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Advances in OBE hadronic interactions:

Toward higher partial waves, 3-body systems, and beyond

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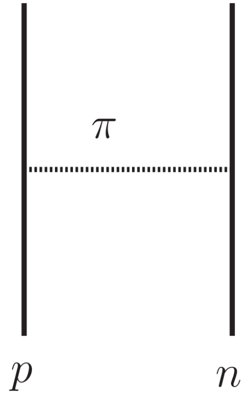
Based on PRL133, 241903 (2024), PRD111, 094022 (2025), PRD111, 094015 (2025)

Contents

Background

Yukawa potential

Yukawa Hideki



On the Interaction of Elementary Particles. I.

By Hideki YUKAWA.

(Read Nov. 17, 1934)

and the multiplicative constants. The potential of force between the neutron and the proton should, however, not be of Coulomb type, but decrease more rapidly with distance. It can be expressed, for example, by

$$+ \text{ or } -g^2\frac{e^{-\lambda r}}{r}, \qquad (2)$$

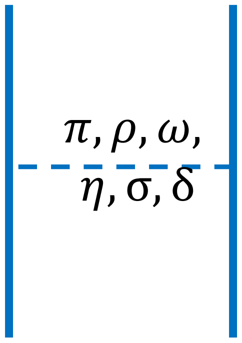
Assuming $\lambda = 5\times10^{12}\text{cm}^{-1}$., we obtain for m_U a value 2×10^2 times as large as the electron mass. As such a quantum with large mass and positive or negative charge has never been found by the experiment, the above theory seems to be on a wrong line. We can show, however, that, in the ordinary nuclear transformation, such a qnantum can not be emitted into outer space.

- Try to explan the interaction between nucleons, predicting pion
- Pion mass~200 times of m_e

Proc. Phys.-Math. Soc. Jpn. 1935, **17** (48)

High precision nuclear force: Bonn potential

- Interaction via one-boson exchange
- Great succuss in nuclear force



δ: $a_0(980)$; σ: $f_0(500)$

	Theory	Experiment
ε_d (MeV)	2.2246	2.224644 ± 0.000046
P_D (%)	4.38	–
Q_d (fm^2)	0.274 [a]	0.2860 ± 0.0015
μ_d (μ_N)	0.8548 [a]	0.857406 ± 0.000001
A_S ($fm^{-1/2}$)	0.8862	0.8846 ± 0.0016
D/S	0.0262	0.0271 ± 0.0008
r_d (fm)	1.9684	1.9660 ± 0.0068
a_s (fm)	−23.744	−23.748 ± 0.010
r_s (fm)	2.704	2.75 ± 0.05
a_t (fm)	5.424	5.424 ± 0.004
r_t (fm)	1.760	1.759 ± 0.005

R. Machleidt, K. Holinde, and C. Elster, Phys. Rept. **149**, 1 (1987).

High precision nuclear force: Bonn potential

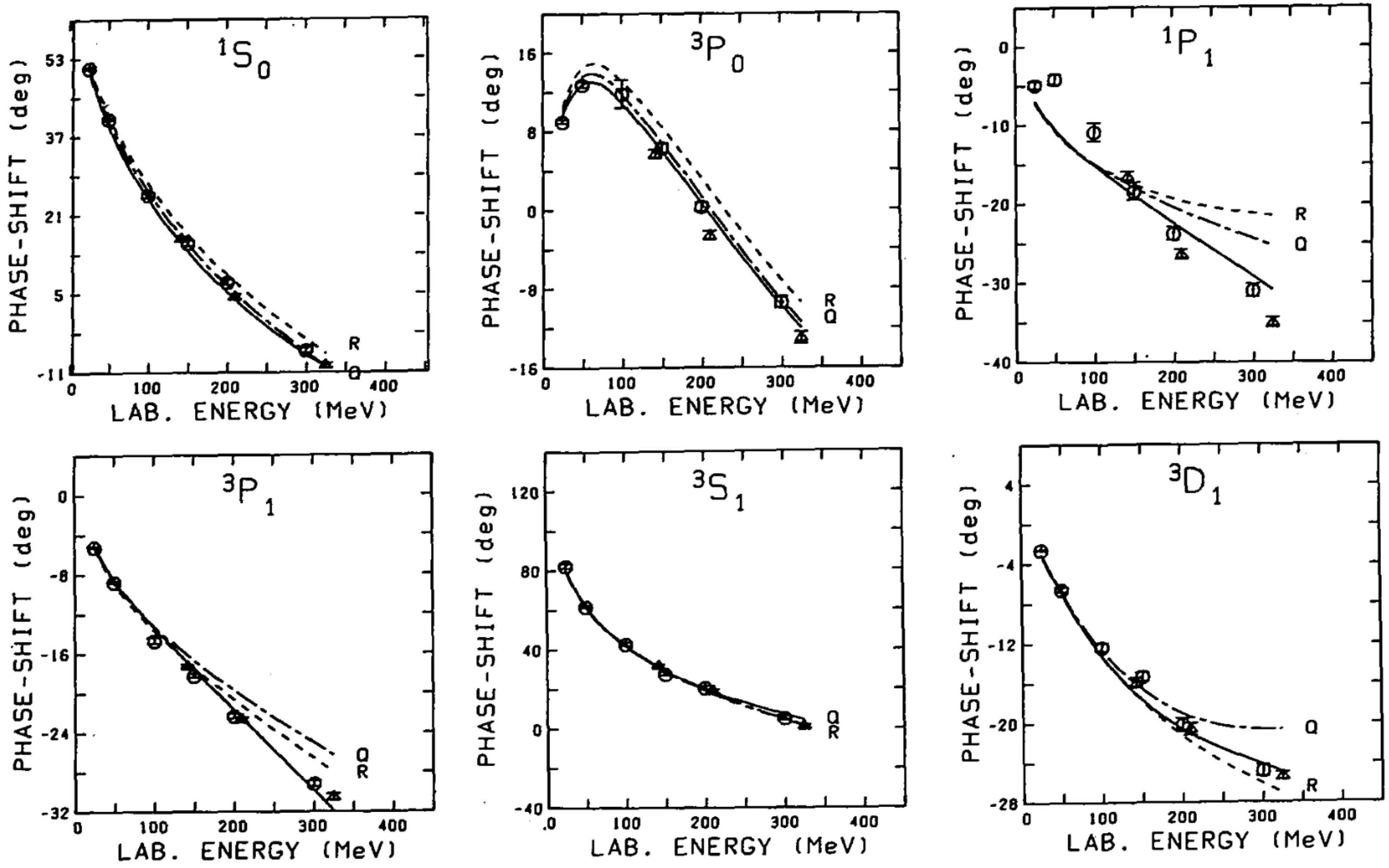


Fig. 17. The description of NN phase shifts by OBEP in comparison to the full model. The solid line represents the result predicted by our full model. The dashed–dotted line refers to the momentum space OBEP(Q) discussed in the section 9.1 with the parameters given in table 5. The dashed line gives the results of the coordinate space OBEP(R) with the parameters given in appendix F, table 14. Error bars as in fig. 15.

R. Machleidt, K. Holinde, and C. Elster, Phys. Rept. **149**, 1 (1987).

Hadronic molecules in OBE

Predictions in OBE model

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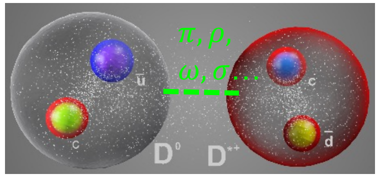
$X(3872)$
The 1st charmonium-like state

$Z_c(3900)$
The 1st manifestly exotic charmonium-like state

$P_{c\bar{c}}$
The 1st pentaquark states

$T_{cc}(3875)$
The 1st open double charm tetraquark state

2003 — 2013 — 2015 — 2019 — 2021



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R.Chen,Q.Huang,X.Liu,andS.-L.Zhu, PRD104,114042(2021)
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R. Chen, X. Liu, X. Q. Li and S.L.Zhu, PRL115, 132002, (2015)
R. Chen, J.He and X. Liu, CPC41, 103105 (2017)
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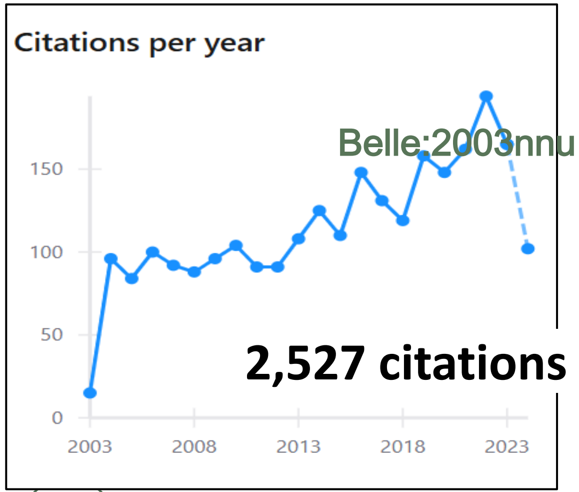
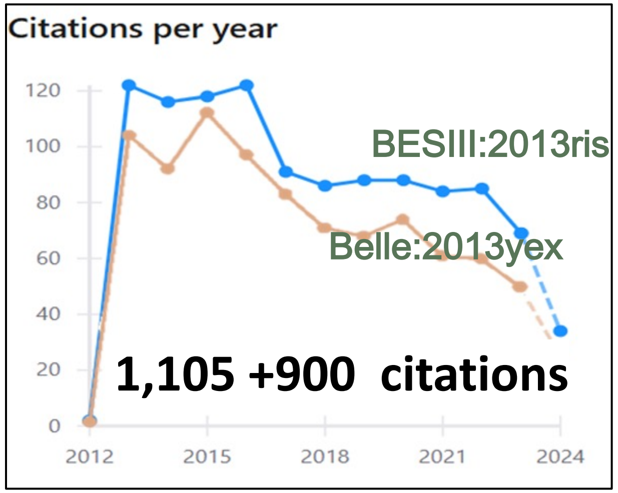
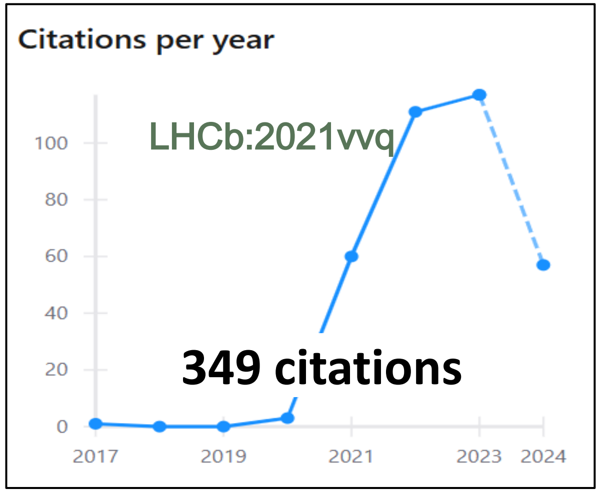
F. L. Wang, R. Chen, Z. W. Liu and X. Liu, PRC101, 025201, (2020)
….

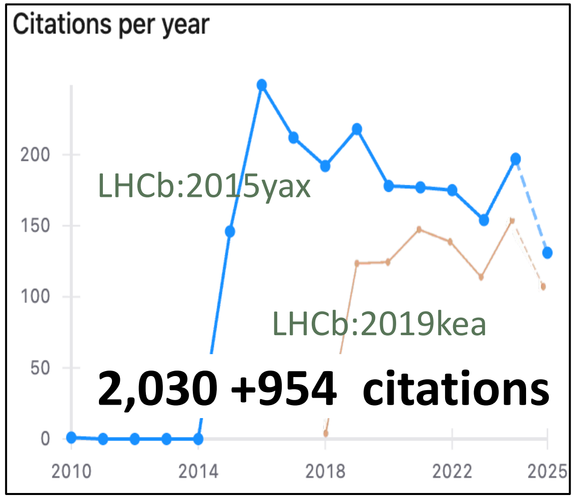
"Superstars"

	Quark contents	$I^G(J^{PC})$	Threshold	ΔM [MeV]	Γ [MeV]
$X(3872)$	$q\bar{q}c\bar{c}/c\bar{c}$	$0^+(1^{++})$	$D^0\bar{D}^{0*}$	$0.0068^{+0.1655}_{-0.17000}$	$0.380^{+0.412}_{-0.322}$
	The 1st charmonium-like state				BESIII:2023hmi
$Z_c(3900)$	$q\bar{q}c\bar{c}$	$1^+(1^{+-})$	$D\bar{D}^*$	11.3 ± 2.6	PDG 28.4 ± 2.6
	The 1st manifestly exotic charmonium-like state				
$T_{cc}(3985)$	$\bar{q}\bar{q}cc$	?	$D^{*+}D^0$	$-0.360^{+0.040}_{-0.040}$	$0.048^{+0.002}_{-0.014}$
	The 1st open double charm tetraquark state				LHCb:2021auc
$P_{c\bar{c}}$ states	$qqqc\bar{c}$	?	$\Sigma_c\bar{D}^{(*)}$	$-5\sim-20$	$\sim -$ **10**
	The 1st pentaquark states				LHCb:2015yax









Components of OBE model

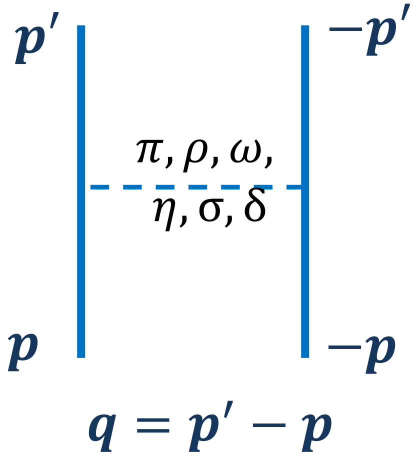
- Exchanged mesons
  - ▶ Hidden local gauge symmetry: vector-meson exchange
  - ▶ π, ρ, ω, σ,…: e.g. Bonn potential
- Determine coupling constants: Exp., Lattice, Models
- Regulator and cutoff

$$\frac{1}{u^2+\boldsymbol{q}^2} \to \frac{e^{-ur}}{4\pi r},$$

$$\frac{\boldsymbol{q}^2}{u^2+\boldsymbol{q}^2} = 1 - \frac{u^2}{u^2+\boldsymbol{q}^2} \to \delta^3(\boldsymbol{r}) - \frac{u^2 e^{-ur}}{4\pi r},$$

$$\frac{\boldsymbol{q}_i \boldsymbol{q}_j}{u^2+\boldsymbol{q}^2} \to -\frac{e^{-ur}}{4\pi r}\left(\frac{u^2}{3}+\frac{u}{r}+\frac{1}{r^2}\right)T_{ij} + \frac{1}{3}\left(\delta^3(\boldsymbol{r}) - \frac{u^2 e^{-ur}}{4\pi r}\right)\delta_{ij},$$

  - ▶ Singular terms: $\delta^3(r)$ and $1/r^3$
  - ▶ Short-range: structure of the hadron become important
- Coupled channel or not?
  - ▶ Hard to incopoerate properly, relativistic effect
  - ▶ Not important? $M_D^* - M_D \sim 130$ MeV, $M_{\Sigma_c} - M_{\Sigma_c^*} \sim 70$ MeV, Neglected in our works



$$V(\boldsymbol{p}',\boldsymbol{p}) \to V(\boldsymbol{p}',\boldsymbol{p}) \frac{\Lambda^2}{\Lambda^2+p^2}\frac{\Lambda^2}{\Lambda^2+p'^2}$$

$$V(\boldsymbol{p}',\boldsymbol{p}) \to V(\boldsymbol{p}',\boldsymbol{p}) \left(\frac{\Lambda^2-u^2}{\Lambda^2+q^2}\right)^2$$

Other options…

Schrodinger/Lippman-Schwinger equation

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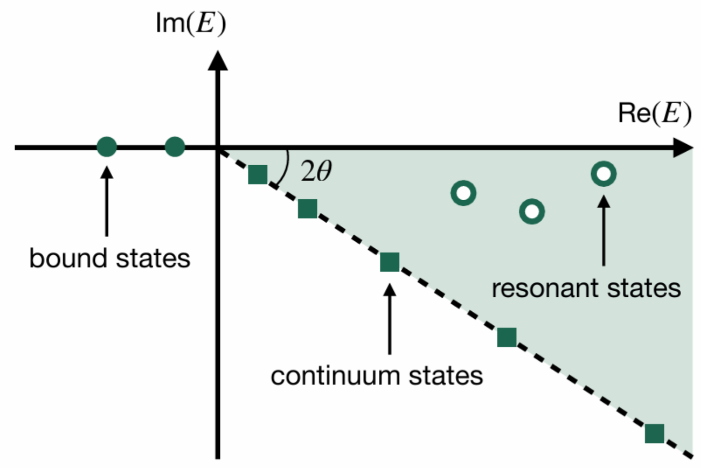
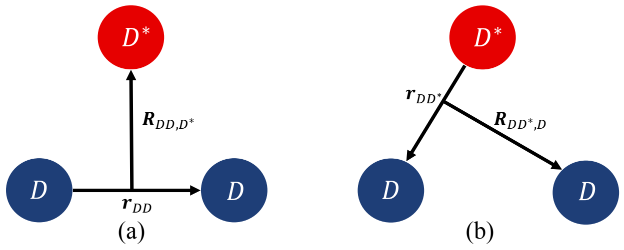
- Momentum space:
  - ▶ **Discrize the momenta**
  - ▶ **Deformed contour**: resonance, virtual state

$$\frac{p^2}{2\mu}\psi(p)+\int_C \frac{dq}{(2\pi)^3} q^2 V_{l=0}(p,q)\psi(q)=E\psi(p),$$

- Coordinate spance
  - ▶ Basis expandion method, Finite element method
  - ▶ Resonant: complex scaling method…
- 3-body/4-body systems
  - ▶ Faddeev equation
  - ▶ **Gaussian expansion method+ Complex scaling**



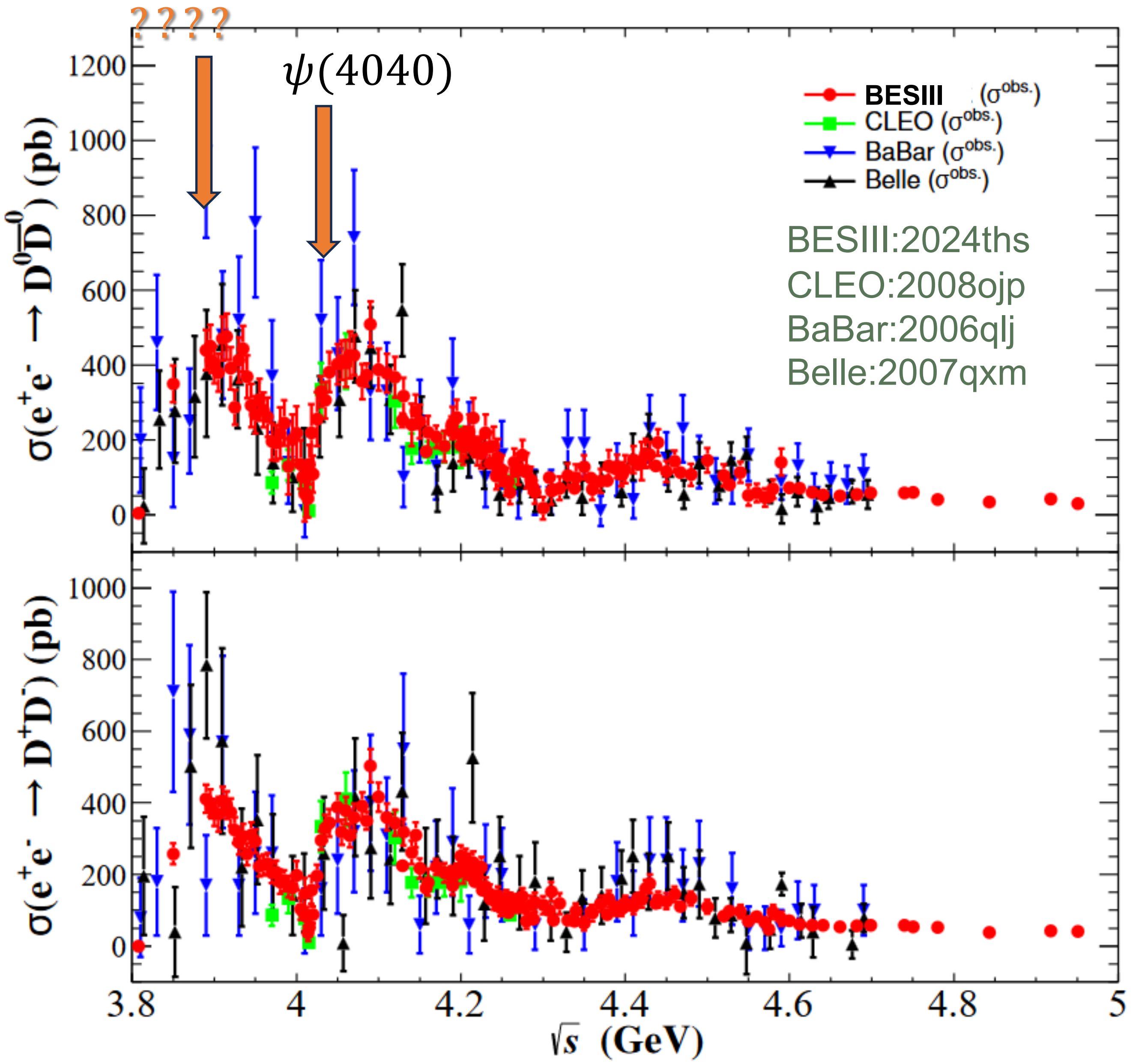
$$\phi_{nlm}(\boldsymbol{r})=\sqrt{\frac{2^{l+5/2}}{\Gamma\left(l+\frac{3}{2}\right) r_n^3}}\left(\frac{r}{r_n}\right)^l e^{-\frac{r^2}{r_n^2}} Y_{lm}(\hat{r}),$$



P-wave DD^* and $\bar{D}^*D/\bar{D}D^*$

$G(3900)$ states

- An enhancement close to 3.9 GeV
  - $D^*\bar{D}$ threshold
  - New $D^*\bar{D}$ resonance?
  - Quantum number $J^{PC}=1^{--}$, from virtual photon
  - P-wave $D^*\bar{D}$ state?



Breit-Wigner fit from BESIII

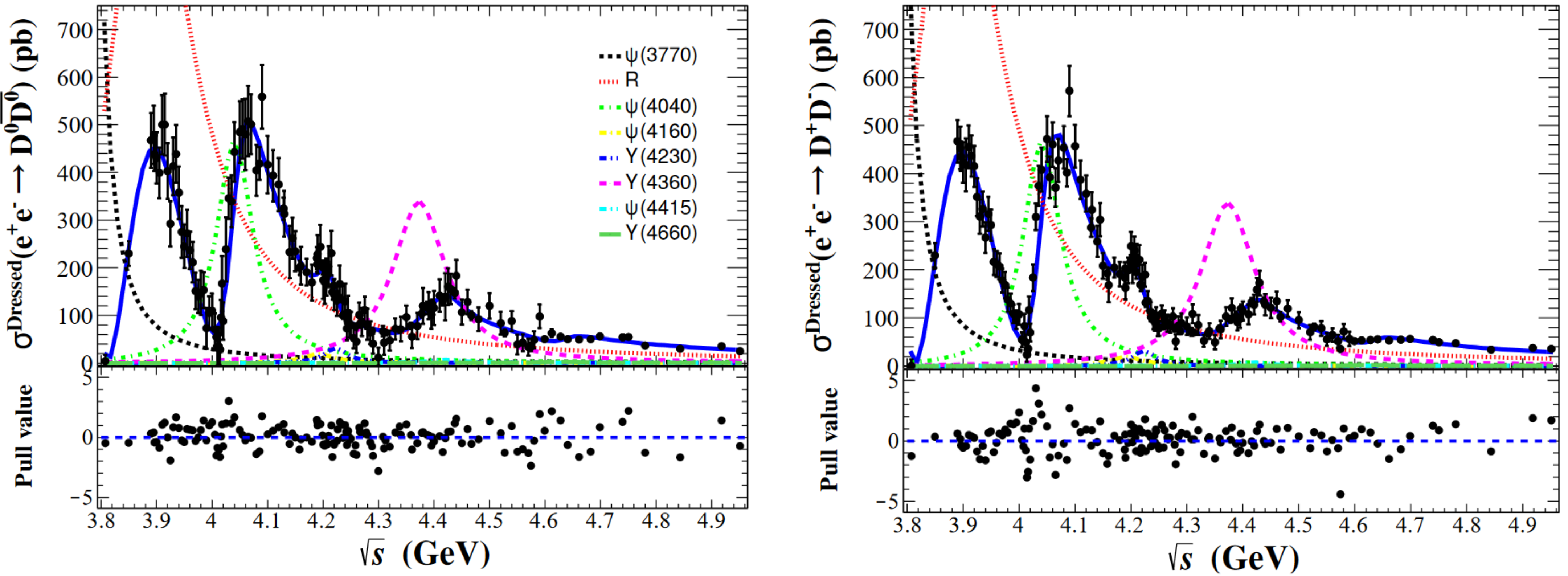
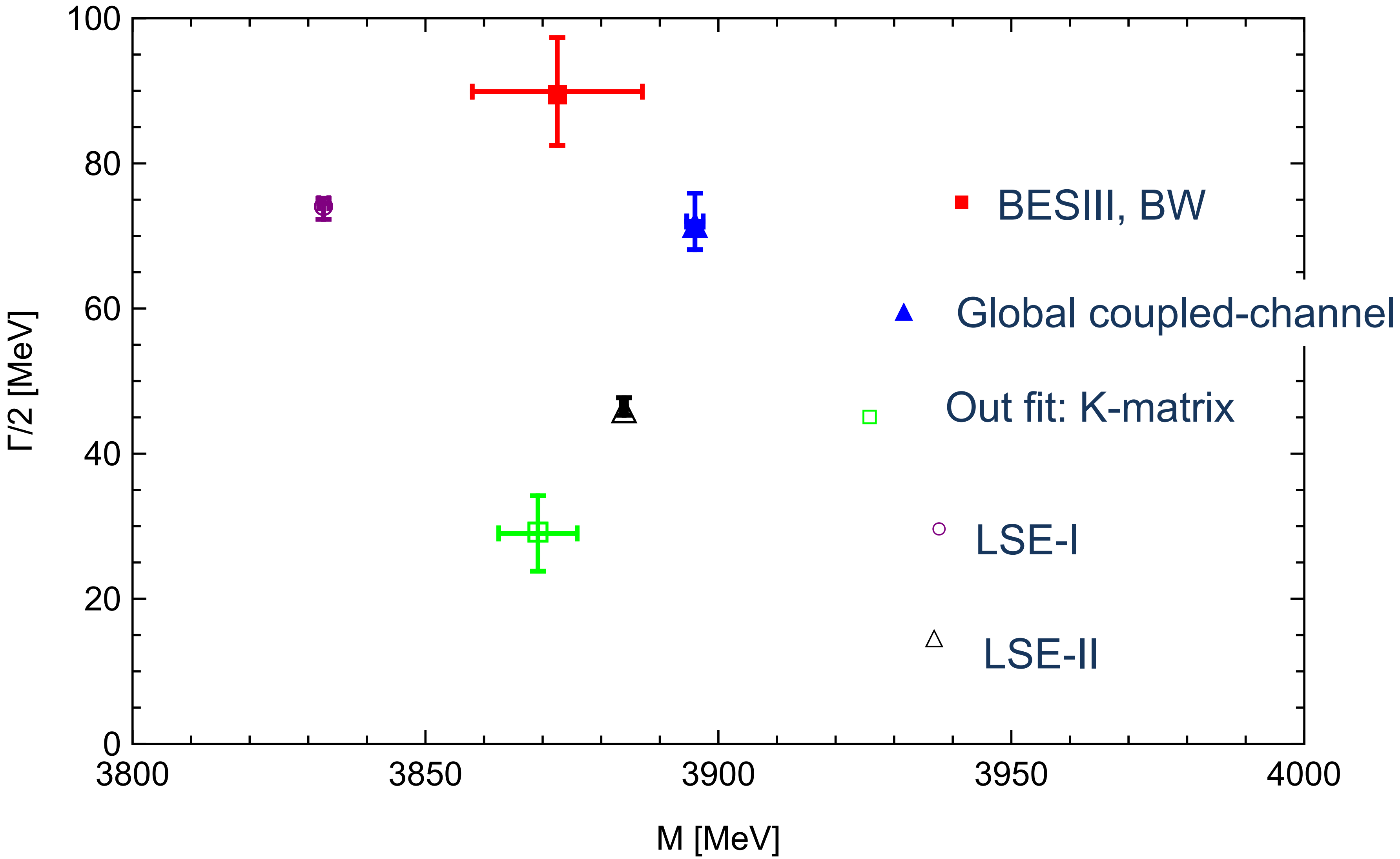


TABLE I. Fit results of the Born cross section, where the first uncertainties are the statistical and the second are systematic and S denotes the significance.

BESIII:2024ths

$e^+e^- \to D\bar{D}$								
Resonance	$\psi(3770)$	R	$\psi(4040)$	$\psi(4160)$	$Y(4230)$	$Y(4360)$	$\psi(4415)$	$Y(4660)$
Mass (MeV/c^2)	3773.7 (fixed)	3872.5±14.2±3.0	4039 (fixed)	4191 (fixed)	4222.5 (fixed)	4374 (fixed)	4421 (fixed)	4630 (fixed)
Width (MeV/c^2)	87.6 (fixed)	179.7±14.1±7.0	80 (fixed)	70 (fixed)	48 (fixed)	118 (fixed)	62 (fixed)	72 (fixed)
$\Gamma_{ee}\mathcal{B}$ (eV)	95-106	202-292	41-44	1-2	1-2	50-144	0-2	0-1
S(σ)	10	> 20	13	7	11	11	4	8
χ^2/d.o.f = 346/275				p-value = 0.002				

Pole positions

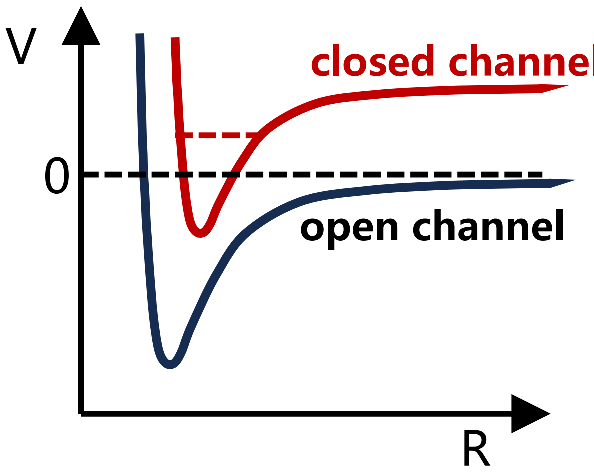


We need:

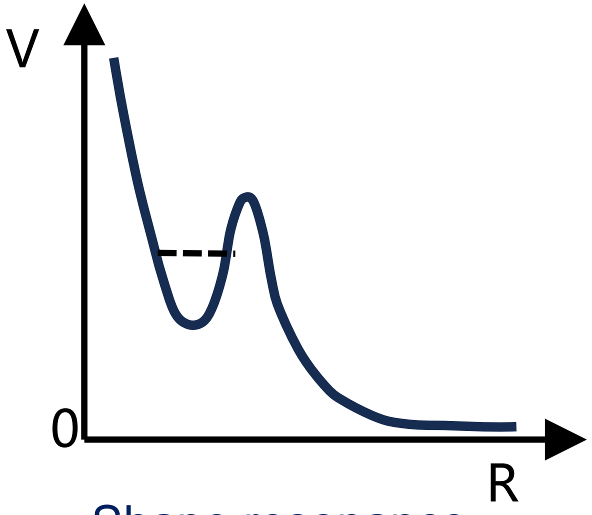
- More Data
- Careful investigation of sys. uncertainties

Q. Ye, Z. Zhang, M.-L. Du, U.-G. Meißner, P.-Y. Niu, and Q. Wang,, Phys. Rev. D **112**, 016015 (2025).

Two types of resonances



Feshbach resonance



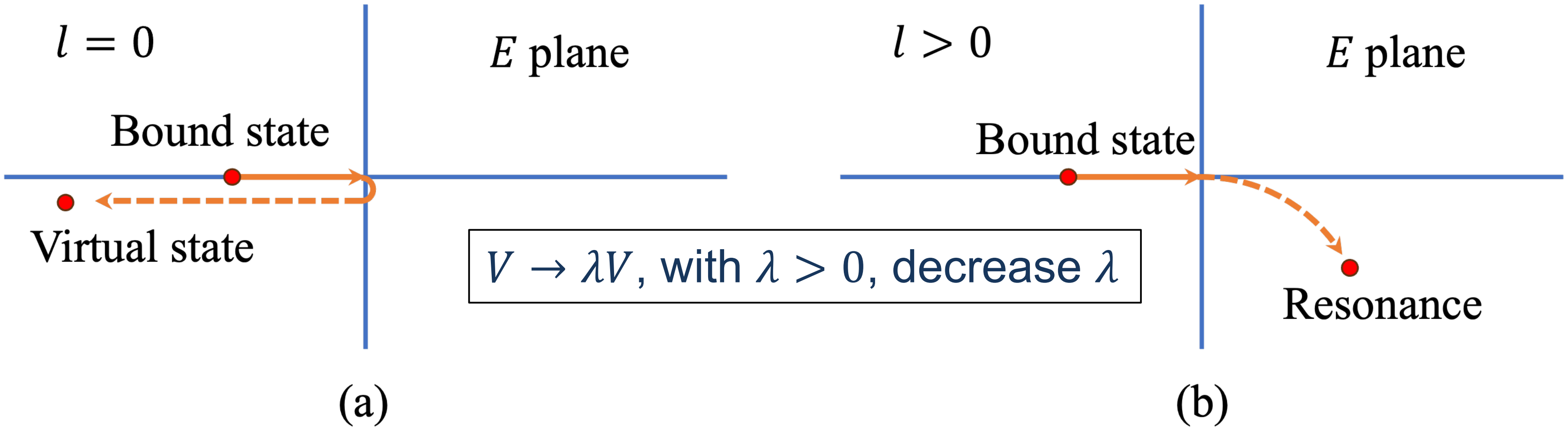
Shape resonance

- Resonance: state with finite lifetime
- Feshbach resonance
  - E.g. P_c states could be bound states of $\Sigma_c \bar{D}^{(*)}$
  - Considering the $J/\psi p$ channel: resonance
- Shape resonance
  - Barrier
- Feshbach or shape? depending on scheme

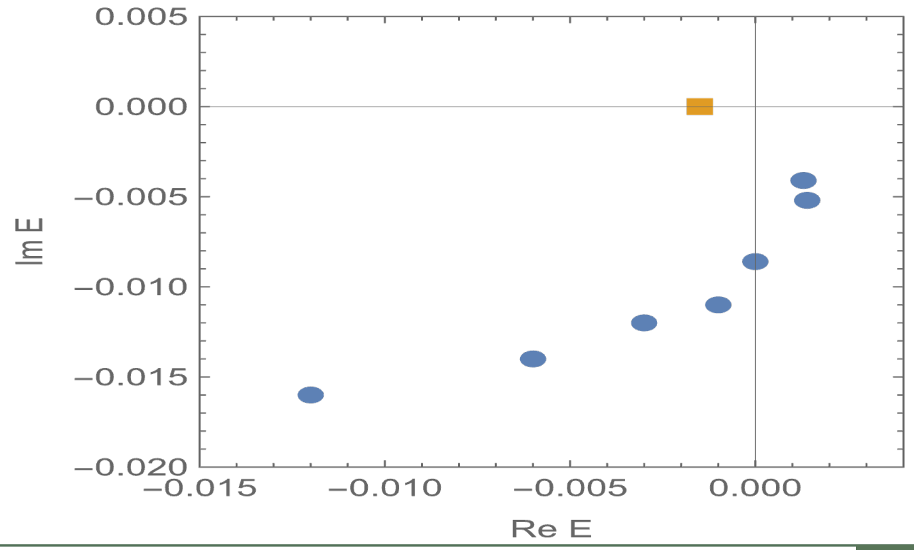
From bound state to…

- From bound state to…

Taylor, scattering theory textbook sec. 13-b P245

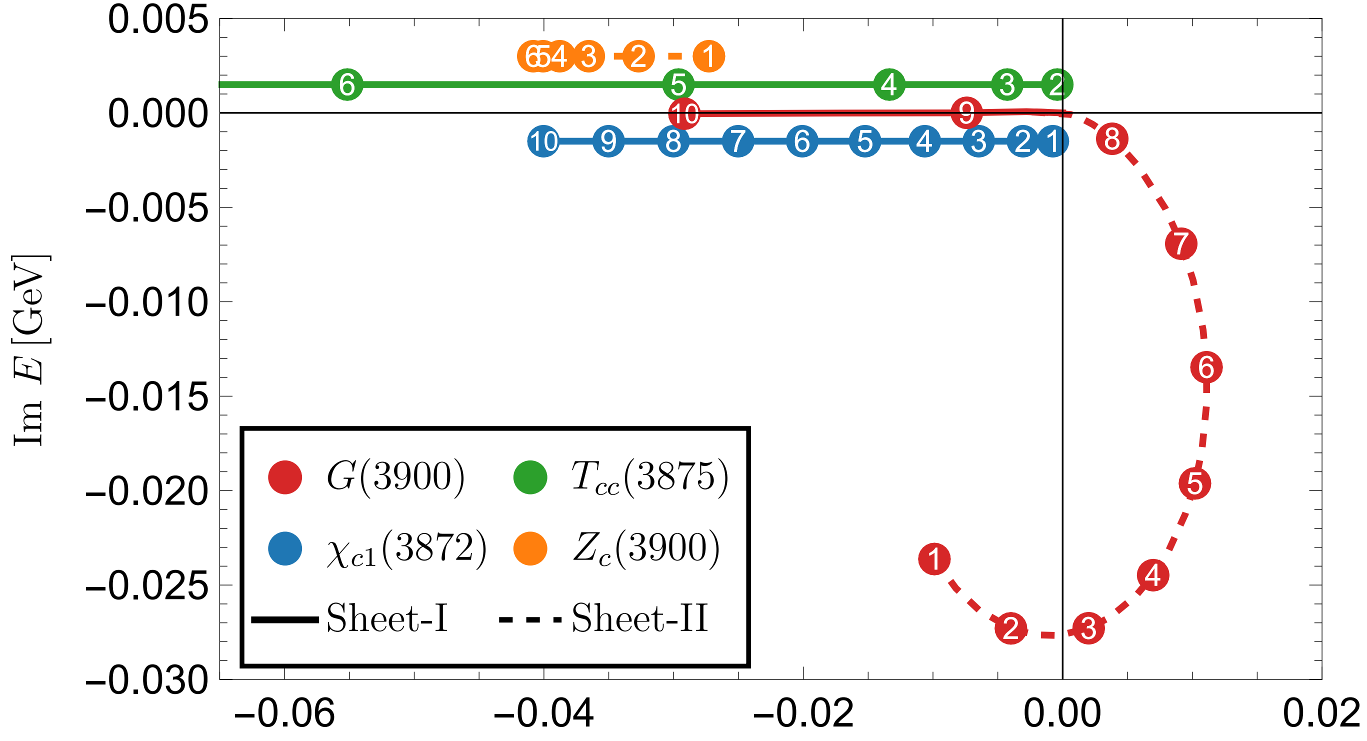


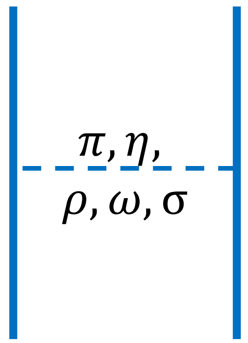
Potential barrier: centrifugal barrier for l>0



- ${}^3P_0$ NN resonance pole: $-0.012 - i0.016$
  - ▶ Were seldom investigated
  - ▶ Hard to detect: no lower coupled-channel

Results





- Vary Λ from 0.4 to 1.3 GeV (①−⑩)
- Constrained by $X(3872)$, $Z_c(3900)$, and $T_{cc}(3875)$
- $X(3872)$ and $T_{cc}(3875)$: bound states
- $Z_c(3900)$: virtual state
- $G(3900)$: resonance → bound state

Experimental Search

		$D\bar{D}^*, C=+$		$D\bar{D}^*, C=-$		DD^*	
		$I=0$	$I=1$	$I=0$	$I=1$	$I=0$	$I=1$
$\Lambda=0.5$GeV	$1^+(^3S_1)$	-3.1^B, $\chi_{c1}(3872)$	-	-1.60^B ②	-35.6^V, $Z_c(3900)$	-0.41^B, $T_{cc}(3875)$	-
	$0^-(^3P_0)$	$-1.5-14.5i$ ③	-	-	-	$-9.6-9.7i$ ④	-
	$1^-(^3P_1)$	-	-	$-4.0-27.3i$, $G(3900)$ ①	-	$-31.7-70.6i$	-
	$2^-(^3P_2)$	$-42.6-39.4i$	-	$-21.3-50.7i$	-	$-37.8-40.9i$	-

B: bound state, V: virtual state

① Precision measurement of $e^+e^- \to \bar{D}D^*$ close to threshold

② Hidden charm final state: $\eta_c\omega$, $J/\psi\eta$, $J/\psi\pi\pi$

③ Hidden charm final state: $J/\psi\omega$, $\eta_c\pi\pi$, $\chi_{c1}\pi\pi$

④ Final state $DD\pi$

Lattice support

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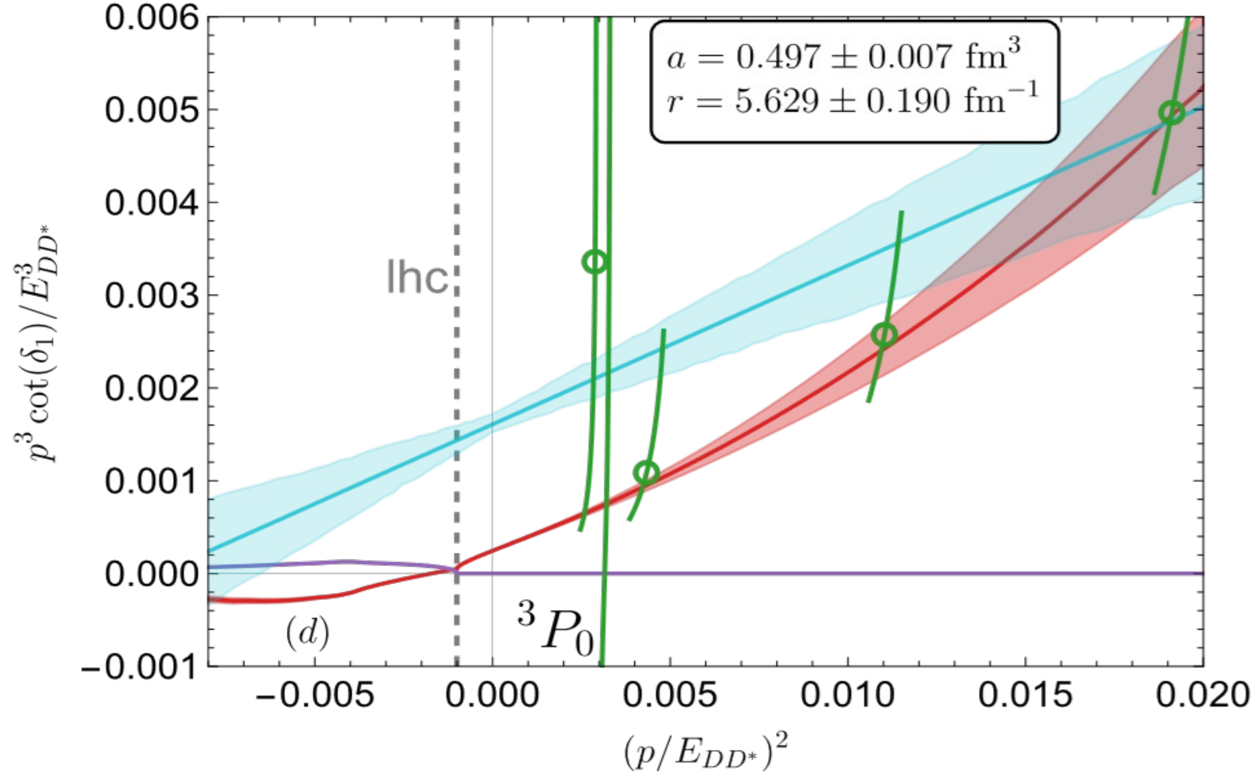
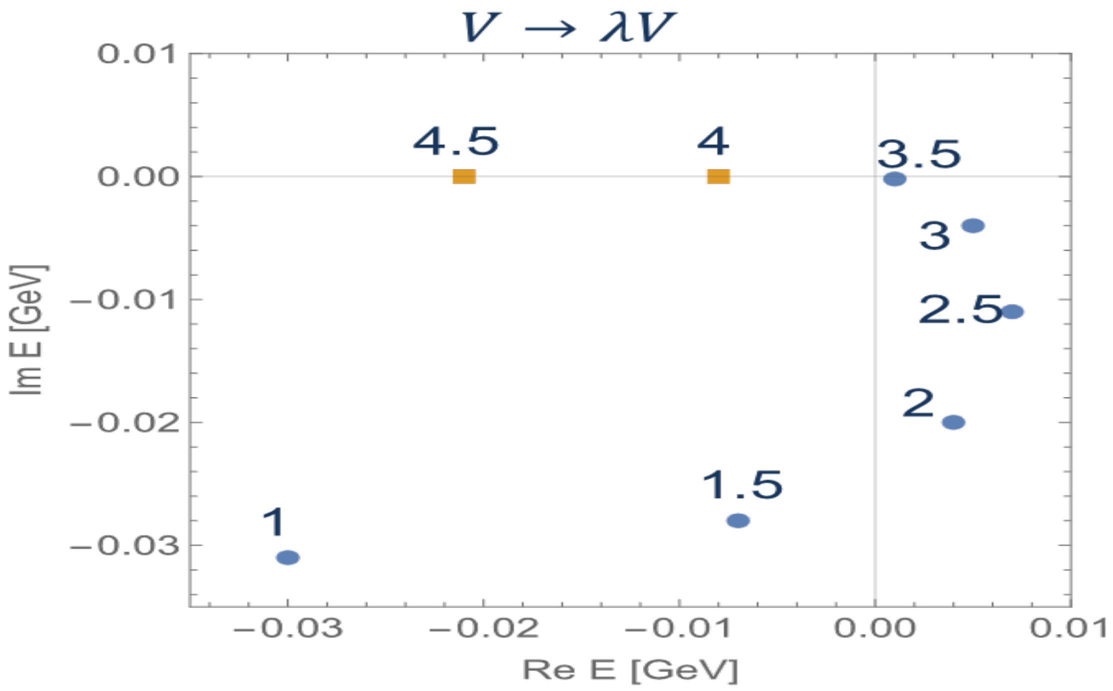
- Lattice QCD: ${}^3P_0\ DD^*$ resonance pole: $-0.030 - i0.031$





L. Meng, V. Baru, E. Epelbaum, A. A. Filin, and A. M. Gasparyan, PRD**109**, L071506 (2024).

DDD^* bound state and $Z_c(3900)$

Uncertain coupling constants in OBE

- 4 coupling constants in SU(3) flavor and heavy quark symmetry
  - ▶ Pseudoscalar: extracted from $D^* \to D\pi$
  - ▶ Three remains unknown: λ, β, g_s
- Determine the coupling
  - ▶ vector meson dominance, Σ-model, light-cone sum rule, LQCD

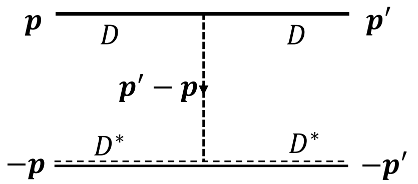
Not in unified framework

- Quantitively different results due to uncertain coupling
  - ▶ Different by 20 times
  - ▶ Both models give the loosely bound $X(3872)$ and $T_{cc}(3875)$
  - ▶ However, different results for $Z_c(3900)$ and DDD^* !!!

	Model-I	Model-II
$DDD^*, S=1, I=1/2$	unbound	binding energy ~1 MeV
$Z_c(3900)$	~ -40 MeV (virtual)	~ -5 MeV (virtual)

Model-I {[53] N. Li and S.L.Zhu, PRD86, 074022 (2012)
[54] N. Li, Z.-F. Sun, X.Liu and S.-L Zhu, PRD88, 114008 (2013)

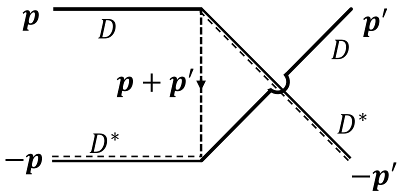
Model-II {[65] T.-W. Wu, Y.-W.Pan, M.-Z.Liu,S.-Q.Luo, L.-S. Geng, X.Liu, PRD105,L031505(2022)
[84] M.-Z.Liu, T.-W. Wu, M.P. Valderrama, J.-J.Xie and L.-S.Geng, PRD99, 094028 (2019)



Direct diagrams

g_s for σ

β for ρ, ω



Cross diagrams

g_a for π,η ;

λ for ρ, ω

V	C_{coupling} Model-I[53, 54]		Model-II[65, 84]	
V_ρ^D	$\frac{\beta^2 g_v^2}{2}$	13.62	$2g_\rho^2$	13.52
V_ω^D	$\frac{\beta^2 g_v^2}{2}$	13.62	$2g_\omega^2$	13.52
V_σ^D	g_s^2	0.58	g_σ^2	11.56
V_π^C	$\frac{g_a^2}{f_\pi^2}$	19.15	$\frac{g^2}{f_\pi^2}$	20.66
V_η^C	$\frac{g_a^2}{f_\pi^2}$	19.15	0	0
V_ρ^C	$2\lambda^2 g_v^2$	21.10	$\frac{f_\rho^2}{2M^2}$	19.64
V_ω^C	$2\lambda^2 g_v^2$	21.10	$\frac{f_\omega^2}{2M^2}$	19.64

Refit the couling

- Use the model-I as the baseline

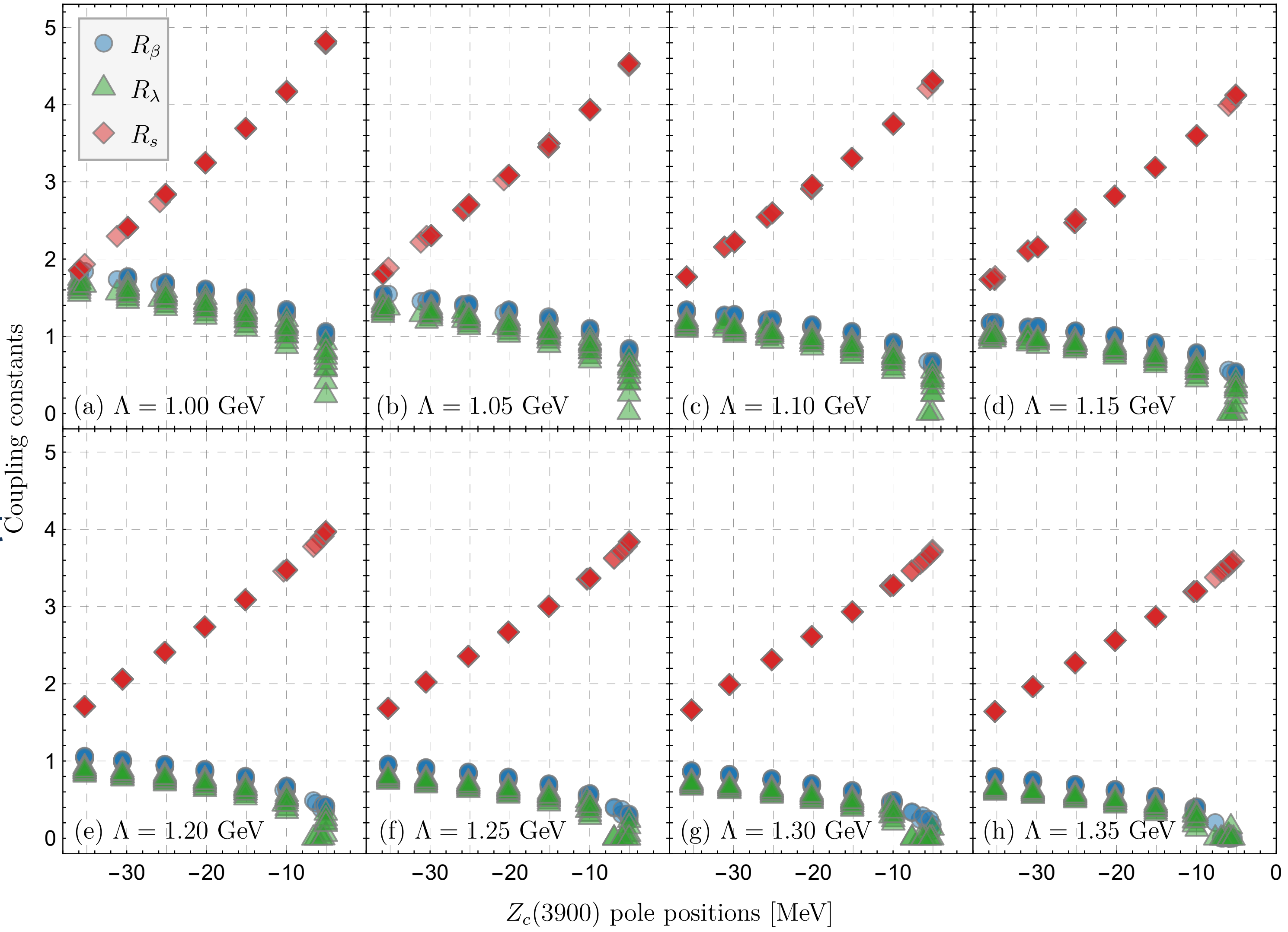
$$\lambda \to \lambda R_\lambda, \quad \beta \to \beta R_\beta, \quad g_s \to g_s R_s$$

- Chosse 63 sets of pole positions

$$\begin{aligned} X(3872) &: \ (-1) \times (0.2, 0.4, 0.6)^B \text{ MeV}, \\ T_{cc}(3875) &: \ (-1) \times (0.2, 0.4, 0.6)^B \text{ MeV}, \\ Z_c(3900) &: \ (-1) \times (5, 10, 15, 20, 25, 30, 35)^V \text{ MeV}, \end{aligned}$$

Large uncertainty in Ex.

- 8 different cutoffs
  - ▶ Systematically take into account the uncertainty from the cutoff
- R_s: dramatic variation
  - ▶ Related to $Z_c(3900)$ ples
- New sets of coupling constants
  - ▶ Used for other systems

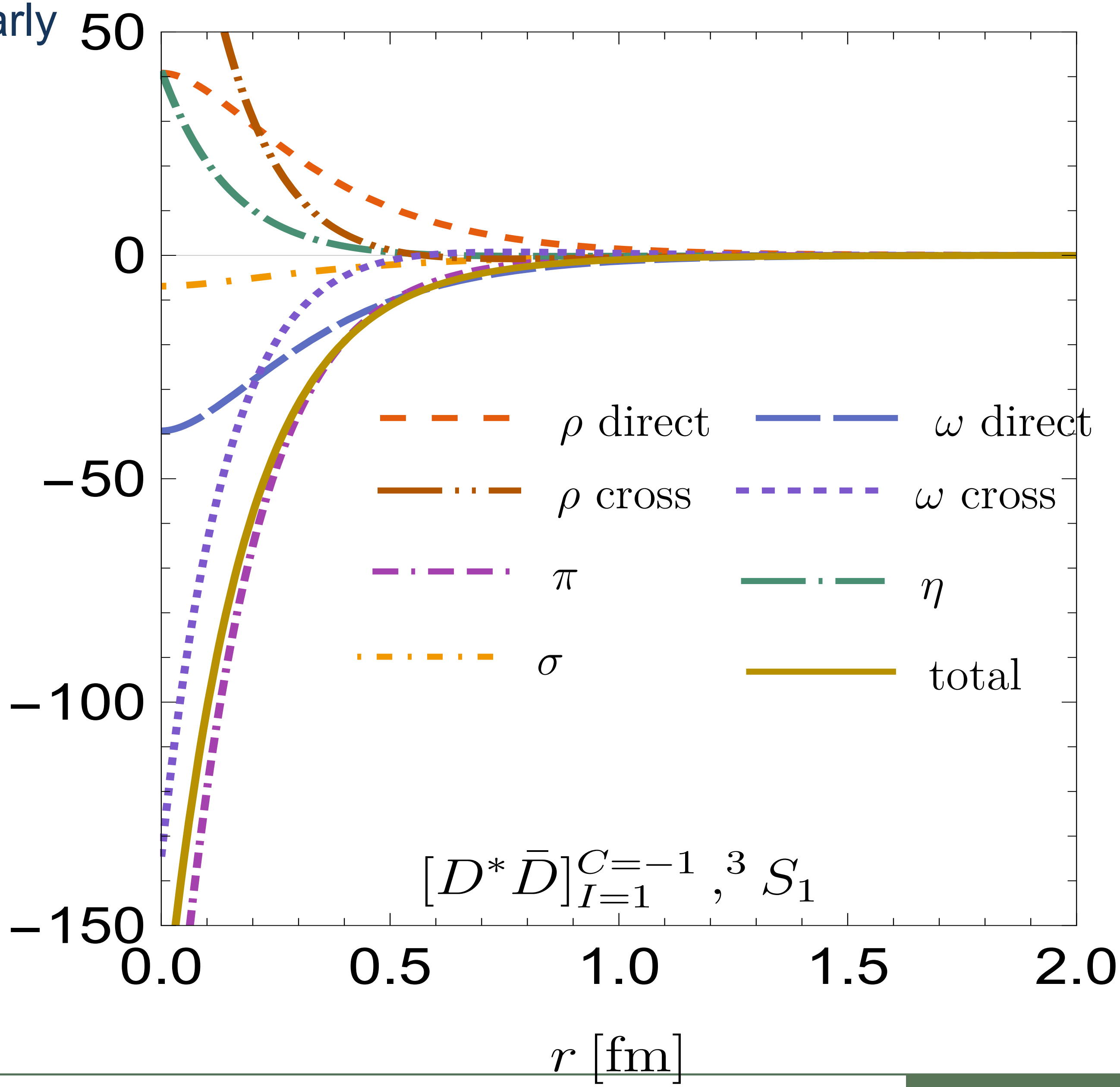


DDD^* state and $Z_c(3900)$

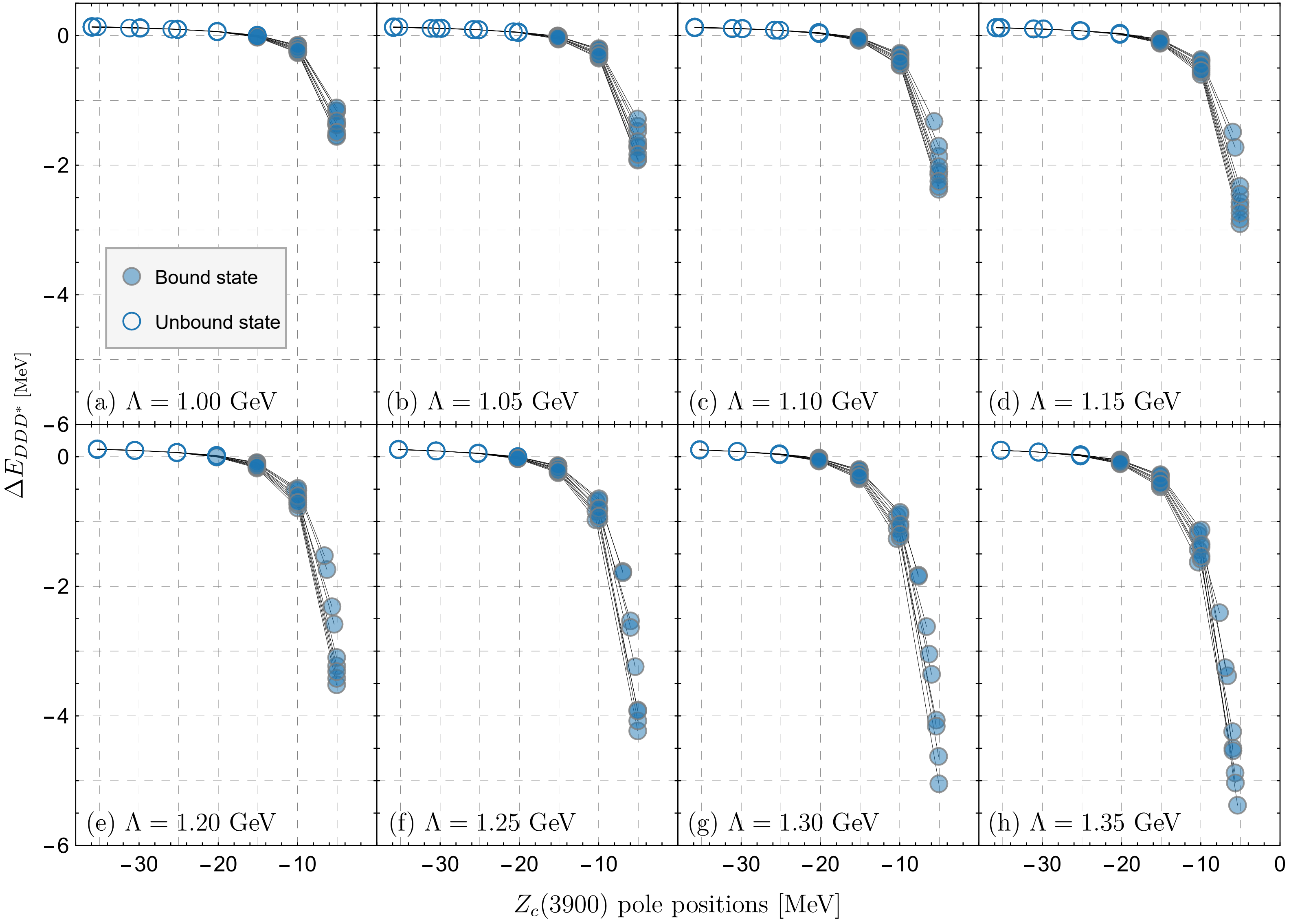
- For $Z_c(3900)$: the ω and ρ exchange are nearly cancle out
  - Exactly cancel out if $m_\omega = m_\rho$
  - The coupling of π and η are fixed
  - Very sensitive to the σ-exchange
- σ-exchange:
  - blind for the spin and isospin
  - Attactive for all pairs
  - 3-body: 3 different pairs
  - 2-body: 1 pair

More easily show the effect

- $Z_c(3900)$ and DDD^* bound state



DDD^* state and $Z_c(3900)$



Stronger σ coupling, toward bound DDD^*

DDD^*

bound-unbound transition point

$Z_c(3900)$

virtual state pole $-15 \sim -20$ MeV

Conclusion

- Remain uncertain:
  - ▶ DDD^* bind or not
  - ▶ $Z_c(3900)$ pole position
- They are closely related
  - ▶ $Z_c(3900)$ pole position
  - ▶ Progress in either direction shed light on the other
- Several sets of coupling

Short-range interaction of $P_{c\bar{c}}$

Pentaquark state in OBE model

- Fail to determine 3 coupling constants by three states

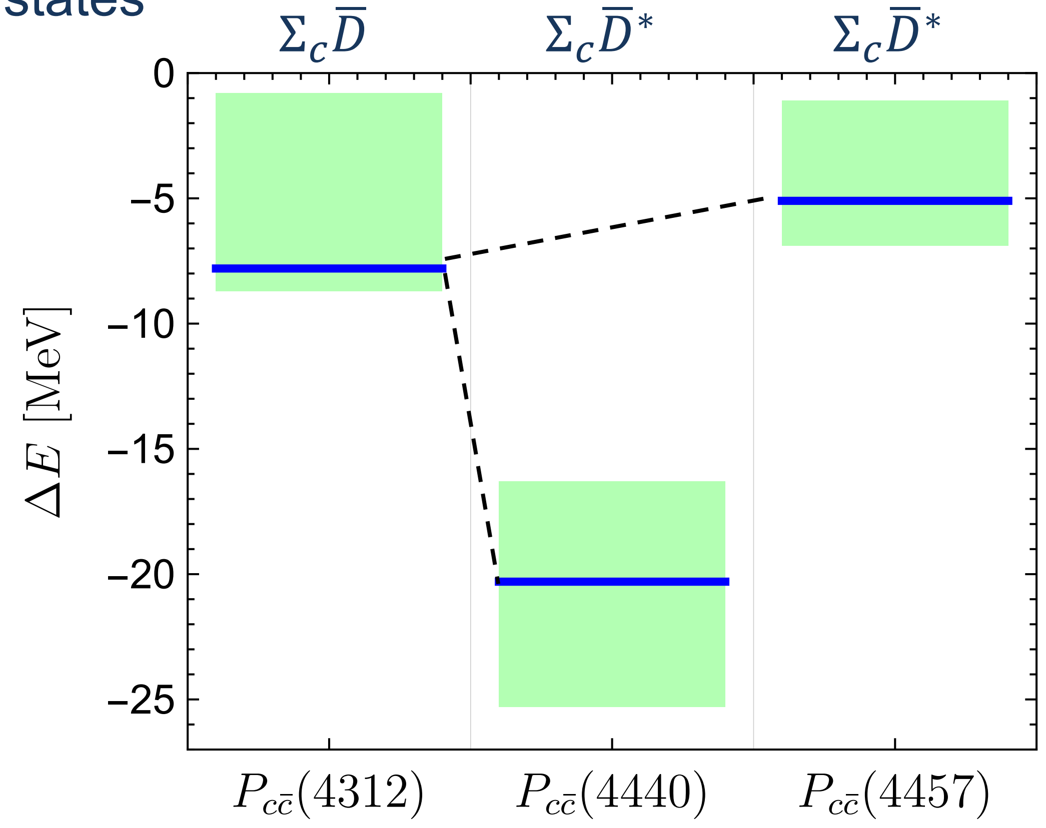
$$\beta\beta_s \to R_\beta\beta\beta_s, \quad l_s g_s \to R_s l_s g_s, \quad \lambda\lambda_s \to R_\lambda\lambda\lambda_s.$$

Why?

- S-wave Interaction: central + spin-depedent

$$V = V_C^{\rho,\omega,\sigma} + V_S^{\rho,\omega} + V_S^{\pi\eta},$$

  - For $\Sigma_c\bar{D}$, only central part
  - The splitting of ΔE relect the V_S

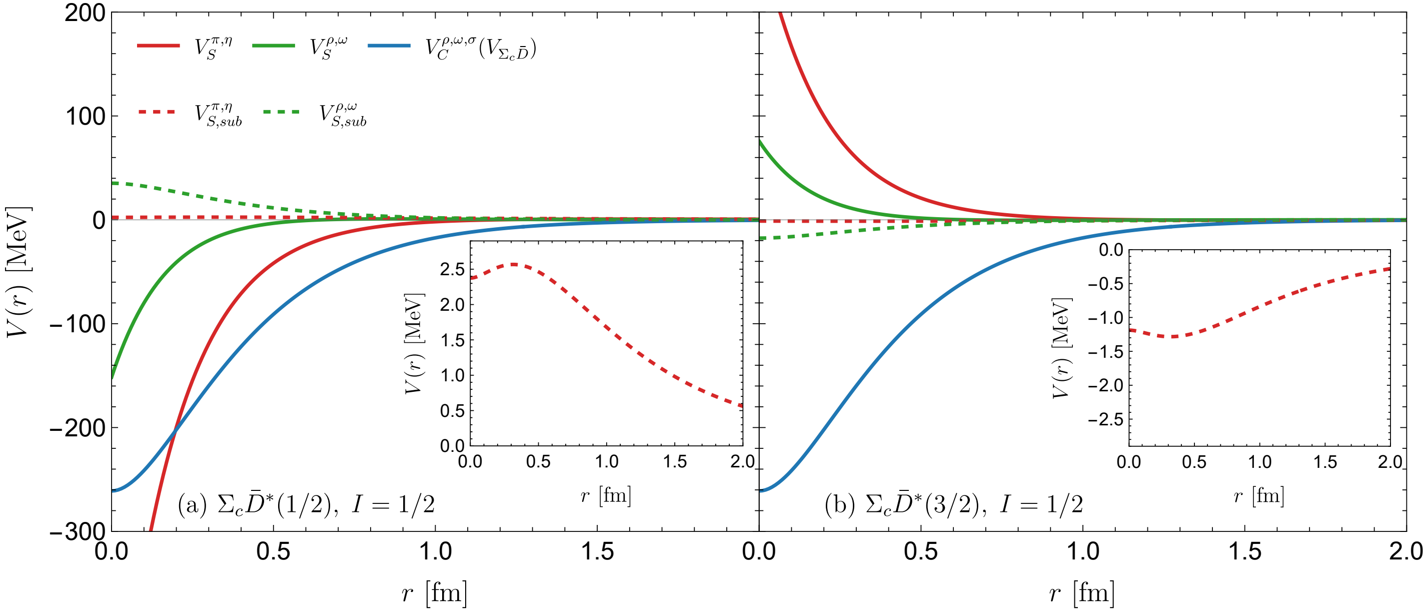


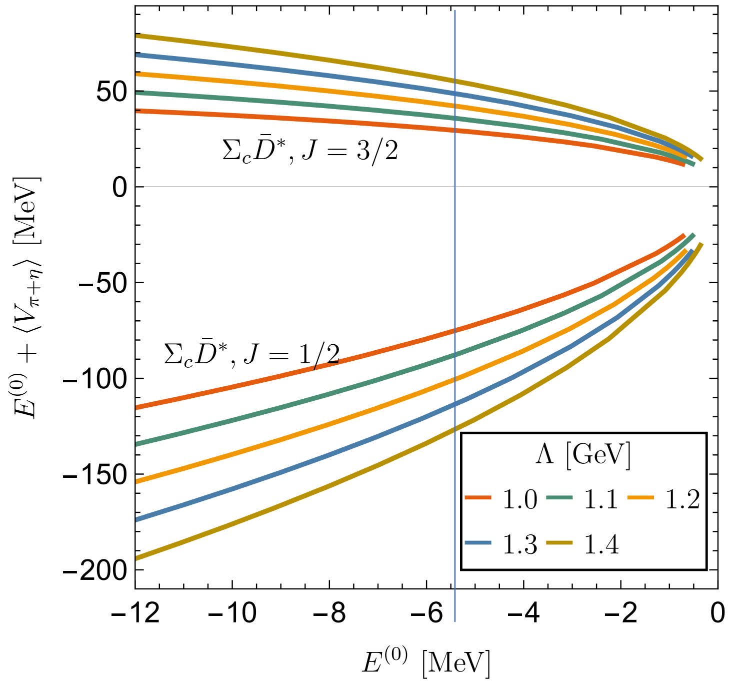
$S = \frac{1}{2}$ $\quad S = \frac{3}{2}$

$S = \frac{3}{2}$ $\quad S = \frac{1}{2}$

Spin-assigenment problem

Spin splitting





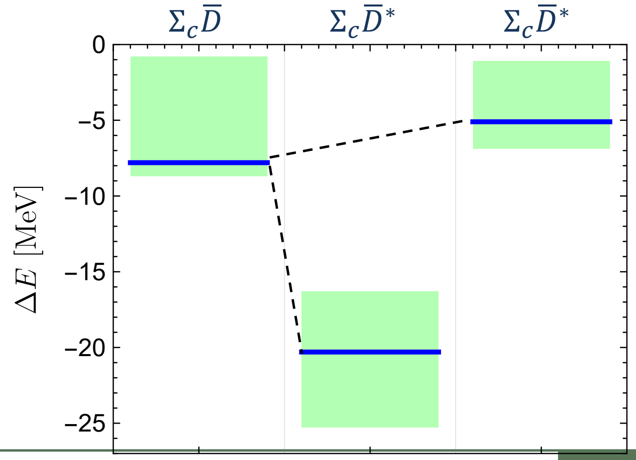
$$V = V_C^{\rho,\omega,\sigma} + V_S^{\rho,\omega} + V_S^{\pi\eta},$$

$$\mathrm{sign}(V_S^{\rho,\omega}) = \mathrm{sign}(V_S^{\pi,\eta})$$

Lower limit of the V_S: $V_S^{\rho,\omega} = 0$

The spin-depdent interaction is too strong to fit the Ex. spin splittiing

→Adjust the short-range interaction



See also: M.-Z. Liu, T.-W. Wu,et al, PRD103, 054004 (2021); T. J. Burns and E. S. Swanson, PRD 100, 114033 (2019); N. Yalikun, Y.-H. Lin, et al PRD104, 094039 (2021)

Stratergy: remove the δ-terms

- Remove the $\delta^3(r)$ terms

$$\frac{\boldsymbol{q}^2}{u^2+\boldsymbol{q}^2} = 1 - \frac{u^2}{u^2+\boldsymbol{q}^2} \to \delta^3(\boldsymbol{r}) - \frac{u^2 e^{-ur}}{4\pi r},$$

However, some queries:

- The regulator has regularized the singularity, no reason to remove it
- Bonn potential: not remove it
- Some ambuigities:
  - ▶ Remove for all mesons or just pion
  - ▶ Remove it or soften it further
- G-paritiy rule: $V(\Sigma_c D^{(*)}) = (-1)^{G_{ex}} V(\Sigma_c D^{(*)})$
  - ▶ Do the removed part satisfy the G-parity rule?

$$\frac{q_i q_j}{u^2+\boldsymbol{q}^2} F(u,\Lambda,q^2)^2 \to -\left[H_3(u,\Lambda,r)T_{ij} + H_1(u,\Lambda,r)\frac{\delta_{ij}}{3}\right],$$

$$H_1(u,\Lambda,r) = \frac{u^3}{4\pi}\left[\frac{e^{-ur}-e^{-\Lambda r}}{ur} - \frac{(\Lambda^2-u^2)\Lambda^2}{2u^3\Lambda}e^{-\Lambda r}\right],$$

$$H_3(u,\Lambda,r) \to \frac{\Lambda^4+u^4-2\Lambda^2u^2}{96\pi} r + \mathcal{O}(r^2).$$

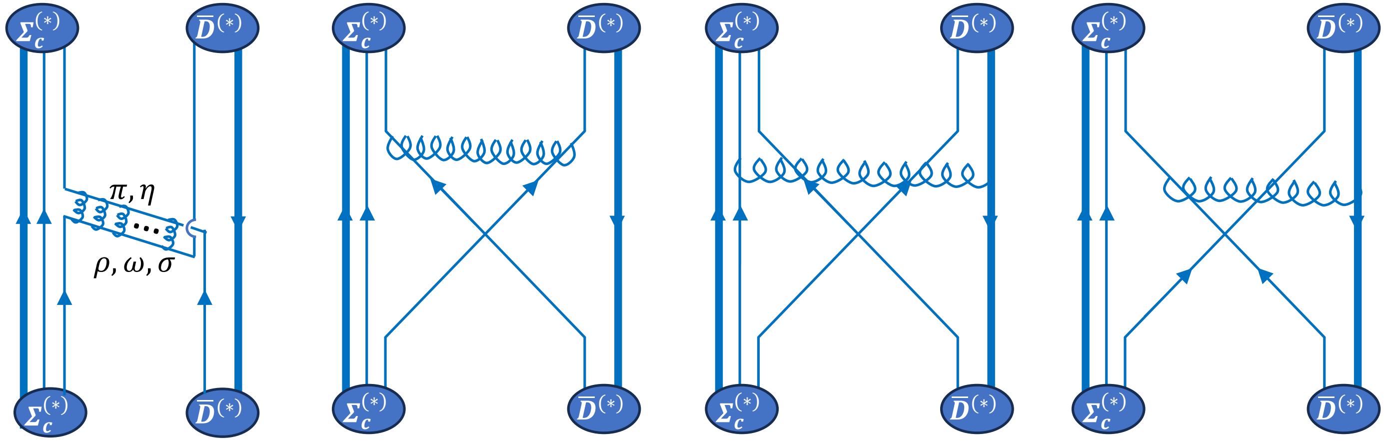
Physical reasonable Stratergy: model the short-range interaction

Our proposal: quark exchange effect

- Quark exchanged effect were used to model the repulsive core of the nuclear force

M. Oka and K. Yazaki, Prog. Theor. Phys. **66**, 556 (1981).

- Quark-exchange effect for $\Sigma_c^{(*)}\bar{D}^{(*)}$



(a) Meson-exchange (b1) (b2) Subleading quark-exchange effect (b3) Leading quark-exchange effect

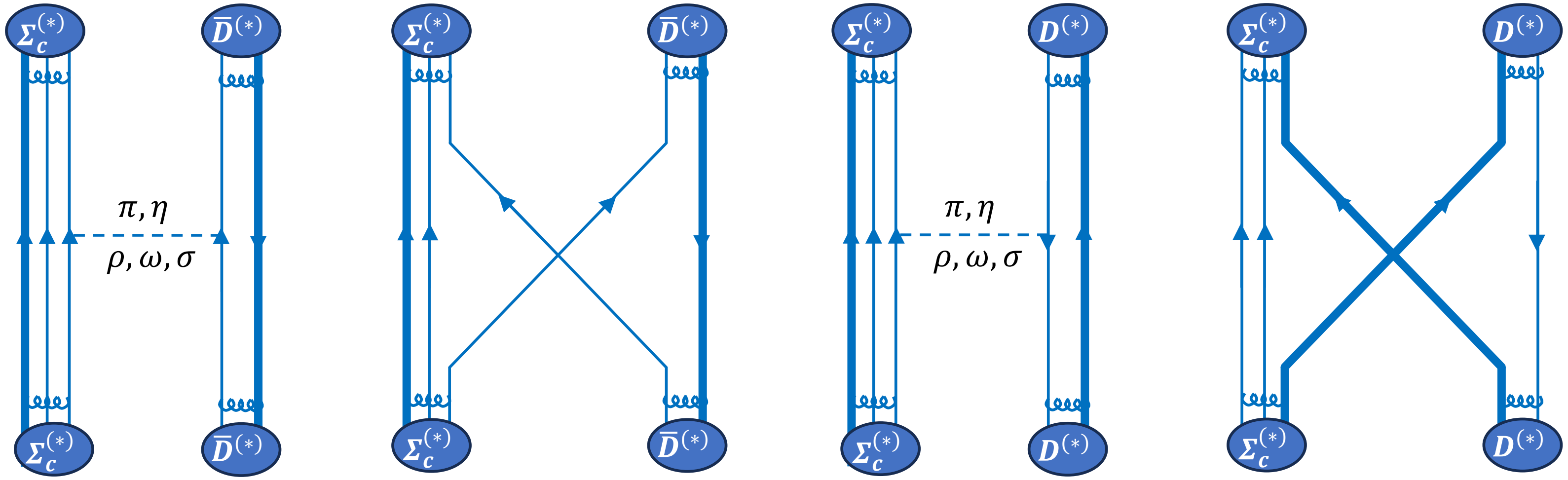
$$V_{qex}(s_1, s_2, J, I) = \frac{16}{27}\left(\boldsymbol{s}_1 \cdot \boldsymbol{s}_2 + \frac{3}{2}\right)\left(\boldsymbol{I}_1 \cdot \boldsymbol{I}_2 + \frac{1}{2}\right)\delta_v$$

$\delta_v > 0$: given no node from ground state wave functions

- The spin-isospin structure of V_{qex} is different from the OBE interaction
- Fo the $I = \frac{1}{2}$ $\Sigma_c\bar{D}^{(*)}$ system, V_{qex} could cancel the spin-depdent interaciton from OPE

G-parity rule

- V_{ex} will not follow the G-parity rule
- Discern the short-range meachnism via $\Sigma_c^{(*)} D^{(*)}$ systems



(a1) (a2) (b1) (b2)

Summary

- P-wave DD^* and $\bar{D}^*D/\bar{D}D^*$
  - ▶ centrifugal barrier for l>0 help form the shape resonance
  - ▶ High-partial wave dynamics sensitive to the long-range interaction
  - ▶ $G(3900)$ and related predictions
- $\boldsymbol{DDD^*}$ bound state and $\boldsymbol{Z_c(3900)}$
  - ▶ Uncertaity of the coupling constants in OBE
  - ▶ Discrepancy of σ-exchange coupling constants
  - ▶ DDD^* bind or not and $Z_c(3900)$ pole position remain uncertain
  - ▶ **$\boldsymbol{DDD^*}$ bound state and $\boldsymbol{Z_c(3900)}$ pole position are closely related**
  - ▶ Data sets **systematically** taking into account the uncertainty from the cutoff
- Short-range interaction of $\boldsymbol{P_{c\bar{c}}}$
  - ▶ Spin-depdent part is too strong to fit the ex. spin splitting of binding energies
  - ▶ Adjust the short-range interaction: quark-exchange model
  - ▶ Different implications from the OBE in open charm systems
- General lessons I learned for **model calculation**:
  - ▶ Use models to test even the wildest ideas
  - ▶ Be cautious when drawing conclusions
  - ▶ Focus on qualitative picture, not decimal precision

Thanks for your attention!

backup

Status of $Z_c(3900)$

Refit Ex. data using amplitudes with exact unitarity.

M_{Z_c} (MeV)	$\Gamma_{Z_c}/2$ (MeV)	Ref.	Final state
3899 ± 6	23 ± 11	[1] (BESIII)	$J/\psi\,\pi$
3895 ± 8	32 ± 18	[2] (Belle)	$J/\psi\,\pi$
3886 ± 5	19 ± 5	[3] (CLEO-c)	$J/\psi\,\pi$
3884 ± 5	12 ± 6	[4] (BESIII)	$\bar{D}^*D$
3882 ± 3	13 ± 5	[5] (BESIII)	$\bar{D}^*D$
3894 ± 6 ± 1	30 ± 12 ± 6	Λ = 1.0 GeV	$J/\psi\,\pi, \bar{D}^*D$
3886 ± 4 ± 1	22 ± 6 ± 4	Λ = 0.5 GeV	$J/\psi\,\pi, \bar{D}^*D$
$3831 \pm 26^{+7}_{-28}$	virtual state	Λ = 1.0 GeV	$J/\psi\,\pi, \bar{D}^*D$
$3844 \pm 19^{+12}_{-21}$	virtual state	Λ = 0.5 GeV	$J/\psi\,\pi, \bar{D}^*D$

solution I: resonance

Solution II: virtual state

Below thresh. 30-40 MeV

M. Albaladejo, F. K. Guo, C. Hidalgo-Duque and J. Nieves, PLB755 (2016), 337-342

Global coupled-channel analysis of $e^+e^- \to c\bar{c}$

TABLE VI. $IJ^{PC} = 11^{+-}$ $D^*\bar{D} - D^*\bar{D}^* - J/\psi\pi - \psi'\pi - h_c\pi - \eta_c\rho$ coupled-channel scattering amplitude poles (unit:MeV). $Z_c(3900)$ and $Z_c(4020)$ are $D^*\bar{D}$ and $D^*\bar{D}^*$ virtual (resonance) poles in this work (PDG [4]).

$E_{Z_c}^{\text{This work}}$	$M_{Z_c}^{\text{PDG}}$	$\Gamma_{Z_c}^{\text{PDG}}$	
$(3837.7\pm7.4)+(19.4\pm1.6)i$	3887.1 ± 2.6	28.4 ± 2.6	$Z_c(3900)$
$(3989.9\pm5.6)+(26.1\pm4.3)i$	4024.1 ± 1.9	13 ± 5	$Z_c(4020)$

Virtual state: below thresh. 30-40 MeV

S. X. Nakamura, X. H.Li, H.P.Peng, Z.T.Sun and X.R.Zhou, PRD112 (2025), 054027

Three-coupled-channel analysis: $D\bar{D}^*$, $J/\psi\,\pi$, and $\rho\eta_c$

	Pole Position	Type	Scheme($\Lambda_{\pi J/\psi}$)
This work	3798.72 - 1.10i	Virtual	1(1.3GeV)
	3798.46 - 1.71i		1(1.5GeV)
	3798.12 - 2.26i		1(1.7GeV)
	3798.27 - 2.02i		2(1.5GeV)
	3797.80 - 2.64i		2(1.7GeV)

Virtual state Pole below threshod 80 MeV

K.Yu, G.J.Wang, J.J.Wu and Z.Yang, PRD110 (2024), 114029

$\pi^+\pi^-$ and $J/\psi\pi^\pm$ mass spectra @ $e^+e^- \to J/\psi\pi^+\pi^-$
D^*D^{*-} mass spectrum @ $e^+e^- \to D^*D^{*-}\,\pi^+$

resonance:

$$(3880.7 \pm 1.7 \pm 22.4) - i(17.9 \pm 0.7 \pm 7.7)\ \text{MeV}\,.$$

Y.H.Chen, M.L.Du and F.K.Guo, , *Sci.China Phys.Mech.Astron.* 67 (2024) 9, 291011

- Cuoff-depedence? Regulator depedence?
- Coupling constants?
- Short-range interaction
- Sigma mass why not physical one?
- Width of the exchanged mesons

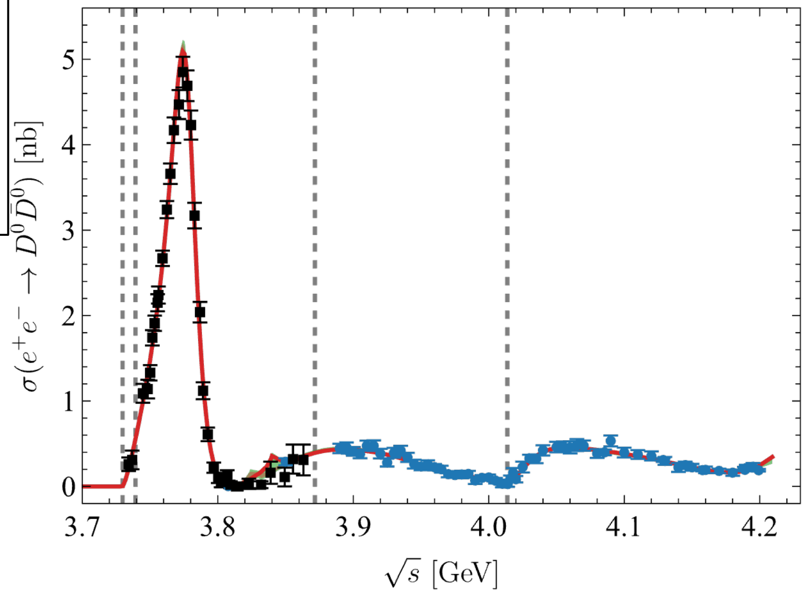
- Annihilation effect?
- Coupled channel effect
- Recoiling effect
- Relativistic effect?

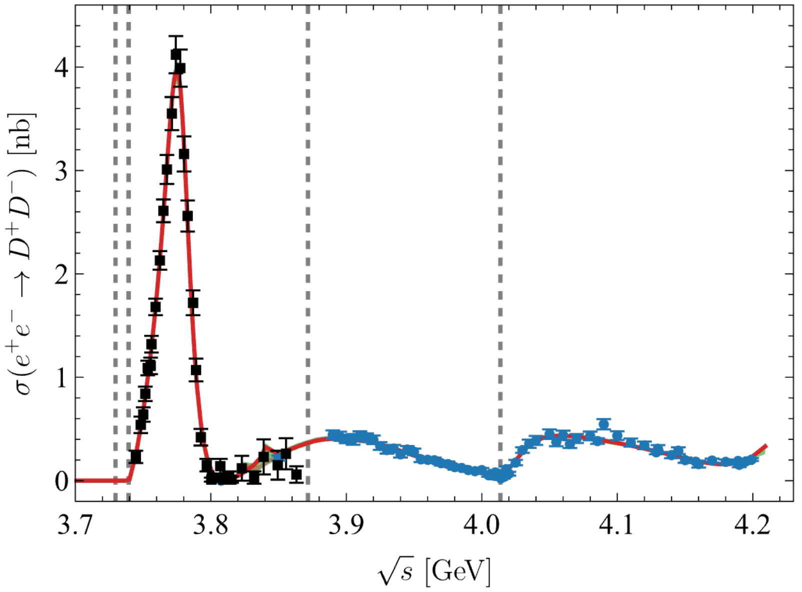
Meson parameters used in the relativistic (energy-independent) momentum space one-boson-exchange potential (OBEPQ)

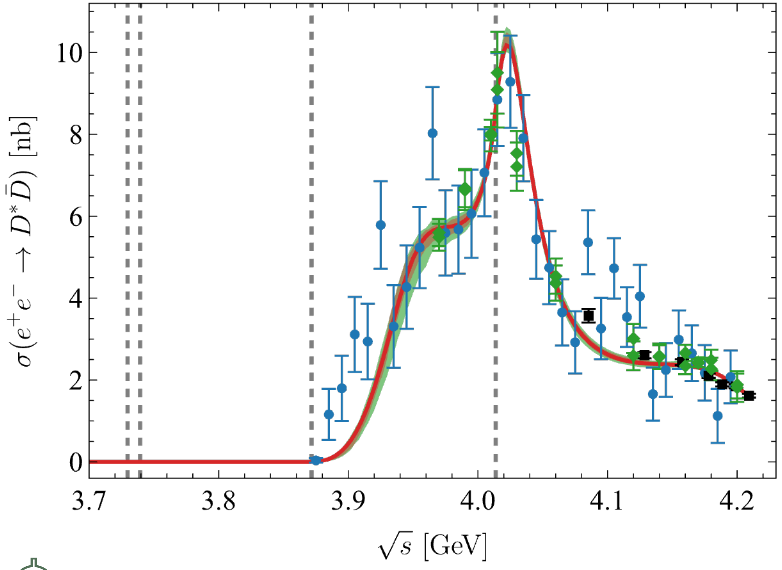
	$g_\alpha^2/4\pi$; $[f_\alpha/g_\alpha]$	$g_\alpha^2/4\pi(k^2=0)$	m_α [MeV]	Λ_α [GeV]	n_α
π	14.6	14.27	138.03	1.3	1
ρ	0.81; [6.1]	0.43	769	2.0	2
η	5	3.75	548.8	1.5	1
ω	20; [0.0]	10.6	782.6	1.5	1
δ	1.1075	0.64	983	2.0	1
σ	8.2797 [a]	7.07	550 [a]	2.0	1

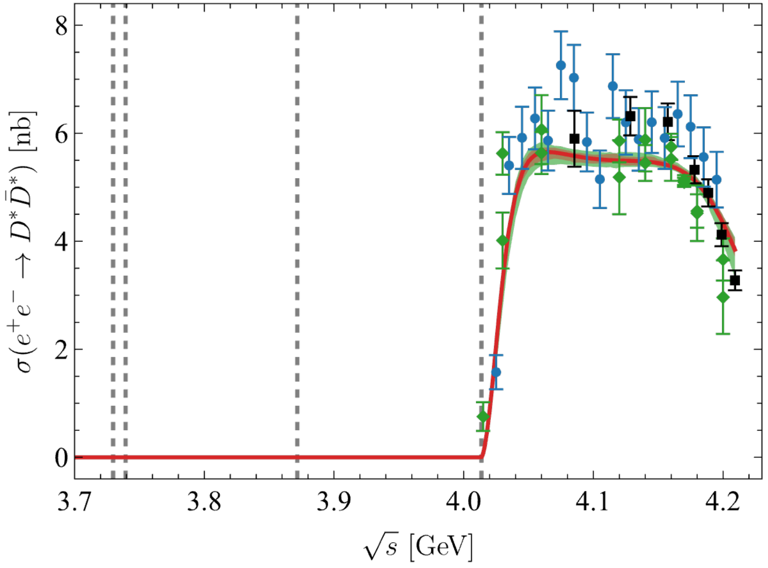
Our fit in the K-matrix formalism

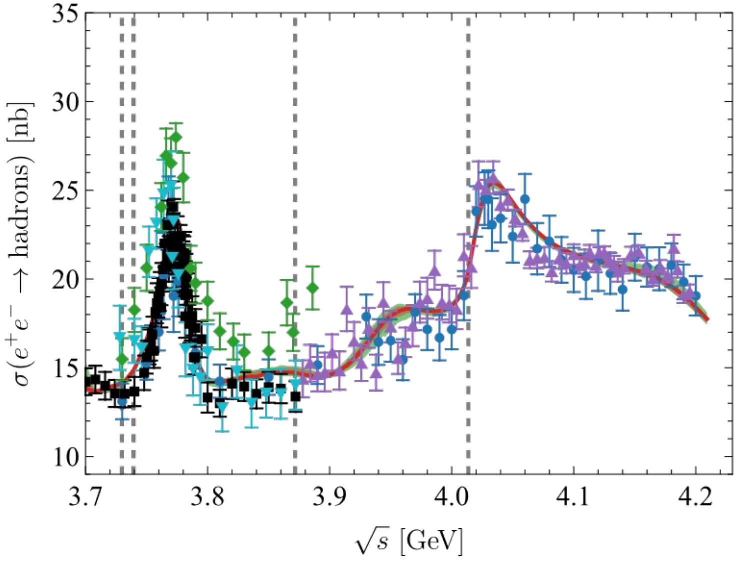
- Repair the three defects
- Our refit results
  - ▶ $\chi^2/\mathrm{dof} = 2.07$



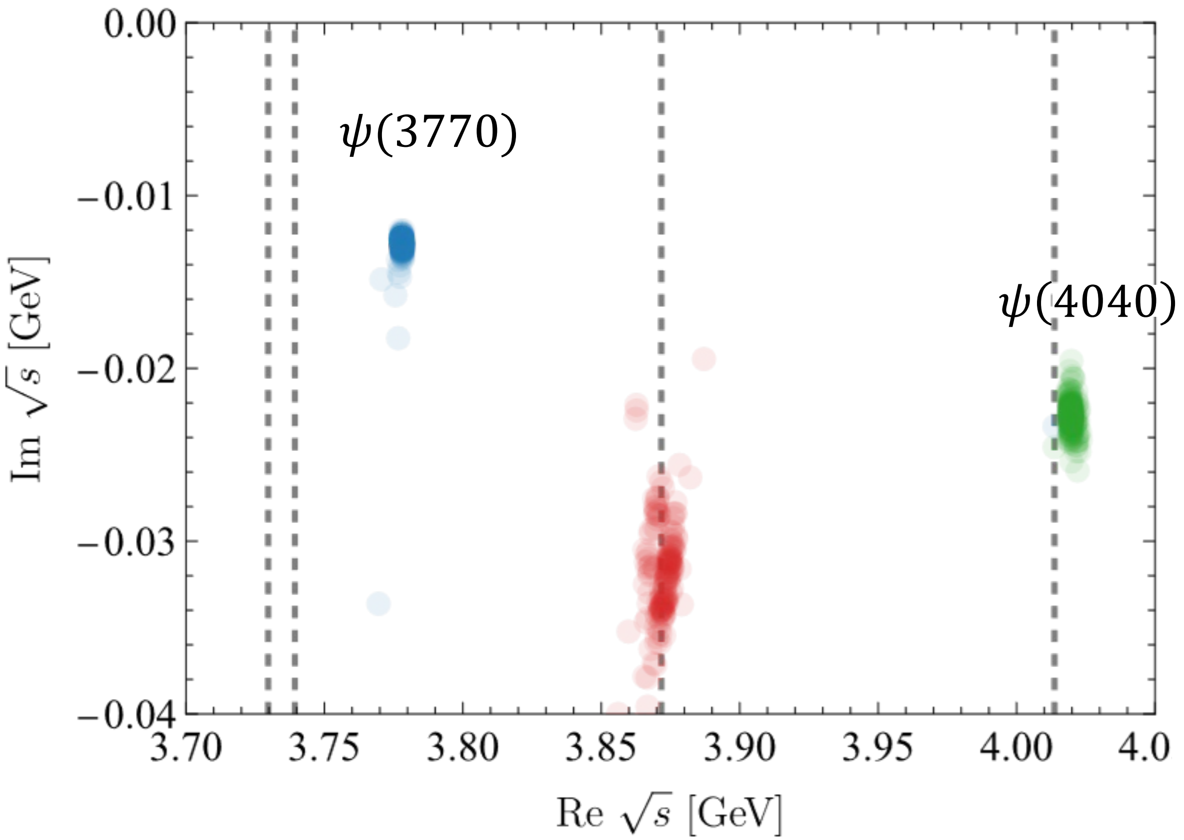








Our fit in the K-matrix formalism



Extra poles

$$3869.2(67) - i29.0(52)$$

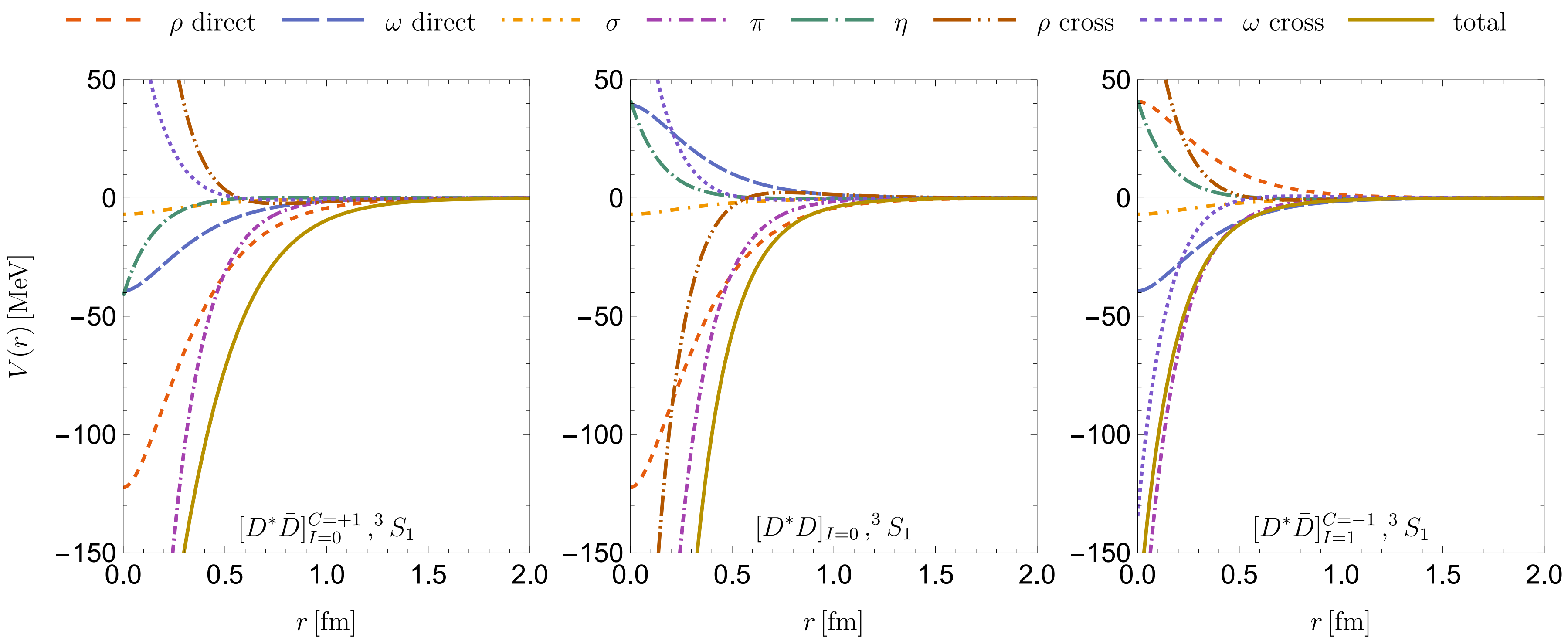
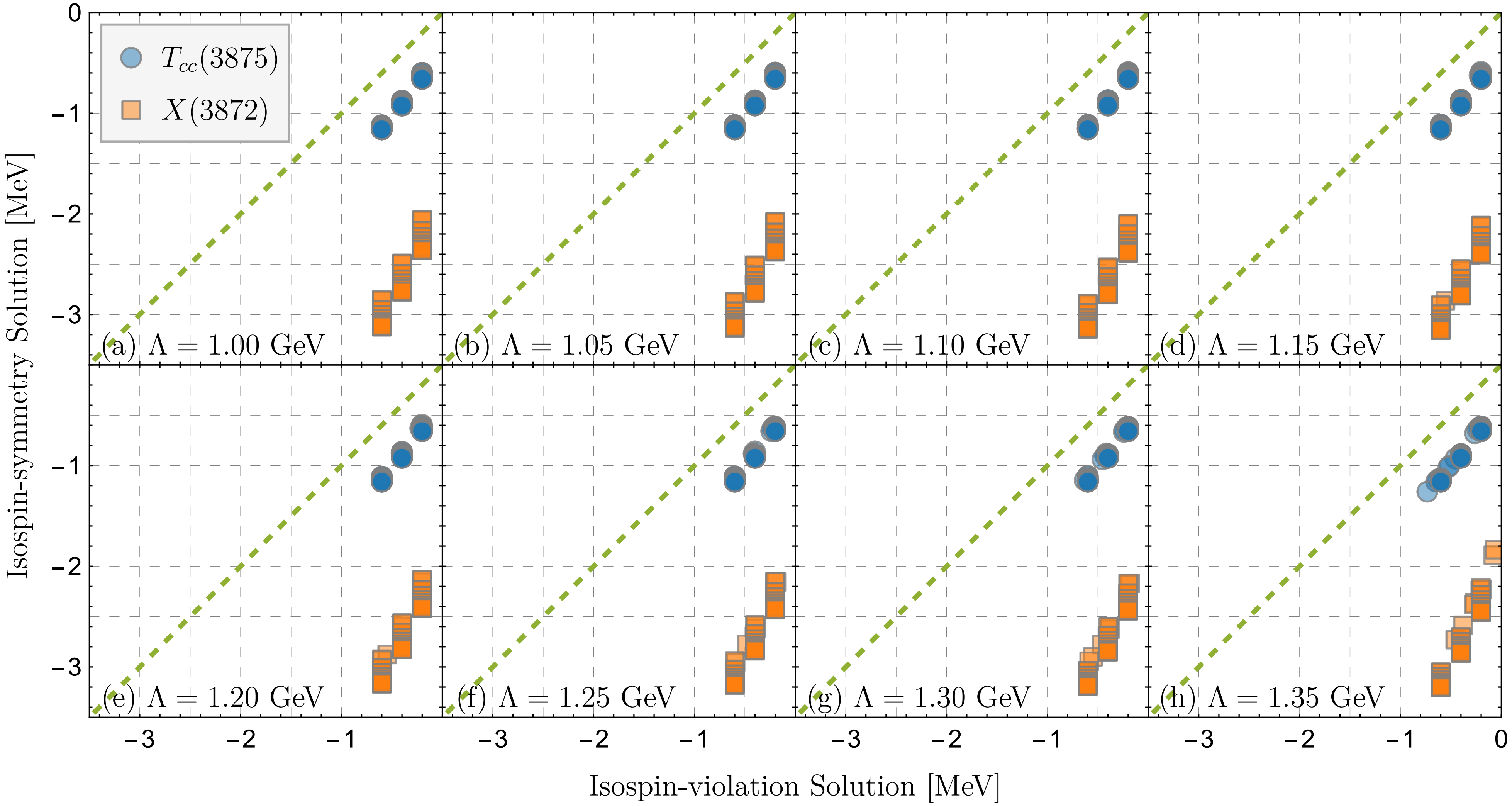
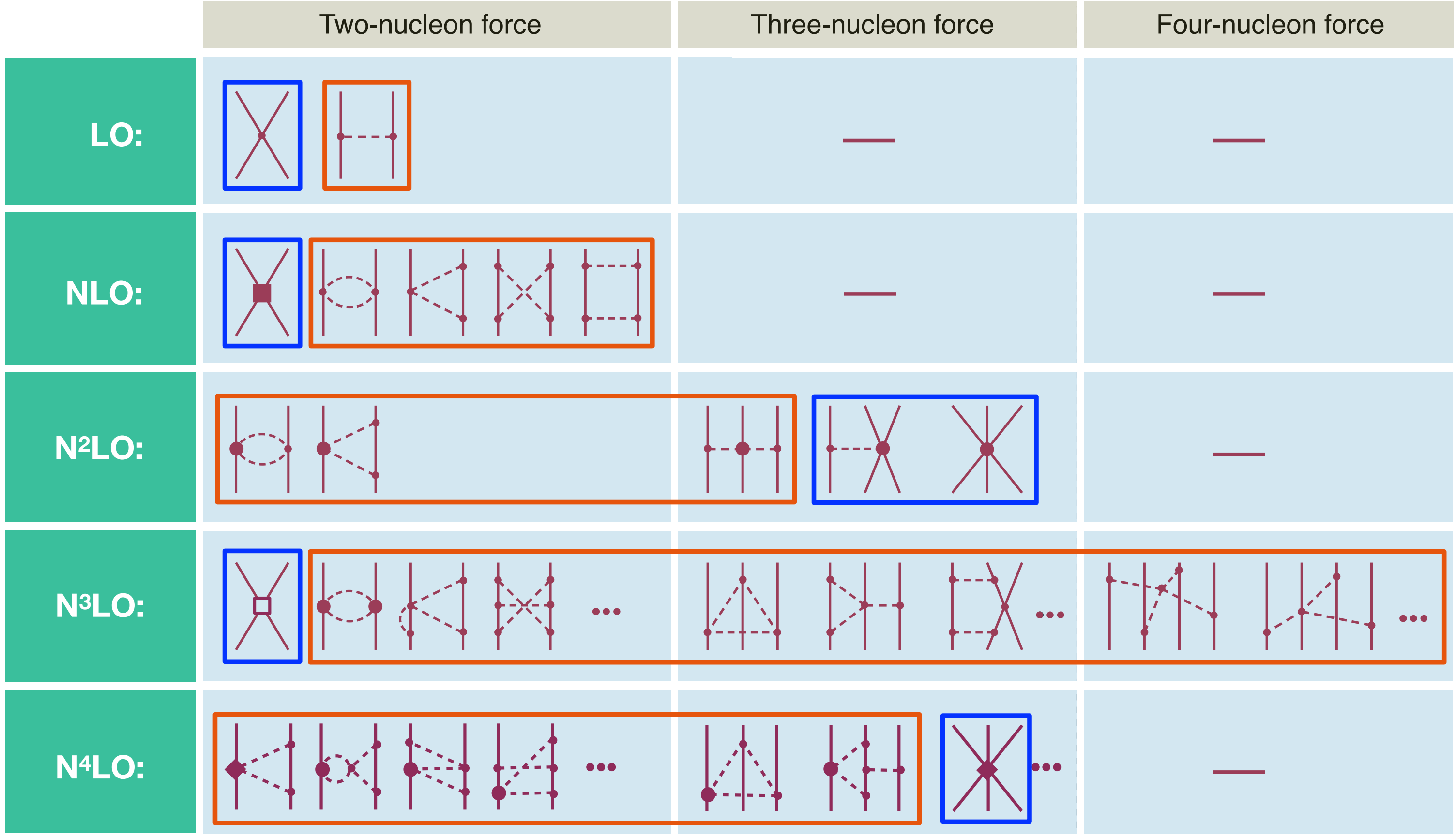


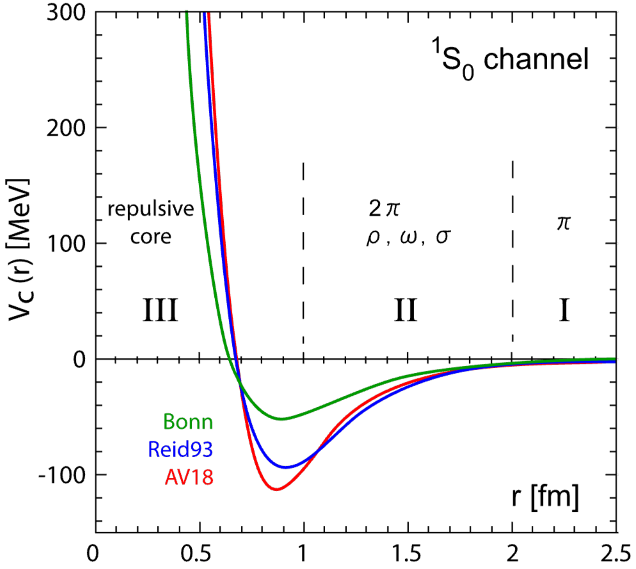
FIG. 2. The coordinate space potentials in the isospin limit for the systems corresponding to the $X(3872)$, $T_{cc}(3875)$, and $Z_c(3900)$ states. The parameter Λ is set to 1.20 GeV, with R_β, R_λ, and R_s fixed at 1.

Isospin violation effect

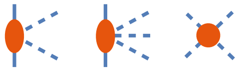


Nuclear force in ChEFT





Chiral dynamics: Long-range interactions are predicted in terms of on-shell amplitudes



Short-range few-N interactions are tuned to experimental data

Chiral nucear force

Phenomenologial nucear force