informed**



- DNNs: approximate high-dimensional functions

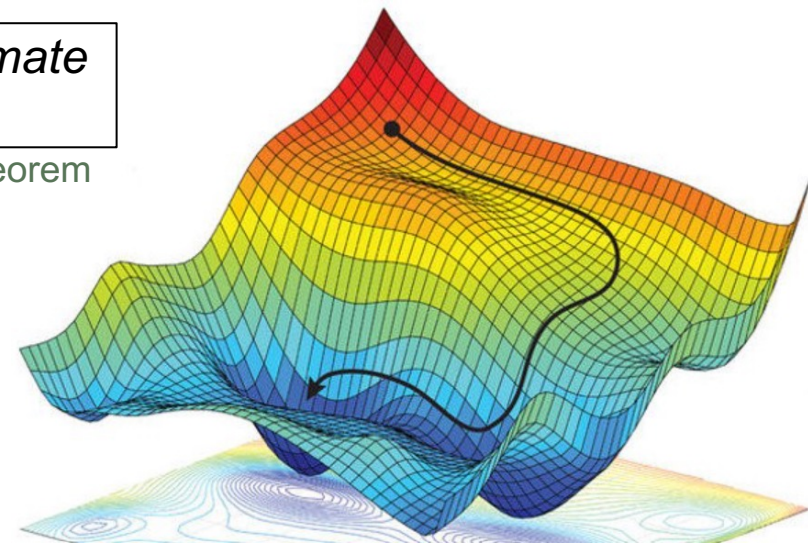
- ▶ **Universal approximation theorems**

→ sufficiently large or deep

Neural networks with a certain structure can, in principle, approximate any continuous function to any desired accuracy

https://en.wikipedia.org/wiki/Universal_approximation_theorem

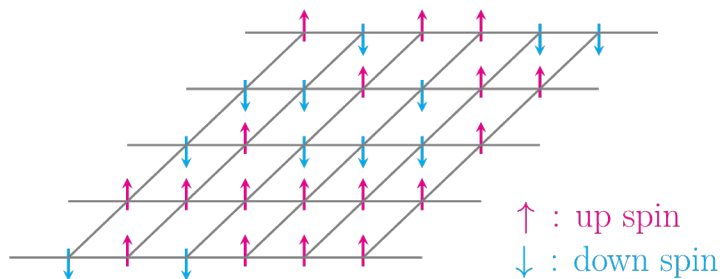
- ▶ Variational principle: the more general the trial function, the more accurate the solution
 - ▶ Unbiasedly distinguish molecular and compact tetraquark states
- VMC: applicable for few-body potential
 - ▶ Flux-tube confinement potential
 - Circumvents the sign problem in imaginary time evolution of DMC
 - Fast optimization: AD+GPUs
 - Easy-to-use open-source frameworks



TensorFlow

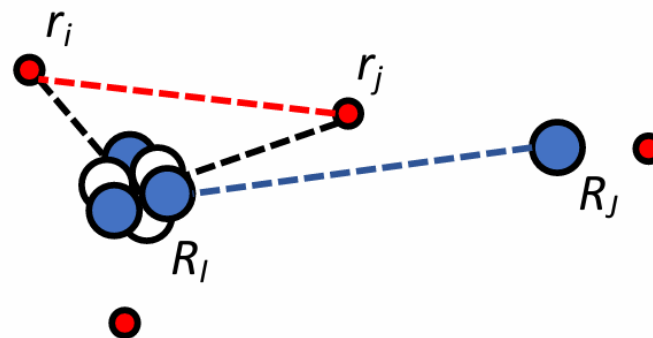


Spin models



G. Carleo and M. Troyer, *Science* 355, (2017).

Electronic systems



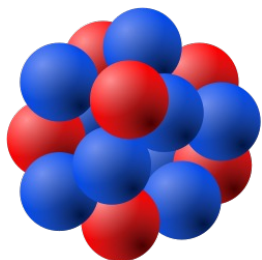
PauliNet: J. Hermann, Z. Schätzle, and F. Noé, *Nat. Chem.* 12, 891 (2020).

FermiNet: D. Pfau et al., *Phys. Rev. Res.* 2, 033429 (2020).

...

Review: J. Hermann et al., *Nat. Rev. Chem.* 7, 692–709 (2023)

Nuclear structure



Deuteron: J. W. T. Keeble and A. Rios, *Phys. Lett. B* 809, 135743 (2020).

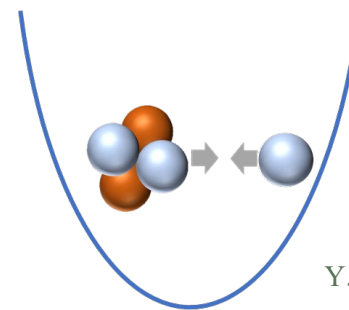
$A \leq 4$: C. Adams et al., *Phys. Rev. Lett.* 127, 022502 (2021).

FeynmanNet: Y. Yang and P. Zhao, *Phys. Rev. C* 107, 034320 (2023).

Hypernuclei: Z.-X. Zhang et al., arXiv:2508.03575

...

Neutron- α scattering



**Chiral Nuclear force
Trapped five-body problems
(DNNs+VMC)**

Y. Yang, E. Epelbaum, J. Meng, LM, and P. Zhao, arXiv:2502.09961

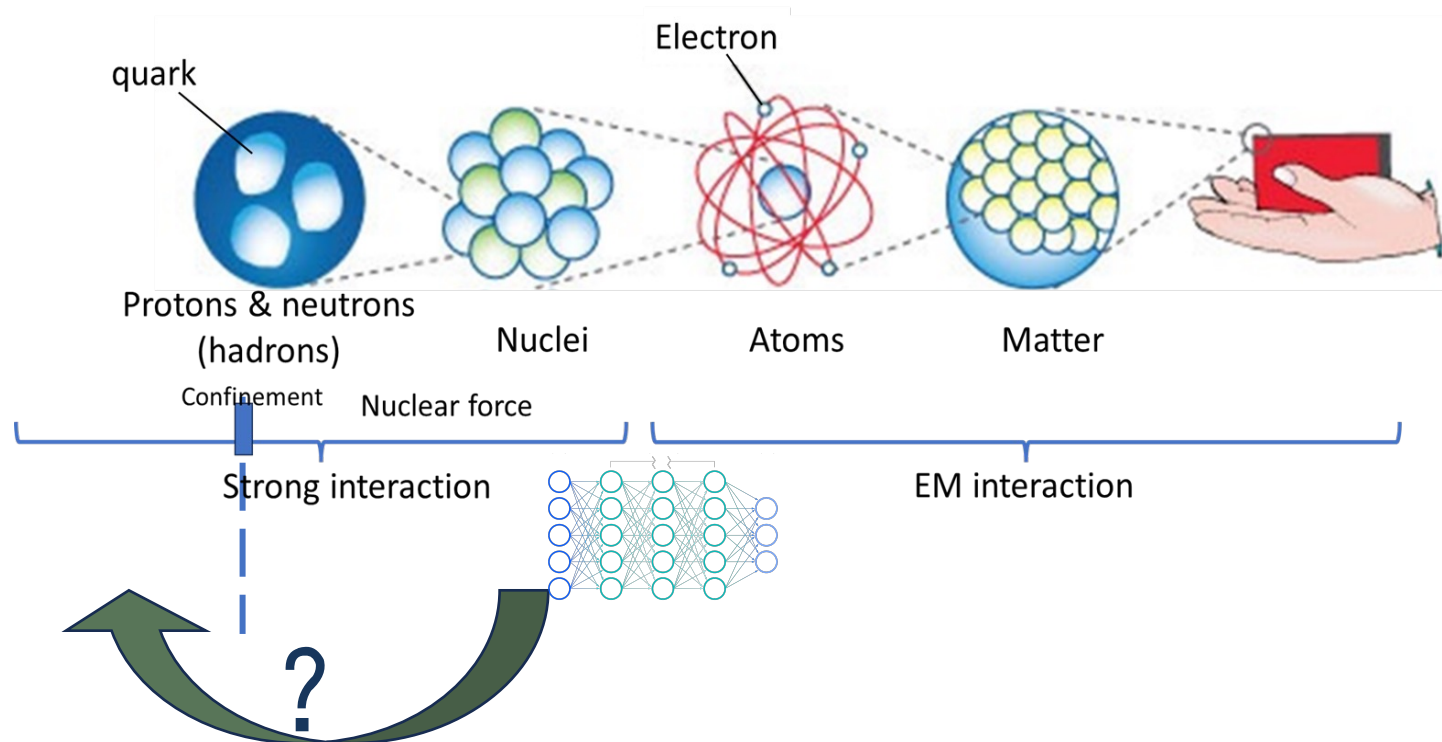
$$V_{ij}(r) = V_{ij}^{OGE} + V_{ij}^{conf.}$$

$$= \left[\frac{\alpha_s}{r} - \frac{8\pi\alpha_s}{3m_Q^2} \delta(\sigma; r) \mathbf{S}_i \cdot \mathbf{S}_j - \frac{3}{4} br \right] \frac{\lambda_i \cdot \lambda_j}{4}$$

Quark potential model

QCD-inspired model

- quarks as the degree of freedom
- SU(3) color symmetry
- Confinement
-



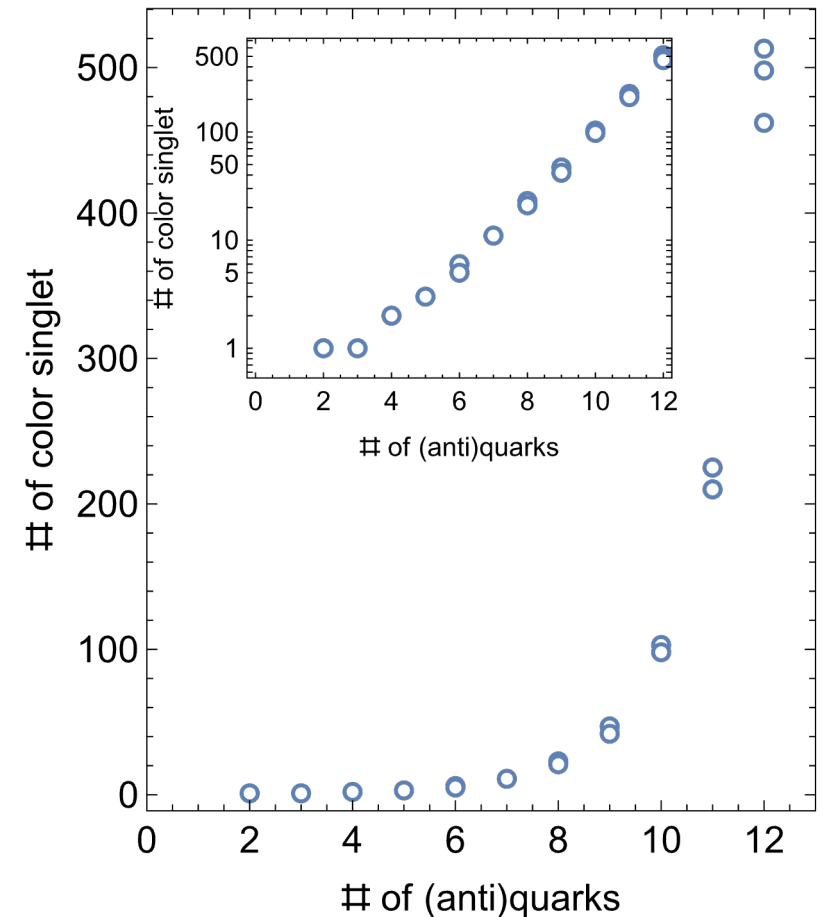
Challenge 1: Spin and color projection

- Conventional spin projection
 - ▶ Option 1: no projection, calculate the ground state
 - ▶ Option 2: penalty terms

$$f_{loss}(\theta) = \langle E \rangle_{\theta} + \langle S^2 \rangle_{\theta}$$

However, for multiquark systems

- State with higher spin could more interesting
 - ▶ $[cc\bar{q}\bar{q}]_{S=0}$: DD threshold; $[cc\bar{q}\bar{q}]_{S=1}$: T_{cc} state
- Color projection is needed
 - ▶ color singlet
 - ▶ $SU(3)$ symmetry

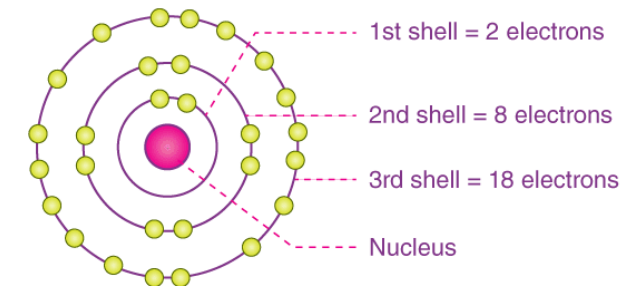


Challenge 2: Strong correlation

- Shell model: good guidance or initial point

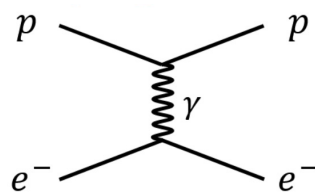
$$\Psi(\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_N) = \frac{1}{\sqrt{N!}} \begin{vmatrix} \chi_1(\mathbf{x}_1) & \chi_2(\mathbf{x}_1) & \cdots & \chi_N(\mathbf{x}_1) \\ \chi_1(\mathbf{x}_2) & \chi_2(\mathbf{x}_2) & \cdots & \chi_N(\mathbf{x}_2) \\ \vdots & \vdots & \ddots & \vdots \\ \chi_1(\mathbf{x}_N) & \chi_2(\mathbf{x}_N) & \cdots & \chi_N(\mathbf{x}_N) \end{vmatrix}$$

For electron and nucleon systems



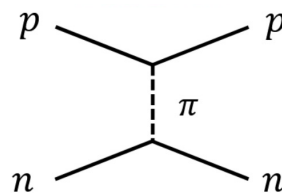
Electron Shell

- Much stronger interaction among quarks
 - ▶ No evidence of a multiquark shell structure
 - ▶ Few-body correlation could be important



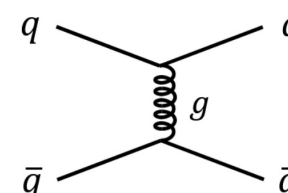
$$e^2 = 0.1$$

$$m_e \approx 0.5 \text{ MeV}$$



$$\frac{m_\pi^2 g_A^2}{16\pi f_\pi^2} \sim 0.1$$

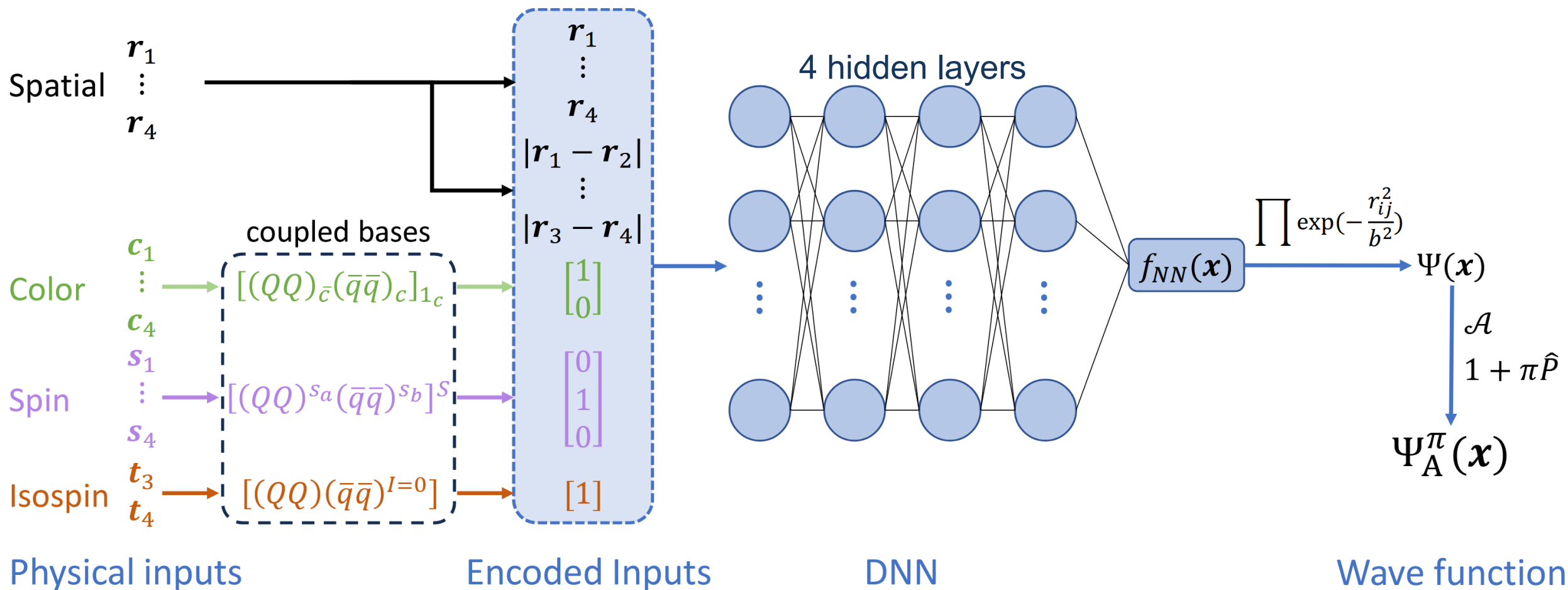
$$m_N = 938 \text{ MeV}$$



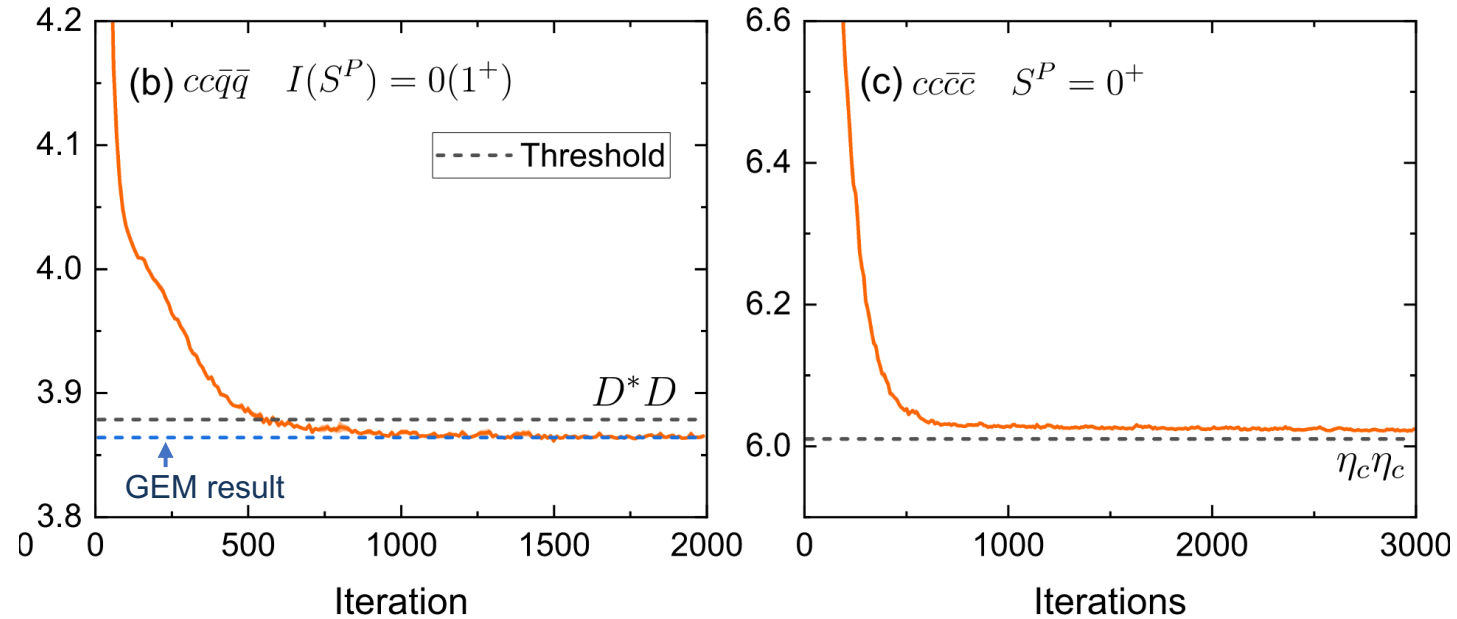
$$\frac{4}{3} \alpha_s(m_c) \sim 0.4$$

$$m_c \approx 1273 \text{ MeV}$$





- Convergence after 1000 iterations
- Statistical errors less than 0.1 MeV
- Competitive performance with GEM
- $cc\bar{q}\bar{q}$: molecular state
- $bb\bar{q}\bar{q}$: compact heavy diquark
- $QQ\bar{Q}\bar{Q}$: no bound states



	$I(S^P)$	Thresh.	ΔE	$P_{\bar{3}_c \otimes 3_c}$	$P_{6 \otimes 6_c}$	r_{QQ}	$r_{\bar{q}\bar{q}}$	$r_{Q\bar{q}}$
$cc\bar{q}\bar{q}$	$0(1^+)$	DD^*	-15	55%	45%	1.24	1.41	1.06
$bb\bar{q}\bar{q}$	$0(1^+)$	$\bar{B}\bar{B}^*$	-153	97%	3%	0.33	0.78	0.69
$QQ\bar{Q}\bar{Q}$	$0(0^+)$ $0(1^+)$ $0(2^+)$	$\eta_c\eta_c$ $\eta_c J/\psi$ $J/\psi J/\psi$	No bound					

ΔE in MeV, r in fm



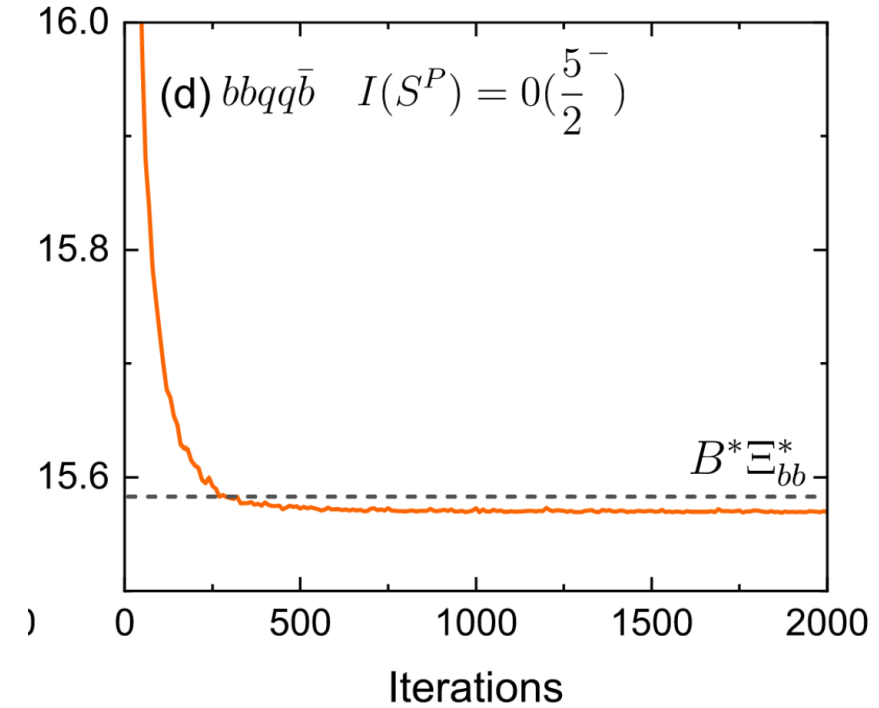
- Exact pentaquark calculations in are computationally prohibitive
 - Approximations in the spatial or color configurations
- Possible bound pentaquark systems
 - Heavy-diquark-antiquark-symmetry: $(QQ)_{\bar{3}_c} \rightarrow \bar{Q}$
 - $\bar{Q}\bar{Q}qq \rightarrow QQ\bar{Q}qq$
 - Given $M_{\bar{Q}\bar{Q}qq} < M_{\bar{Q}q} + M_{\bar{Q}q}$, $M_{QQ\bar{Q}qq} < M_{QQq} + M_{\bar{Q}q}$?
- Exact calculation via DeepQuark

$$\chi_{\bar{3}_c \otimes \bar{3}_c} = \left\{ [(QQ)_{\bar{3}_c} (qq)_{\bar{3}_c}]_{3_c} \bar{Q} \right\}_{1_c},$$

$$\chi_{\bar{3}_c \otimes 6_c} = \left\{ [(QQ)_{\bar{3}_c} (qq)_{6_c}]_{3_c} \bar{Q} \right\}_{1_c},$$

$$\chi_{6_c \otimes \bar{3}_c} = \left\{ [(QQ)_{6_c} (qq)_{\bar{3}_c}]_{3_c} \bar{Q} \right\}_{1_c}.$$

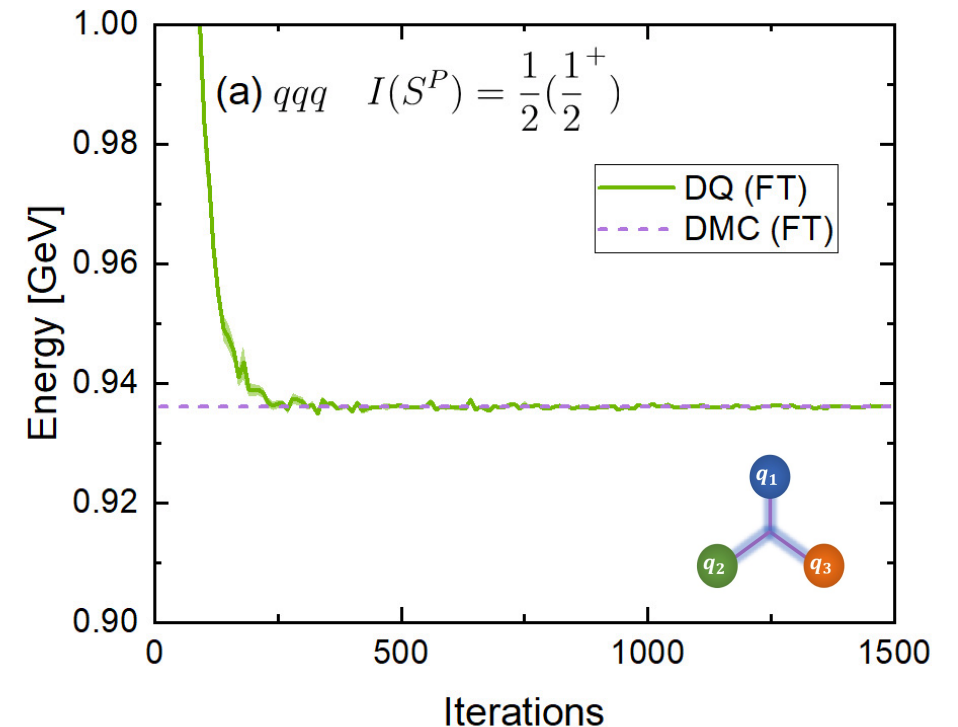
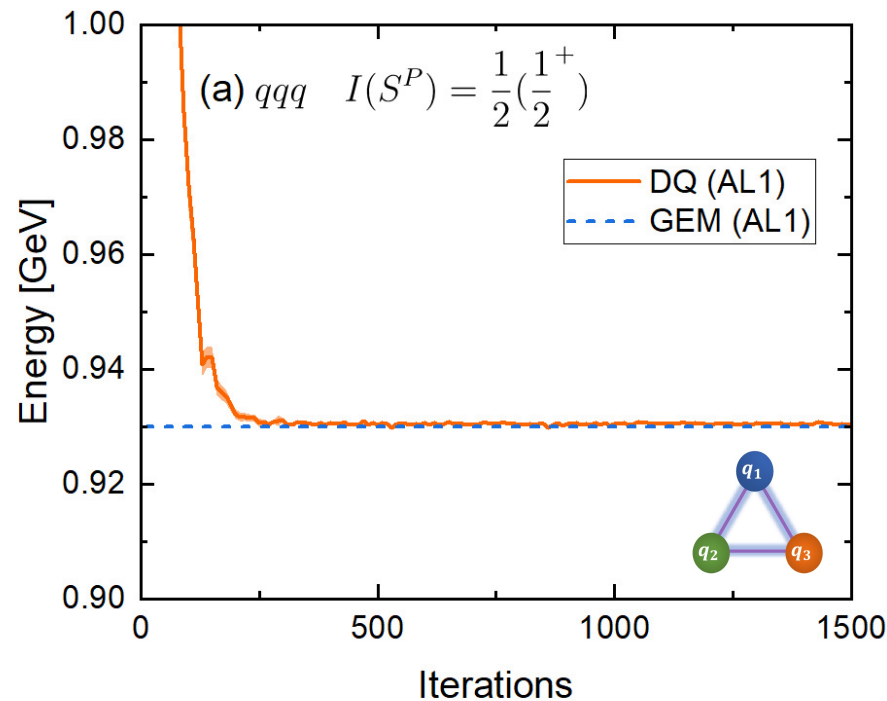
- Moderate increase in comput. cost relative to the tetraquark



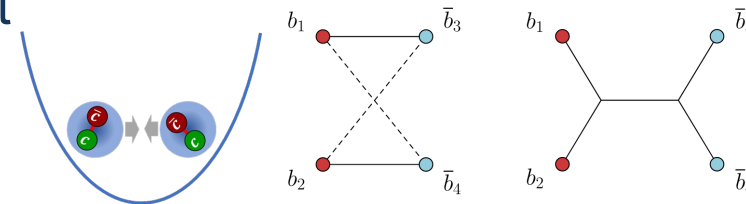
	S^P	Thresholds	ΔE	$\chi_{\bar{3}_c \otimes \bar{3}_c}$	$\chi_{\bar{3}_c \otimes 6_c}$	$\chi_{6_c \otimes \bar{3}_c}$	r_{QQ}	r_{Qq}	r_{qq}	$r_{Q\bar{Q}}$	$r_{q\bar{Q}}$
$ccqq\bar{c}$	$\frac{1}{2}^-, \frac{3}{2}^-$	$\eta_c \Lambda_c, J/\psi \Lambda_c$	NB	$\sim 35\%$	0%	$\sim 65\%$					
	$\frac{5}{2}^-$	$\bar{D}^* \Xi_{cc}^*$	-3	27%	73%	0%	0.50	1.39	1.90	1.73	1.38
$bbqq\bar{b}$	$\frac{1}{2}^-, \frac{3}{2}^-$	$\eta_b \Lambda_b, \Upsilon \Lambda_b$	NB	$\sim 35\%$	0%	$\sim 65\%$					
	$\frac{5}{2}^-$	$B^* \Xi_{bb}^*$	-14	19%	80%	1%	0.30	0.89	1.22	0.88	0.88



- Lattice QCD supports the flux-tube potential for qqq baryons and tetraquarks
- Baryons: Y-type interaction, junction position to minimize the total string length
 - ▶ Small impact on baryon spectrum, but complicate dramatically matrix element calculations
- DeepQuark: incorporate Y-type interaction without additional cost, thanks to the VMC
- Ready for tetraquark systems, where different confinement patterns lead to distinct signatures



- We still need quark models to understand the **pattens** of multi-quark states
 - ▶ **Difficulties:** interactions (confinement) and solving few-body (resonant) problem
 - ▶ **Chances:** All-charm tetraquark family \Rightarrow confinement pictures
- Bound states:
 - ▶ No bound $QQ\bar{Q}\bar{Q}$ for **pairwise** interaction
- Resonances: complex scaling method
 - ▶ Exp. $cc\bar{c}\bar{c}$ results: disfavor the pairwise confinement interaction
 - ▶ Too strong coupling between meson and diquark-antidiquark (?)
- DeepQuark: DNN-based framework
 - ▶ Surpass traditional methods starting from pentaquark states
 - ▶ Ready for flux-tube confinement
- Outlook:
 - ▶ DNNs: excited states, scattering, resonances
 - ▶ All-charm tetraquark with flux-tube confinement



**Thanks for
your attention!**

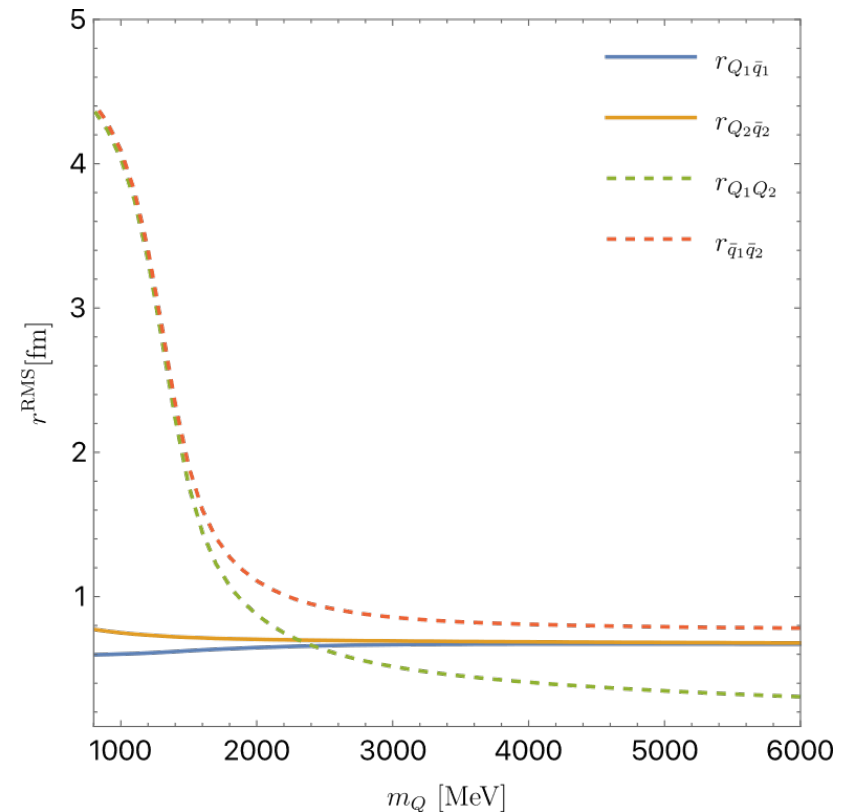
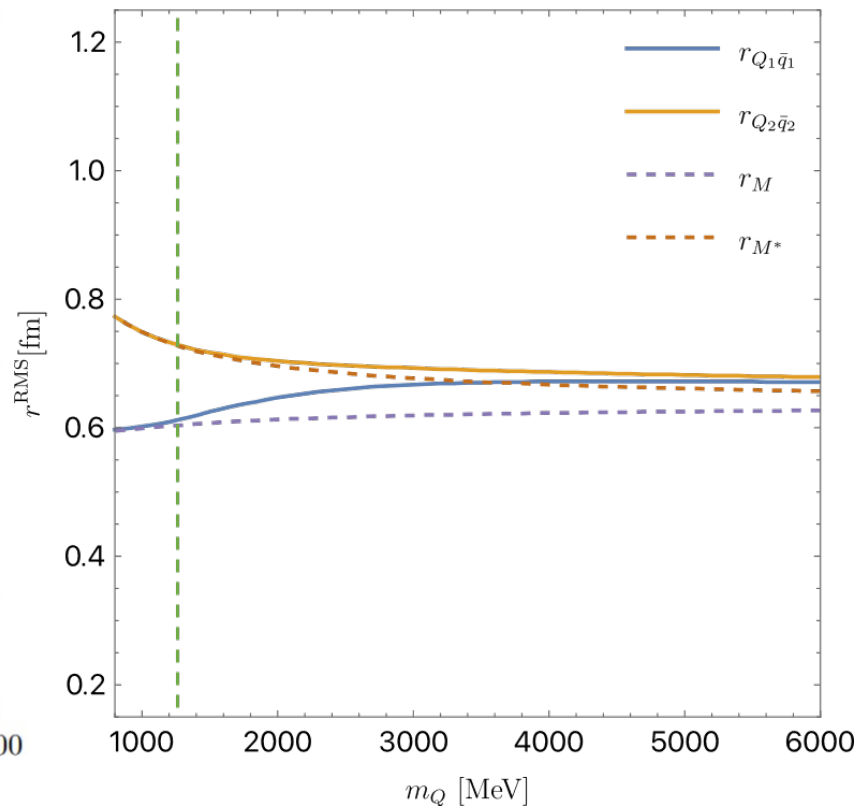
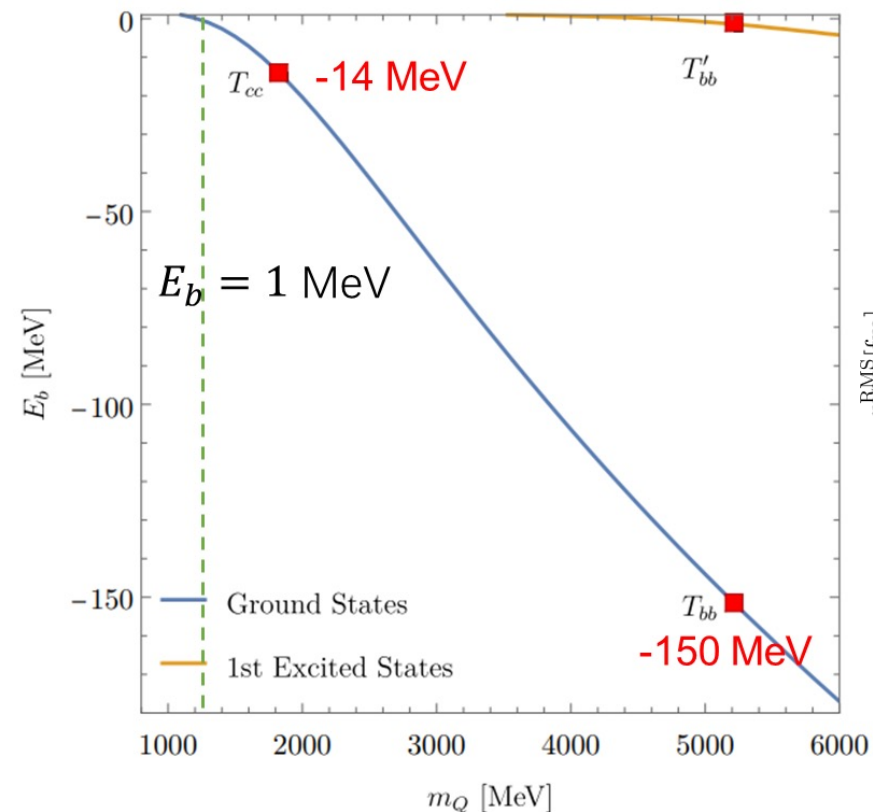




Backup



Molecular or compact ?



- Tuning the m_Q to m_b : (bb) compact diquark
- Tuning the m_Q to make $E_b < 1$ MeV: molecular states

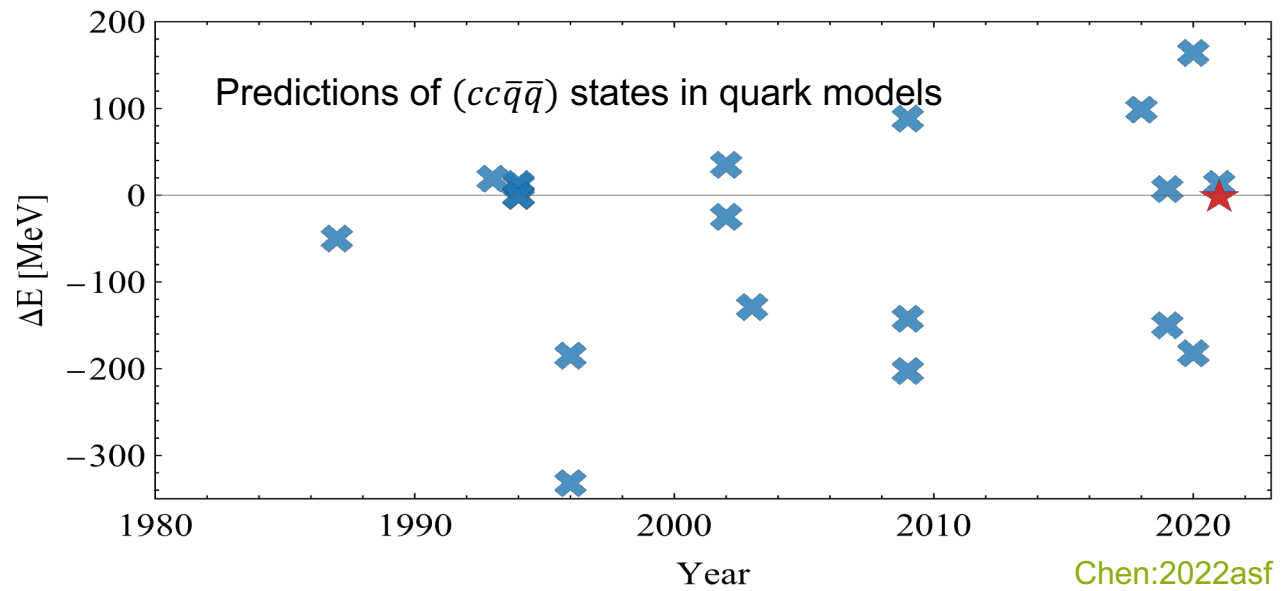


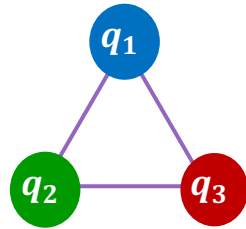
- Ref.[Ortega:2022efc]: Same SLM interactions
 - ▶ $[bb\bar{q}\bar{q}]^{I=1}$ bound state! **Our results: there is no isospin vector states**
 - ▶ Get a $J^P(I) = 0^+(0)$ state dominated by S-wave BB states!
Violating Boson principle

Mass	E_B	$\mathcal{P}_{B^0B^{*+}}$	$\mathcal{P}_{B+B^{*0}}$	$\mathcal{P}_{B^{*+}B^{*0}}$	$\mathcal{P}_{I=0}$	$\mathcal{P}_{I=1}$
10582.2	21.9	47.8	50.0	2.2	99.99	0.01
10593.5	10.5	51.0	48.6	0.4	0.02	99.98

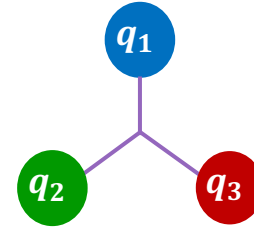
J^P	I	Mass	Width	E_B	\mathcal{P}_{BB}	$\mathcal{P}_{B^*B^*}$	Γ_{BB}	$\Gamma_{B^*B^*}$
0^+	0	10553.0	0	6.0	92%	8%	0	0
		10640.7	2.8	8.7	76%	24%	2.8	0
	1	10545.9	0	13.1	93%	7%	0	0
		10672.6	72.0	-23.2	39%	61%	30.7	41.3
2^+	1	10642.3	0	7.1	-	100%	-	0







$$V_{\text{conf}}^{\Delta} = \sigma_{\Delta} \sum_{i < j} r_{ij}$$



$$V_{\text{conf}}^Y = \sigma_Y L_{\text{min}}$$

$$0.5 \leq \frac{L_{\text{min}}}{L_{\Delta}} \leq 0.58$$



- Activation function: $\sigma(x) = \tanh(x)$
- Optimizer

$$\boldsymbol{\theta}^{i+1} = \boldsymbol{\theta}^i - \eta(S + \epsilon I)^{-1} \nabla_{\boldsymbol{\theta}^i} E_{\boldsymbol{\theta}^i}, \quad (8)$$

where i is the iteration step, η is the learning rate, $\epsilon = 10^{-3}$ is taken for numerical stability, and S is the Quantum Fisher information matrix,

$$S_{ab} = \frac{\langle \partial_{\theta_a} \psi_{\boldsymbol{\theta}} | \partial_{\theta_b} \psi_{\boldsymbol{\theta}} \rangle}{\langle \psi_{\boldsymbol{\theta}} | \psi_{\boldsymbol{\theta}} \rangle} - \frac{\langle \partial_{\theta_a} \psi_{\boldsymbol{\theta}} | \psi_{\boldsymbol{\theta}} \rangle \langle \psi_{\boldsymbol{\theta}} | \partial_{\theta_b} \psi_{\boldsymbol{\theta}} \rangle}{\langle \psi_{\boldsymbol{\theta}} | \psi_{\boldsymbol{\theta}} \rangle^2}. \quad (9)$$



TABLE V. The number of nodes in each hidden layer and the total number of variational parameters in the DNN for different systems.

Systems	S^P	Nodes	Parameters
e^+e^-	0^+	(16, 16, 16, 16)	961
$e^+e^-e^-$	0^+	(16, 16, 16, 16)	1041
$e^+e^+e^-e^-$	0^+	(16, 16, 16, 16)	1137
qqq	$\frac{1}{2}^+$	(16, 16, 16, 16)	1105
$QQ\bar{q}\bar{q}$	1^+	(32, 16, 16, 16)	1889
$QQ\bar{Q}\bar{Q}$	0^+	(32, 16, 16, 16)	1825
$QQ\bar{Q}\bar{Q}$	1^+	(32, 16, 16, 16)	1857
$QQ\bar{Q}\bar{Q}$	2^+	(32, 16, 16, 16)	1793
$QQqq\bar{Q}$	$\frac{1}{2}^-$	(40, 20, 20, 20)	3081
$QQqq\bar{Q}$	$\frac{3}{2}^-$	(40, 20, 20, 20)	3041
$QQqq\bar{Q}$	$\frac{5}{2}^-$	(40, 20, 20, 20)	2921



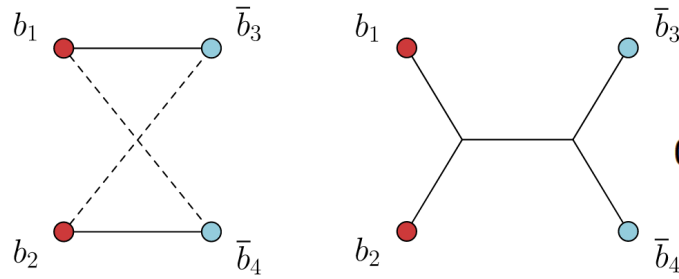


Fig. 1. Left panel: the flip-flop configuration of disconnected di-mesons. Right panel: the butterfly configuration with two connected diquarks. The two middle connecting points are chosen to minimize the total path.

$$V^{4Q} \equiv \min(V^{\text{flip-flop}}, V^{\text{butterfly}}) + \text{DMC}$$

0^{++} state has a mass of 18.69 ± 0.03 GeV, which is around 100 MeV below twice the η_b mass.

Y. Bai, S. Lu, and J. Osborne, Beauty-full tetraquarks, Phys. Lett. B **798**, 134930 (2019).

$$V = \Theta(V_{MM} - V_{YY})\Theta(V_{MM'} - V_{YY})(V_{YY}|\bar{\mathbf{3}}_{12}\mathbf{3}_{34}\rangle\langle\bar{\mathbf{3}}_{12}\mathbf{3}_{34}| + \min(V_{MM}, V_{MM'})|\mathbf{6}_{12}\bar{\mathbf{6}}_{34}\rangle\langle\mathbf{6}_{12}\bar{\mathbf{6}}_{34}|) \\ + \Theta(V_{YY} - V_{MM})\Theta(V_{MM'} - V_{MM})(V_{MM}|\mathbf{1}_{13}\mathbf{1}_{24}\rangle\langle\mathbf{1}_{13}\mathbf{1}_{24}| + \min(V_{YY}, V_{MM'})|\mathbf{8}_{13}\mathbf{8}_{24}\rangle\langle\mathbf{8}_{13}\mathbf{8}_{24}|) \\ + \Theta(V_{YY} - V_{MM'})\Theta(V_{MM} - V_{MM'})(V_{MM'}|\mathbf{1}_{14}\mathbf{1}_{23}\rangle\langle\mathbf{1}_{14}\mathbf{1}_{23}| + \min(V_{YY}, V_{MM})|\mathbf{8}_{14}\mathbf{8}_{23}\rangle\langle\mathbf{8}_{14}\mathbf{8}_{23}|),$$

+ multidimensional numerical integration
+ analytical continuation

$qq\bar{Q}\bar{Q}$ systems

P. Bicudo and M. Cardoso, Phys. Rev. D **94**, 094032 (2016).

