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Advances in OBE hadronic interactions: Toward higher partial waves, 3-body systems, and beyond

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Based on [PRL133, 241903 \(2024\)](#), [PRD111, 094022 \(2025\)](#), [PRD111, 094015 \(2025\)](#)

- Background

- P-wave DD^* and $\bar{D}^*D/\bar{D}D^*$

合作者：林子阳，王俊璋，程建波，孟璐，朱世琳

- DDD^* bound state and $Z_c(3900)$

合作者：朱海翔，孟璐，马尧，李宁，陈伟，朱世琳

- Short-range interaction of $P_{c\bar{c}}$

合作者：徐儒，孟璐，李宁，陈伟

- Summary

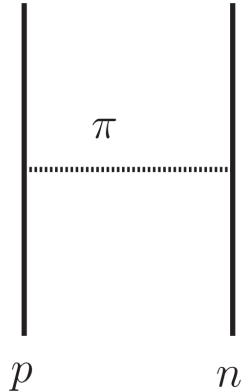


Background





Yukawa Hideki



On the Interaction of Elementary Particles. I.

By Hideki YUKAWA.

(Read Nov. 17, 1934)

and the multiplicative constants. The potential of force between the neutron and the proton should, however, not be of Coulomb type, but decrease more rapidly with distance. It can be expressed, for example, by

$$+ \text{ or } -g^2 \frac{e^{-\lambda r}}{r}, \quad (2)$$

Assuming $\lambda = 5 \times 10^{12} \text{cm}^{-1}$, we obtain for m_π a value 2×10^2 times as large as the electron mass. As such a quantum with large mass and positive or negative charge has never been found by the experiment, the above theory seems to be on a wrong line. We can show, however, that, in the ordinary nuclear transformation, such a quantum can not be emitted into outer space.

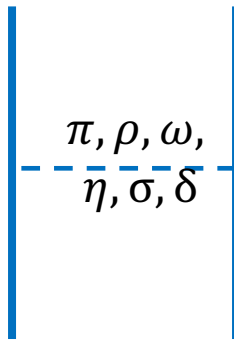
- Try to explain the interaction between nucleons, predicting pion
- Pion mass ~ 200 times of m_e



Proc. Phys.-Math. Soc. Jpn. 1935, 17 (48)



- Interaction via one-boson exchange
- Great success in nuclear force



δ : $a_0(980)$; σ : $f_0(500)$

	Theory	Experiment
ϵ_d (MeV)	2.2246	2.224644 ± 0.000046
P_D (%)	4.38	—
Q_d (fm ²)	0.274^a	0.2860 ± 0.0015
μ_d (μ_N)	0.8548^a	0.857406 ± 0.000001
A_S (fm ^{-1/2})	0.8862	0.8846 ± 0.0016
D/S	0.0262	0.0271 ± 0.0008
r_d (fm)	1.9684	1.9660 ± 0.0068
a_s (fm)	-23.744	-23.748 ± 0.010
r_s (fm)	2.704	2.75 ± 0.05
a_t (fm)	5.424	5.424 ± 0.004
r_t (fm)	1.760	1.759 ± 0.005

R. Machleidt, K. Holinde, and C. Elster, Phys. Rept. **149**, 1 (1987).



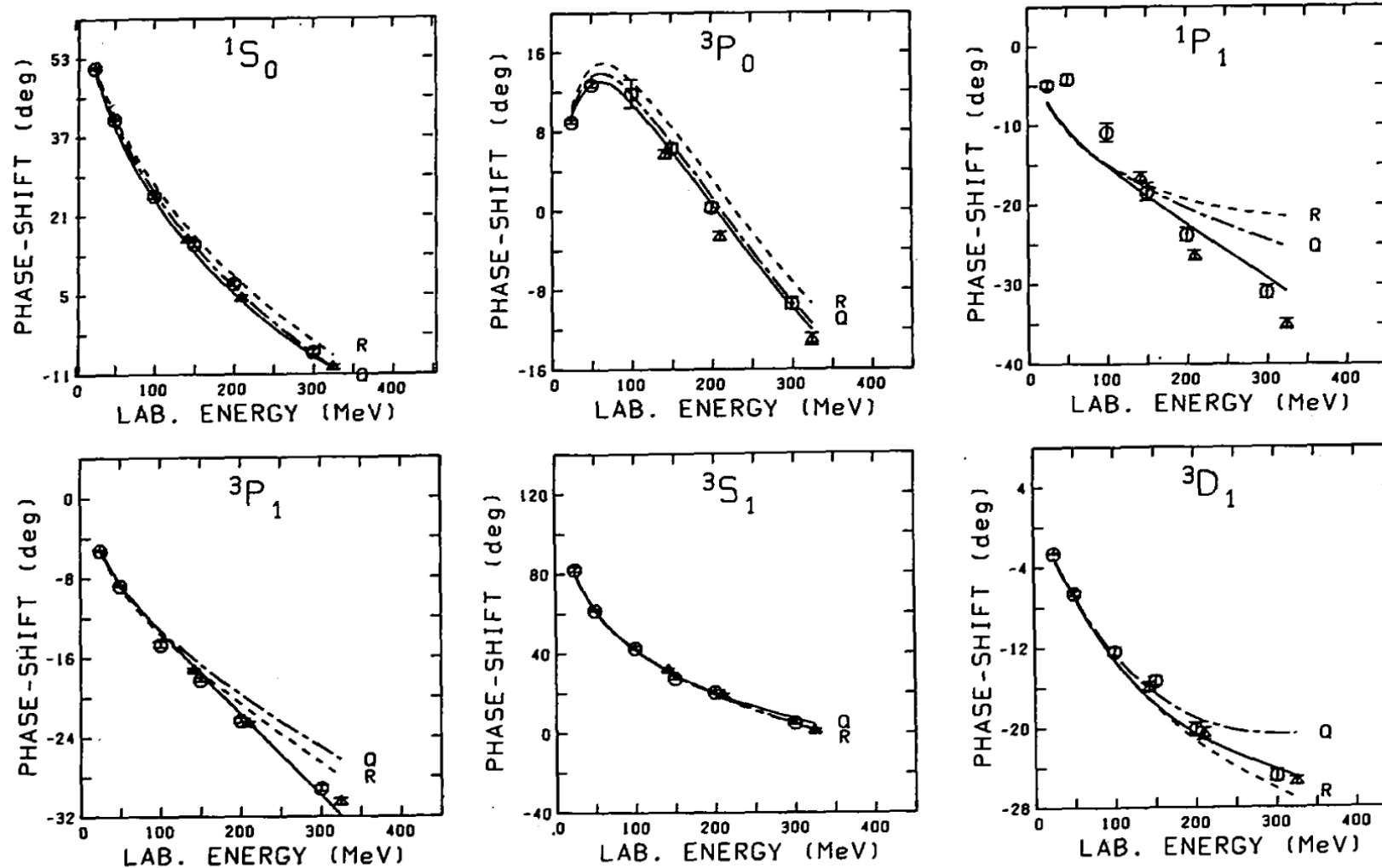
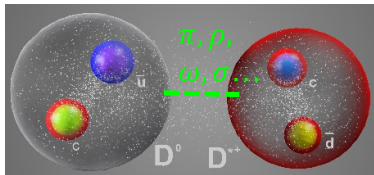
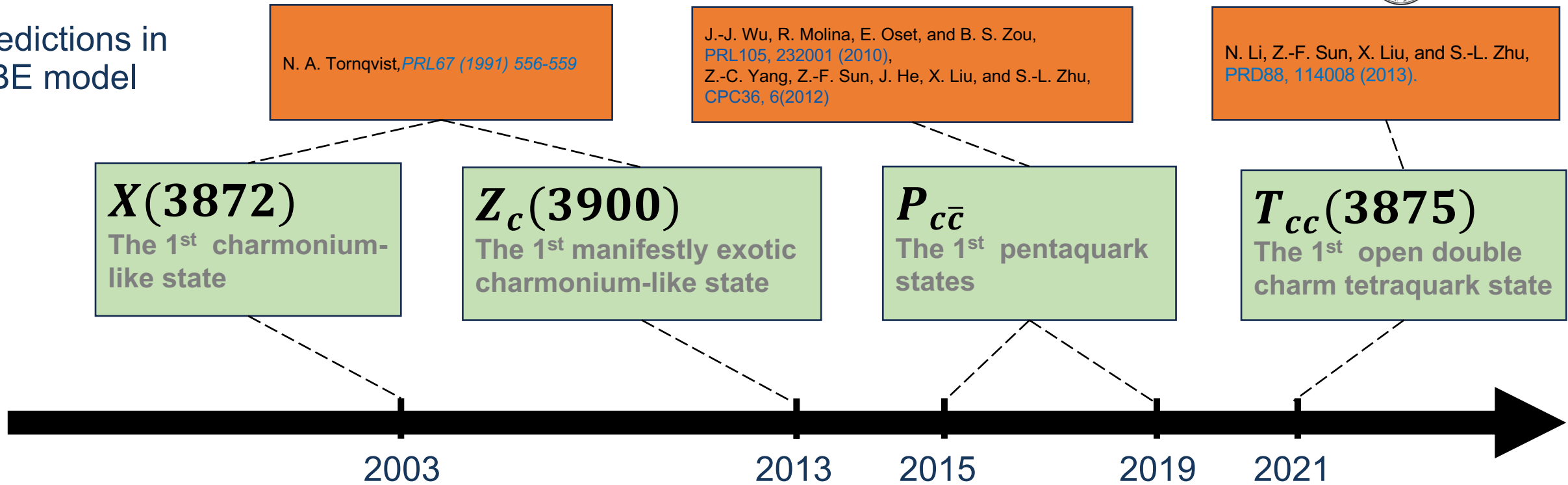


Fig. 17. The description of NN phase shifts by OBEP in comparison to the full model. The solid line represents the result predicted by our full model. The dashed-dotted line refers to the momentum space OBEP(Q) discussed in the section 9.1 with the parameters given in table 5. The dashed line gives the results of the coordinate space OBEP(R) with the parameters given in appendix F, table 14. Error bars as in fig. 15.

R. Machleidt, K. Holinde, and C. Elster, Phys. Rept. **149**, 1 (1987).

Predictions in OBE model

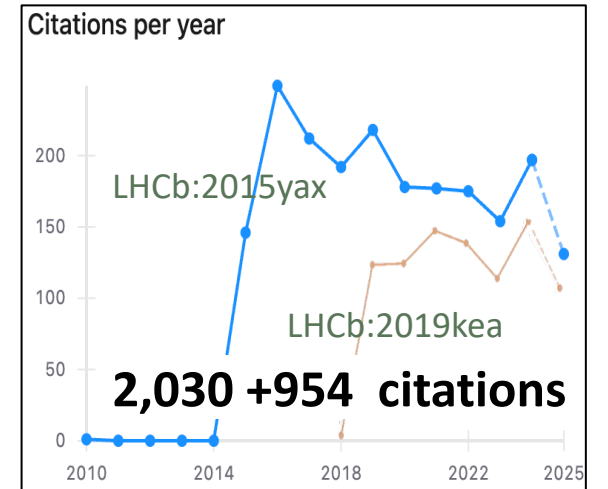
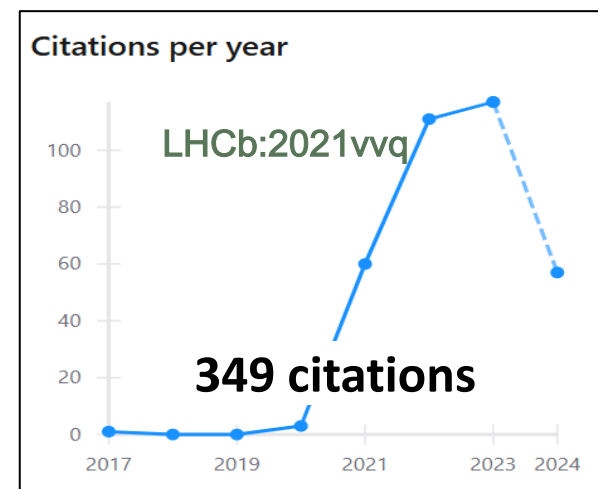
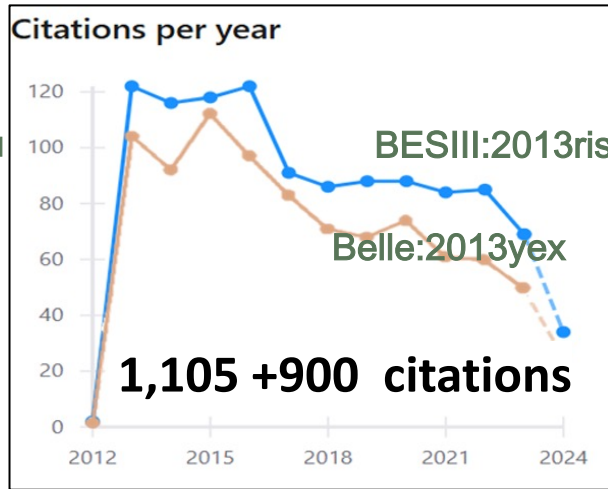
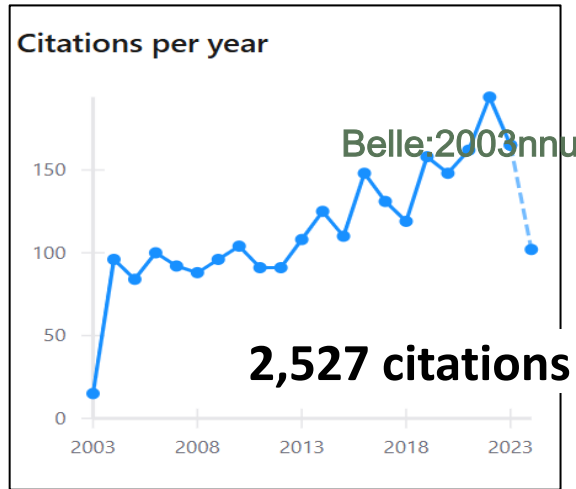


Hadronic molecules
in OBE

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-

"Superstars"

	Quark contents	$I^G(J^{PC})$	Threshold	ΔM [MeV]	Γ [MeV]
X(3872)	$q\bar{q}c\bar{c}/c\bar{c}$	$0^+(1^{++})$	$D^0\bar{D}^{0*}$	$0.0068^{+0.1655}_{-0.17000}$	$0.380^{+0.412}_{-0.322}$
	The 1 st charmonium-like state				BESIII:2023hmt
$Z_c(3900)$	$q\bar{q}c\bar{c}$	$1^+(1^{+-})$	$D\bar{D}^*$	11.3 ± 2.6	28.4 ± 2.6
	The 1 st manifestly exotic charmonium-like state				PDG
$T_{cc}(3985)$	$\bar{q}\bar{q}cc$	?	$D^{*+}D^0$	$-0.360^{+0.040}_{-0.040}$	$0.048^{+0.002}_{-0.014}$
	The 1 st open double charm tetraquark state				LHCb:2021auc
$P_{c\bar{c}}$ states	$qqqc\bar{c}$	?	$\Sigma_c\bar{D}^{(*)}$	$-5 \sim -20$	~ -10
	The 1 st pentaquark states				LHCb:2015yax



- Exchanged mesons
 - ▶ Hidden local gauge symmetry: vector-meson exchange
 - ▶ $\pi, \rho, \omega, \sigma, \dots$: e.g. Bonn potential
- Determine coupling constants: Exp., Lattice, Models

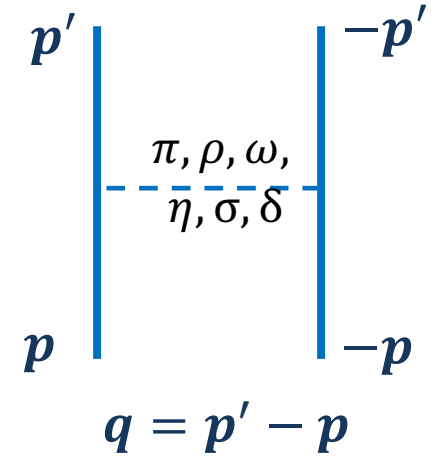
- Regulator and cutoff

$$\frac{1}{u^2 + q^2} \rightarrow \frac{e^{-ur}}{4\pi r},$$

$$\frac{q^2}{u^2 + q^2} = 1 - \frac{u^2}{u^2 + q^2} \rightarrow \delta^3(\mathbf{r}) - \frac{u^2 e^{-ur}}{4\pi r},$$

$$\frac{\mathbf{q}_i \mathbf{q}_j}{u^2 + q^2} \rightarrow -\frac{e^{-ur}}{4\pi r} \left(\frac{u^2}{3} + \frac{u}{r} + \frac{1}{r^2} \right) T_{ij} + \frac{1}{3} \left(\delta^3(\mathbf{r}) - \frac{u^2 e^{-ur}}{4\pi r} \right) \delta_{ij},$$

- ▶ Singular terms: $\delta^3(\mathbf{r})$ and $1/r^3$
 - ▶ Short-range: structure of the hadron become important
- Coupled channel or not?
 - ▶ Hard to incorporate properly, relativistic effect
 - ▶ Not important? $M_D^* - M_D \sim 130 \text{ MeV}$, $M_{\Sigma_c^*} - M_{\Sigma_c} \sim 70 \text{ MeV}$, Neglected in our works



$$V(\mathbf{p}', \mathbf{p}) \rightarrow V(\mathbf{p}', \mathbf{p}) \frac{\Lambda^2}{\Lambda^2 + p^2} \frac{\Lambda^2}{\Lambda^2 + p'^2}$$

$$V(\mathbf{p}', \mathbf{p}) \rightarrow V(\mathbf{p}', \mathbf{p}) \left(\frac{\Lambda^2 - u^2}{\Lambda^2 + q^2} \right)^2$$

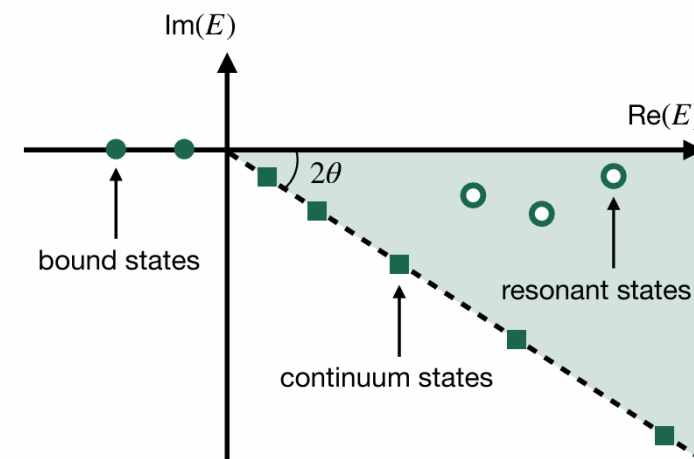
Other options...



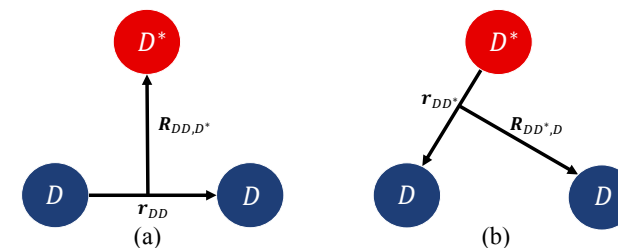
- Momentum space:
 - ▶ **Discretize the momenta**
 - ▶ **Deformed contour:** resonance, virtual state

$$\frac{p^2}{2\mu}\psi(p) + \int_C \frac{dq}{(2\pi)^3} q^2 V_{l=0}(p, q) \psi(q) = E\psi(p),$$

- Coordinate space
 - ▶ Basis expansion method, Finite element method
 - ▶ Resonant: complex scaling method...
- 3-body/4-body systems
 - ▶ Faddeev equation
 - ▶ **Gaussian expansion method+ Complex scaling**



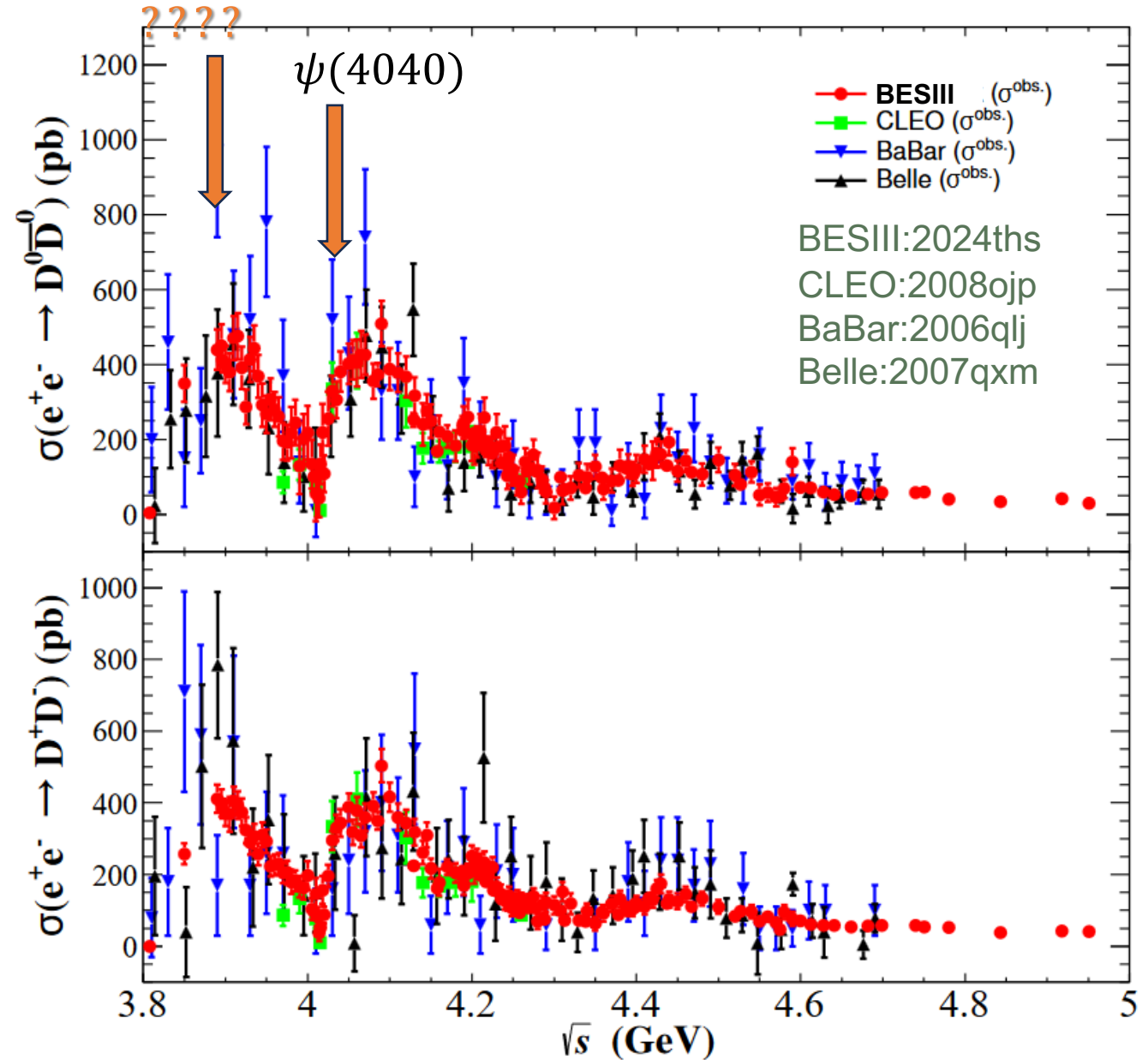
$$\phi_{nlm}(\mathbf{r}) = \sqrt{\frac{2^{l+5/2}}{\Gamma(l + \frac{3}{2}) r_n^3}} \left(\frac{r}{r_n}\right)^l e^{-\frac{r^2}{r_n^2}} Y_{lm}(\hat{r}),$$



P-wave DD^* and $\bar{D}^*D/\bar{D}D^*$



- An enhancement close to 3.9 GeV
 - ▶ $D^*\bar{D}$ threshold
 - ▶ New $D^*\bar{D}$ resonance?
 - ▶ Quantum number $J^{PC} = 1^{--}$, from virtual photon
 - ▶ P-wave $D^*\bar{D}$ state?



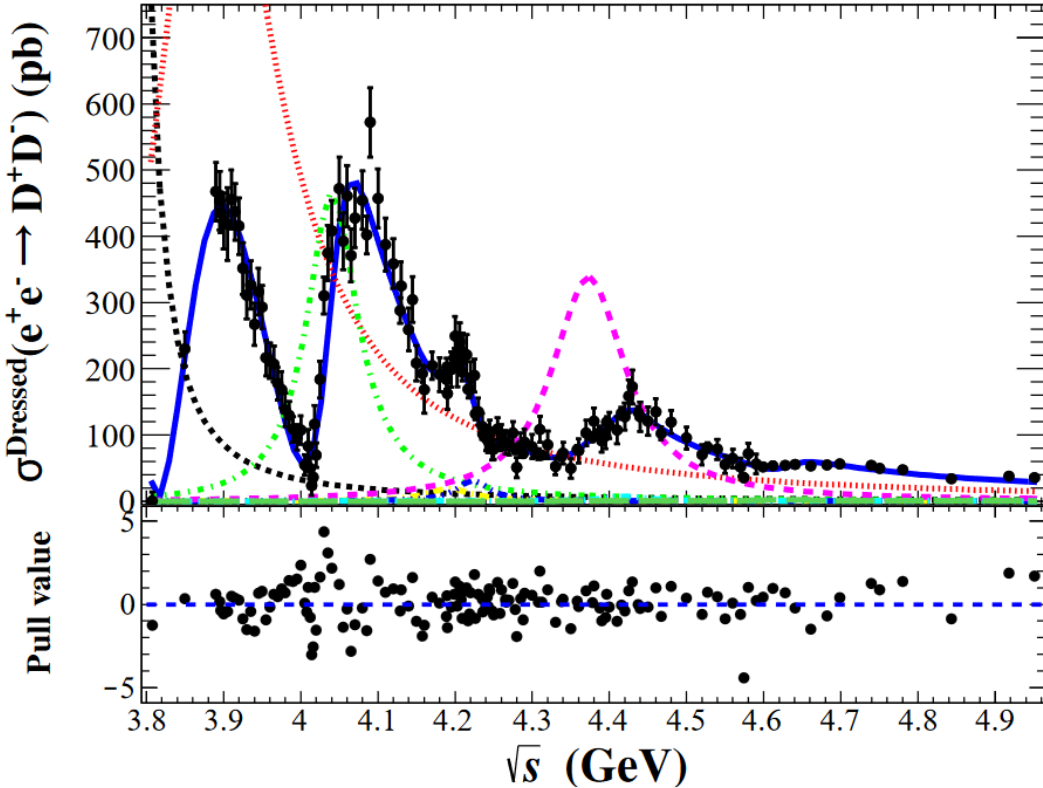
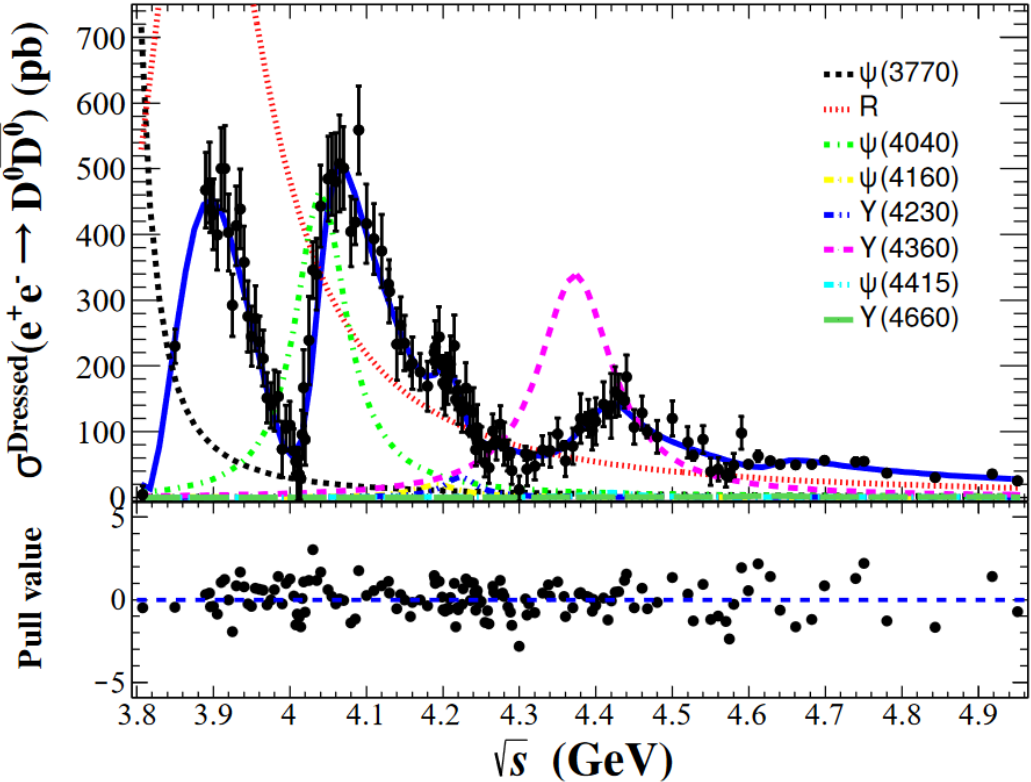
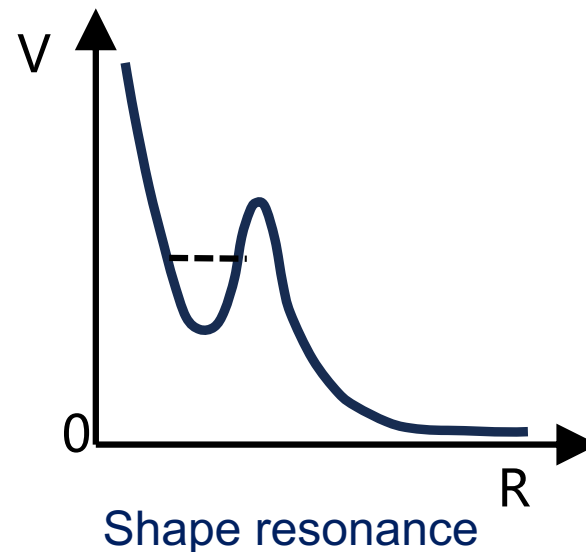
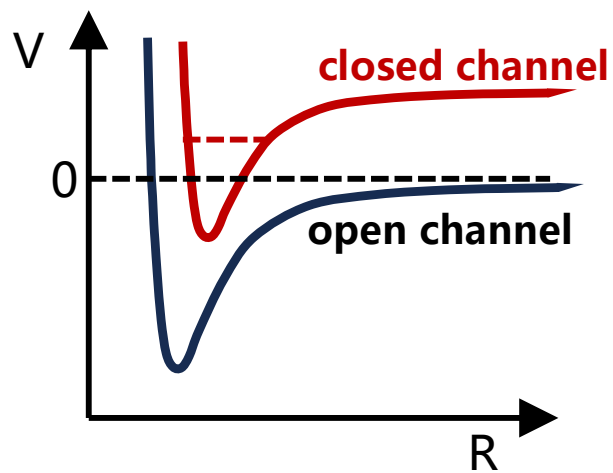


TABLE I. Fit results of the Born cross section, where the first uncertainties are the statistical and the second are systematic and S denotes the significance.

BESIII:2024ths

$e^+e^- \rightarrow DD$								
Resonance	$\psi(3770)$	R	$\psi(4040)$	$\psi(4160)$	$Y(4230)$	$Y(4360)$	$\psi(4415)$	$Y(4660)$
Mass (MeV/ c^2)	3773.7 (fixed)	$3872.5 \pm 14.2 \pm 3.0$	4039 (fixed)	4191 (fixed)	4222.5 (fixed)	4374 (fixed)	4421 (fixed)	4630 (fixed)
Width (MeV/ c^2)	87.6 (fixed)	$179.7 \pm 14.1 \pm 7.0$	80 (fixed)	70 (fixed)	48 (fixed)	118 (fixed)	62 (fixed)	72 (fixed)
$\Gamma_{ee}B$ (eV)	95-106	202-292	41-44	1-2	1-2	50-144	0-2	0-1
S(σ)	10	> 20	13	7	11	11	4	8
$\chi^2/\text{d.o.f} = 346/275$			p-value = 0.002					

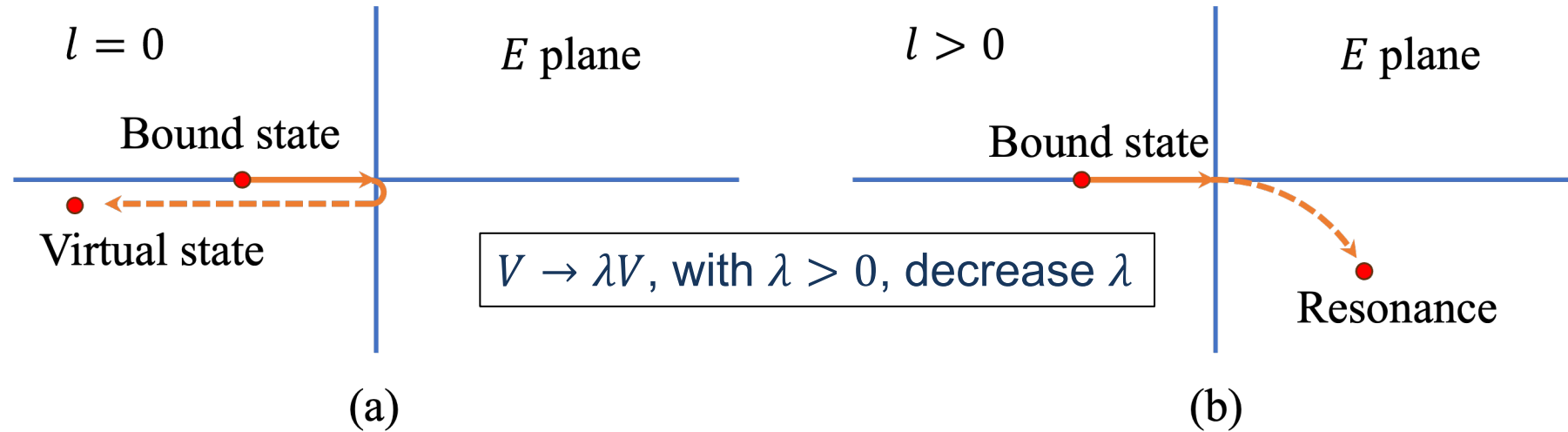


- Resonance: state with finite lifetime
- Feshbach resonance
 - ▶ E.g. P_c states could be bound states of $\Sigma_c \bar{D}^{(*)}$
 - ▶ Considering the $J/\psi p$ channel: resonance
- Shape resonance
 - ▶ Barrier
- Feshbach or shape? depending on scheme



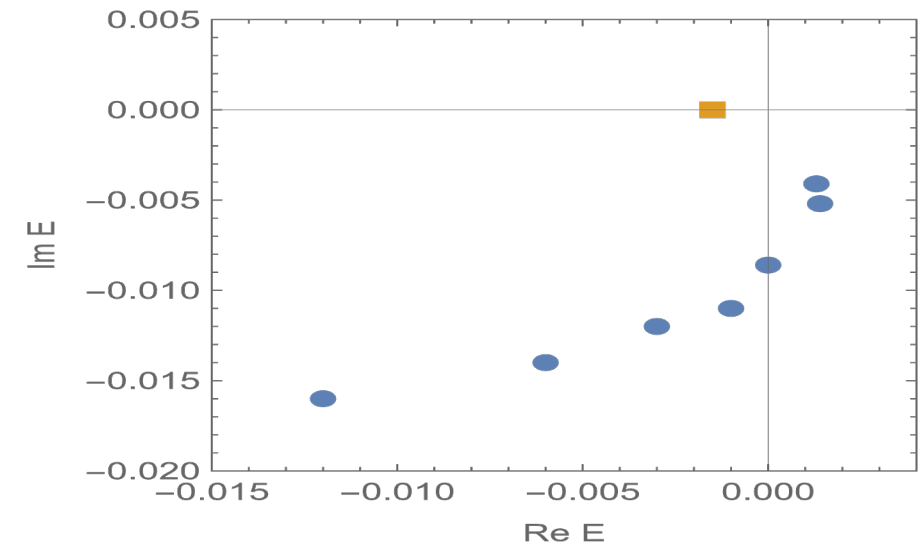
- From bound state to...

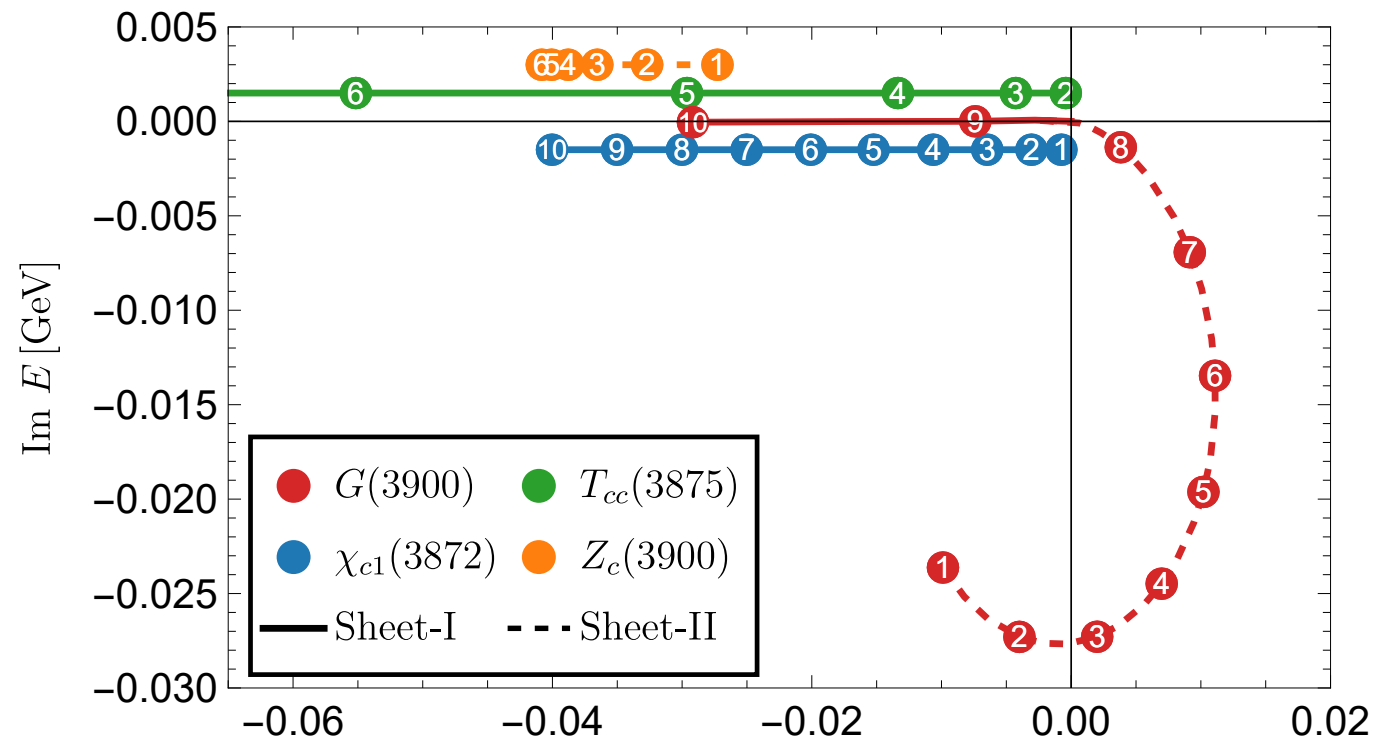
Taylor, scattering theory textbook sec. 13-b P245



- 3P_0 NN resonance pole: $-0.012 - i0.016$
 - Were seldom investigated
 - Hard to detect: no lower coupled-channel

Potential barrier: centrifugal barrier for $l > 0$





$\pi, \eta,$
 ρ, ω, σ

- Vary Λ from 0.4 to 1.3 GeV (①–⑩)
- Constrained by $X(3872)$, $Z_c(3900)$, and $T_{cc}(3875)$
- $X(3872)$ and $T_{cc}(3875)$: bound states
- $Z_c(3900)$: virtual state
- $G(3900)$: resonance $\rightarrow$ bound state



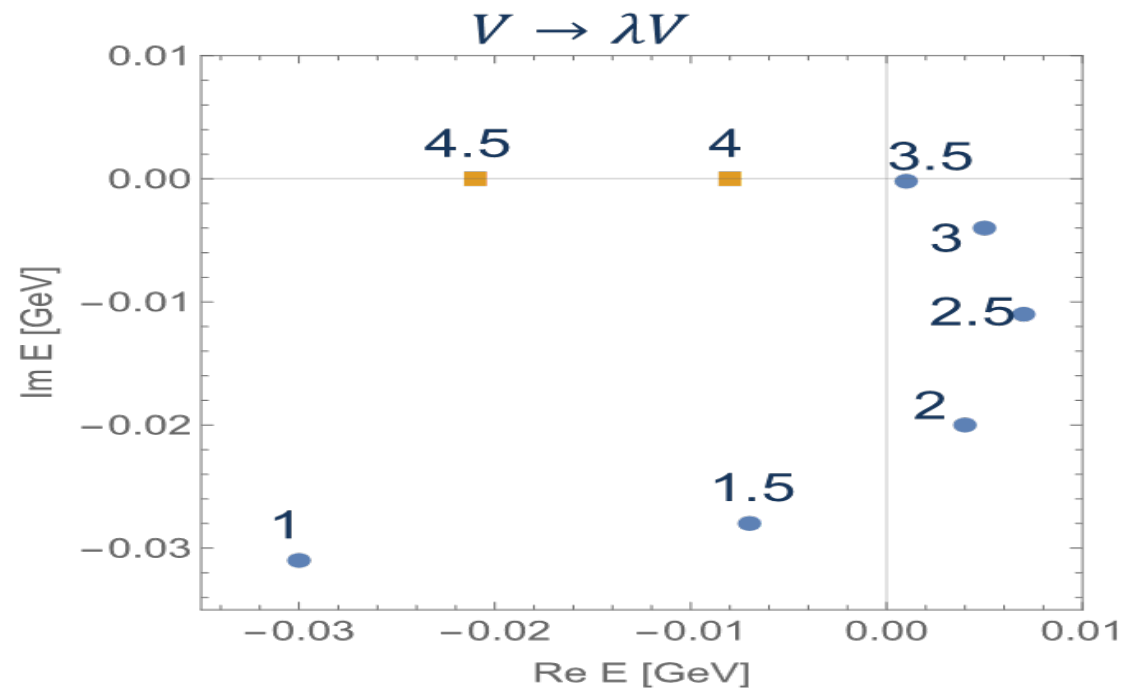
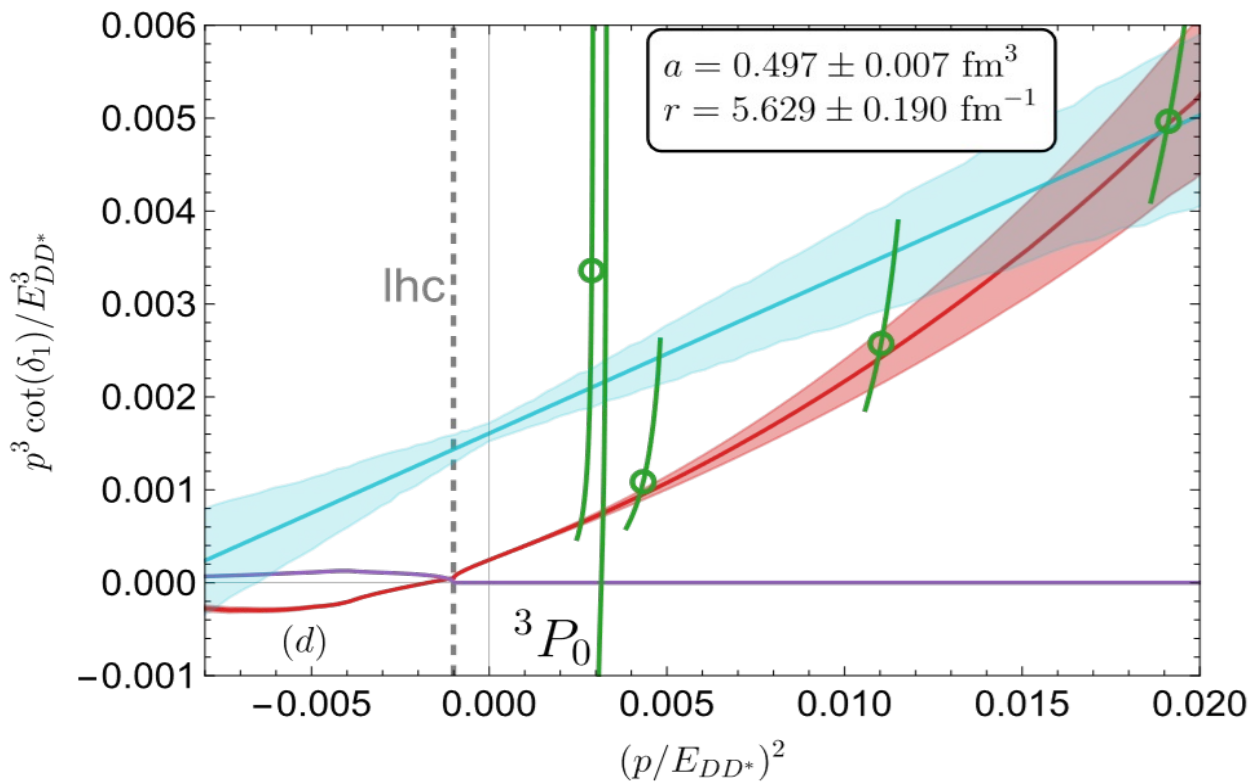
		$D\bar{D}^*, C = +$		$D\bar{D}^*, C = -$		DD^*	
		$I = 0$	$I = 1$	$I = 0$	$I = 1$	$I = 0$	$I = 1$
$\Lambda = 0.5\text{GeV}$	$1^+(^3S_1)$	$-3.1^B, \chi_{c1}(3872)$	-	-1.60^B ②	$-35.6^V, Z_c(3900)$	$-0.41^B, T_{cc}(3875)$	-
	$0^-(^3P_0)$	$-1.5 - 14.5i$ ③	-	-	-	$-9.6 - 9.7i$ ④	-
	$1^-(^3P_1)$	-	-	$-4.0 - 27.3i, G(3900)$ ①	-	$-31.7 - 70.6i$	-
	$2^-(^3P_2)$	$-42.6 - 39.4i$	-	$-21.3 - 50.7i$	-	$-37.8 - 40.9i$	-

B: bound state, V: virtual state

- ① Precision measurement of $e^+e^- \rightarrow \bar{D}D^*$ close to threshold
- ② Hidden charm final state: $\eta_c\omega, J/\psi\eta, J/\psi\pi\pi$
- ③ Hidden charm final state: $J/\psi\omega, \eta_c\pi\pi, \chi_{c1}\pi\pi$
- ④ Final state $DD\pi$



- Lattice QCD: 3P_0 DD^* resonance pole: $-0.030 - i0.031$



L. Meng, V. Baru, E. Epelbaum, A. A. Filin, and A. M. Gasparyan, PRD**109**, L071506 (2024).



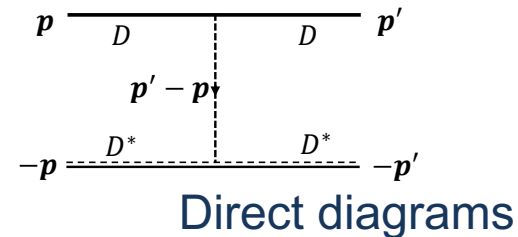
DDD^* bound state and $Z_c(3900)$



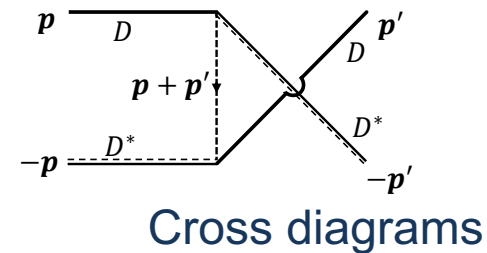
Uncertain coupling constants in OBE

- 4 coupling constants in SU(3) flavor and heavy quark symmetry
 - ▶ Pseudoscalar: extracted from $D^* \rightarrow D\pi$
 - ▶ Three remains unknown: λ, β, g_s
- Determine the coupling
 - ▶ vector meson dominance, Σ -model, light-cone sum rule, LQCD
- Quantitatively different results due to uncertain coupling
 - ▶ Different by 20 times
 - ▶ Both models give the loosely bound $X(3872)$ and $T_{cc}(3875)$
 - ▶ However, different results for $Z_c(3900)$ and DDD^* !!!

Not in unified framework



g_s for σ
 β for ρ, ω



g_a for π, η ;
 λ for ρ, ω

	Model-I	Model-II
$DDD^*, S = 1, I = 1/2$	unbound	binding energy ~ 1 MeV
$Z_c(3900)$	~ -40 MeV (virtual)	~ -5 MeV (virtual)

V	C _{coupling}			
		Model-I[53, 54]	Model-II[65, 84]	
V_ρ^D	$\frac{\beta^2 g_v^2}{2}$	13.62	$2g_\rho^2$	13.52
V_ω^D	$\frac{\beta^2 g_v^2}{2}$	13.62	$2g_\omega^2$	13.52
V_σ^D	g_s^2	0.58	g_σ^2	11.56
V_π^C	$\frac{g_a^2}{f_\pi^2}$	19.15	$\frac{g_a^2}{f_\pi^2}$	20.66
V_η^C	$\frac{g_a^2}{f_\pi^2}$	19.15	0	0
V_ρ^C	$2\lambda^2 g_v^2$	21.10	$\frac{f_\rho^2}{2M^2}$	19.64
V_ω^C	$2\lambda^2 g_v^2$	21.10	$\frac{f_\omega^2}{2M^2}$	19.64

- Model-I { [53] N. Li and S.L.Zhu, PRD86, 074022 (2012)
 [54] N. Li, Z.-F. Sun, X.Liu and S.-L. Zhu, PRD88, 114008 (2013)
- Model-II { [65] T.-W. Wu, Y.-W. Pan, M.-Z. Liu, S.-Q. Luo, L.-S. Geng, X. Liu, PRD105, L031505 (2022)
 [84] M.-Z. Liu, T.-W. Wu, M.P. Valderrama, J.-J. Xie and L.-S. Geng, PRD99, 094028 (2019)

- Use the model-I as the baseline

$$\lambda \rightarrow \lambda R_\lambda, \quad \beta \rightarrow \beta R_\beta, \quad g_s \rightarrow g_s R_s$$

- Chosse 63 sets of pole positions

$$\begin{aligned} X(3872) &: (-1) \times (0.2, 0.4, 0.6)^B \text{ MeV}, \\ T_{cc}(3875) &: (-1) \times (0.2, 0.4, 0.6)^B \text{ MeV}, \\ Z_c(3900) &: (-1) \times (5, 10, 15, 20, 25, 30, 35)^V \text{ MeV}, \\ &\text{Large uncertainty in Ex.} \end{aligned}$$

- 8 different cutoffs

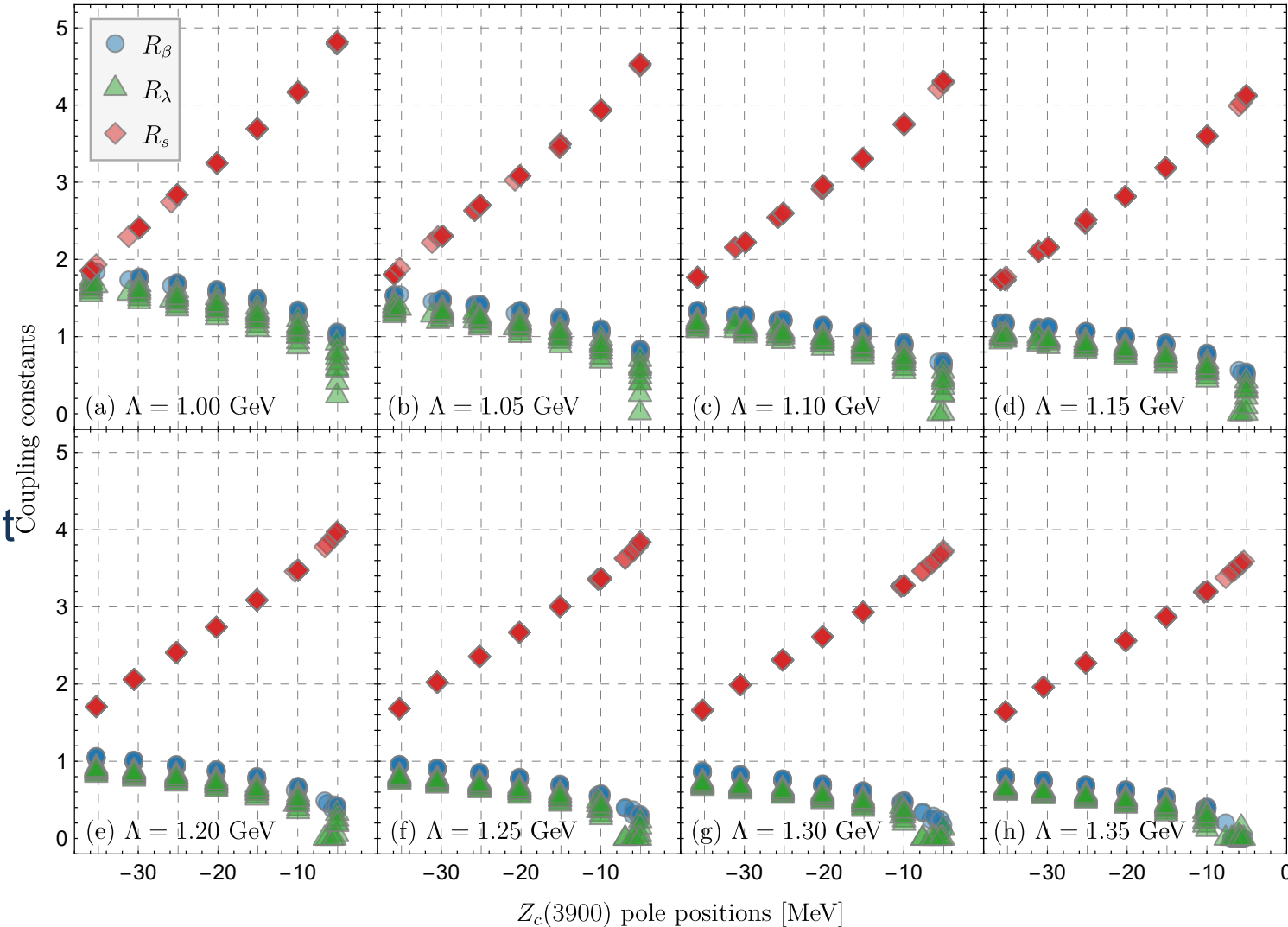
- ▶ Systematically take into account the uncertainty from the cutoff

- R_s : dramatic variation

- ▶ Related to $Z_c(3900)$ ples

- New sets of coupling constants

- ▶ Used for other systems



Refit Ex. data using amplitudes with exact unitarity.

solution I: resonance

Solution II: virtual state

Below thresh. 30-40 MeV

M. Albaladejo, F. K. Guo, C. Hidalgo-Duque and J. Nieves, PLB755 (2016), 337-342

M_{Z_c} (MeV)	$\Gamma_{Z_c}/2$ (MeV)	Ref.	Final state
3899 ± 6	23 ± 11	[1] (BESIII)	$J/\psi \pi$
3895 ± 8	32 ± 18	[2] (Belle)	$J/\psi \pi$
3886 ± 5	19 ± 5	[3] (CLEO-c)	$J/\psi \pi$
3884 ± 5	12 ± 6	[4] (BESIII)	$\bar{D}^* D$
3882 ± 3	13 ± 5	[5] (BESIII)	$\bar{D}^* D$
$3894 \pm 6 \pm 1$	$30 \pm 12 \pm 6$	$\Lambda = 1.0$ GeV	$J/\psi \pi, \bar{D}^* D$
$3886 \pm 4 \pm 1$	$22 \pm 6 \pm 4$	$\Lambda = 0.5$ GeV	$J/\psi \pi, \bar{D}^* D$
$3831 \pm 26^{+7}_{-28}$	virtual state	$\Lambda = 1.0$ GeV	$J/\psi \pi, \bar{D}^* D$
$3844 \pm 19^{+12}_{-21}$	virtual state	$\Lambda = 0.5$ GeV	$J/\psi \pi, \bar{D}^* D$

Global coupled-channel analysis of $e^+e^- \rightarrow c\bar{c}$

TABLE VI. $IJ^{PC} = 11^{+-}$ $D^* \bar{D} - D^* \bar{D}^* - J/\psi \pi - \psi' \pi - h_c \pi - \eta_c \rho$ coupled-channel scattering amplitude poles (unit:MeV). $Z_c(3900)$ and $Z_c(4020)$ are $D^* \bar{D}$ and $D^* \bar{D}^*$ virtual (resonance) poles in this work (PDG [4]).

$E_{Z_c}^{\text{This work}}$	$M_{Z_c}^{\text{PDG}}$	$\Gamma_{Z_c}^{\text{PDG}}$	
$(3837.7 \pm 7.4) + (19.4 \pm 1.6)i$	3887.1 ± 2.6	28.4 ± 2.6	$Z_c(3900)$
$(3989.9 \pm 5.6) + (26.1 \pm 4.3)i$	4024.1 ± 1.9	13 ± 5	$Z_c(4020)$

Virtual state: below thresh. 30-40 MeV

S. X. Nakamura, X. H.Li, H.P.Peng, Z.T.Sun and X.R.Zhou, PRD112 (2025), 054027

Three-coupled-channel analysis: $D\bar{D}^*$, $J/\psi \pi$, and $\rho\eta_c$

	Pole Position	Type	Scheme($\Lambda_{\pi J/\psi}$)
This work	$3798.72 - 1.10i$	Virtual	$1(1.3\text{GeV})$
	$3798.46 - 1.71i$		$1(1.5\text{GeV})$
	$3798.12 - 2.26i$		$1(1.7\text{GeV})$
	$3798.27 - 2.02i$		$2(1.5\text{GeV})$
	$3797.80 - 2.64i$		$2(1.7\text{GeV})$

Virtual state Pole below threshod 80 MeV

K.Yu, G.J.Wang, J.J.Wu and Z.Yang, PRD110 (2024), 114029

$\pi^+\pi^-$ and $J/\psi\pi^\pm$ mass spectra @ $e^+e^- \rightarrow J/\psi\pi^+\pi^-$
 D^*D^{*-} mass spectrum @ $e^+e^- \rightarrow D^*D^{*-}\pi^+$

resonance:

$(3880.7 \pm 1.7 \pm 22.4) - i(17.9 \pm 0.7 \pm 7.7)$ MeV .

Y.H.Chen, M.L.Du and F.K.Guo, , Sci.China Phys.Mech.Astron. 67 (2024) 9, 291011



- For $Z_c(3900)$: the ω and ρ exchange are nearly cancel out

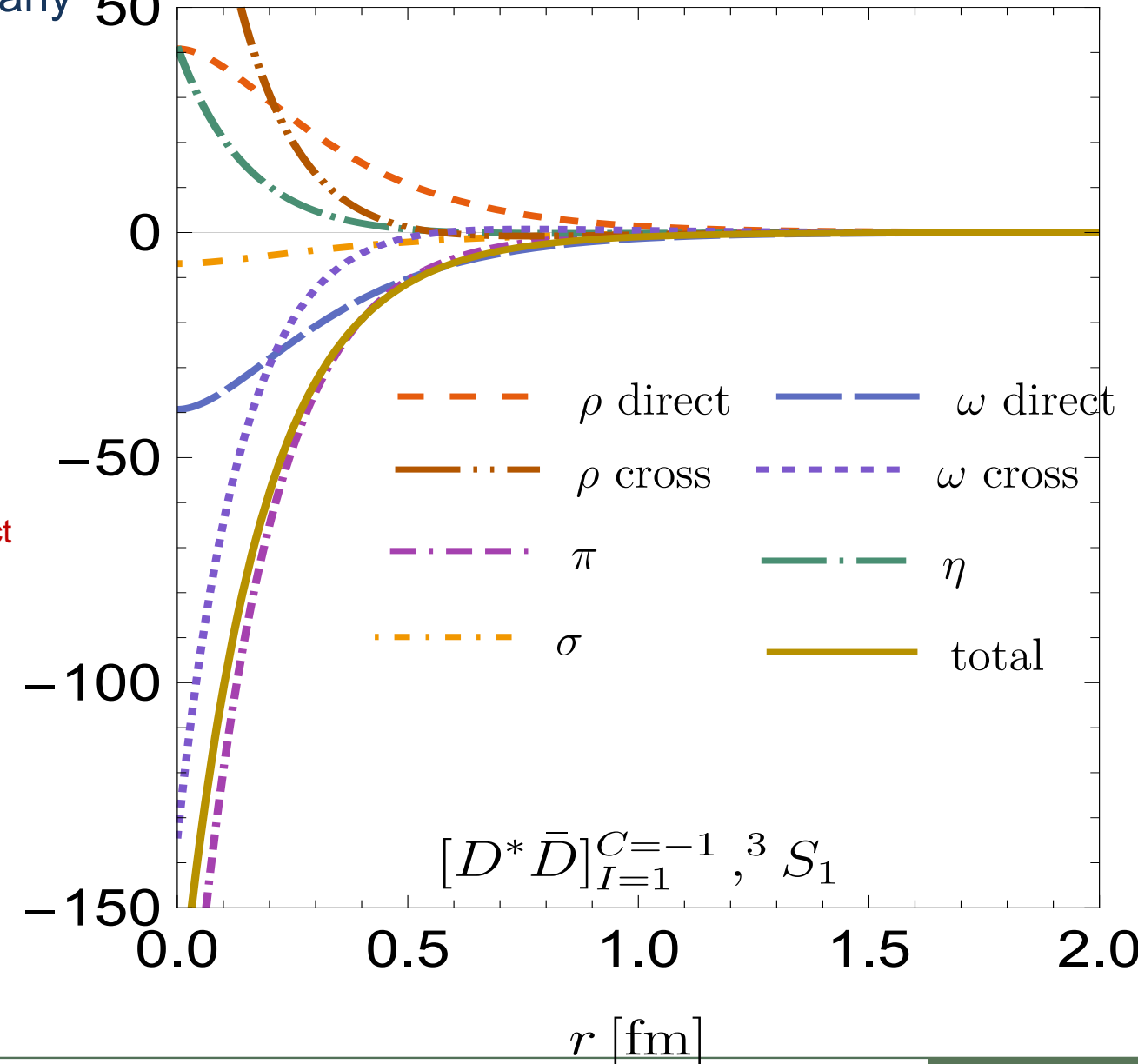
- ▶ Exactly cancel out if $m_\omega = m_\rho$
- ▶ The coupling of π and η are fixed
- ▶ Very sensitive to the σ -exchange

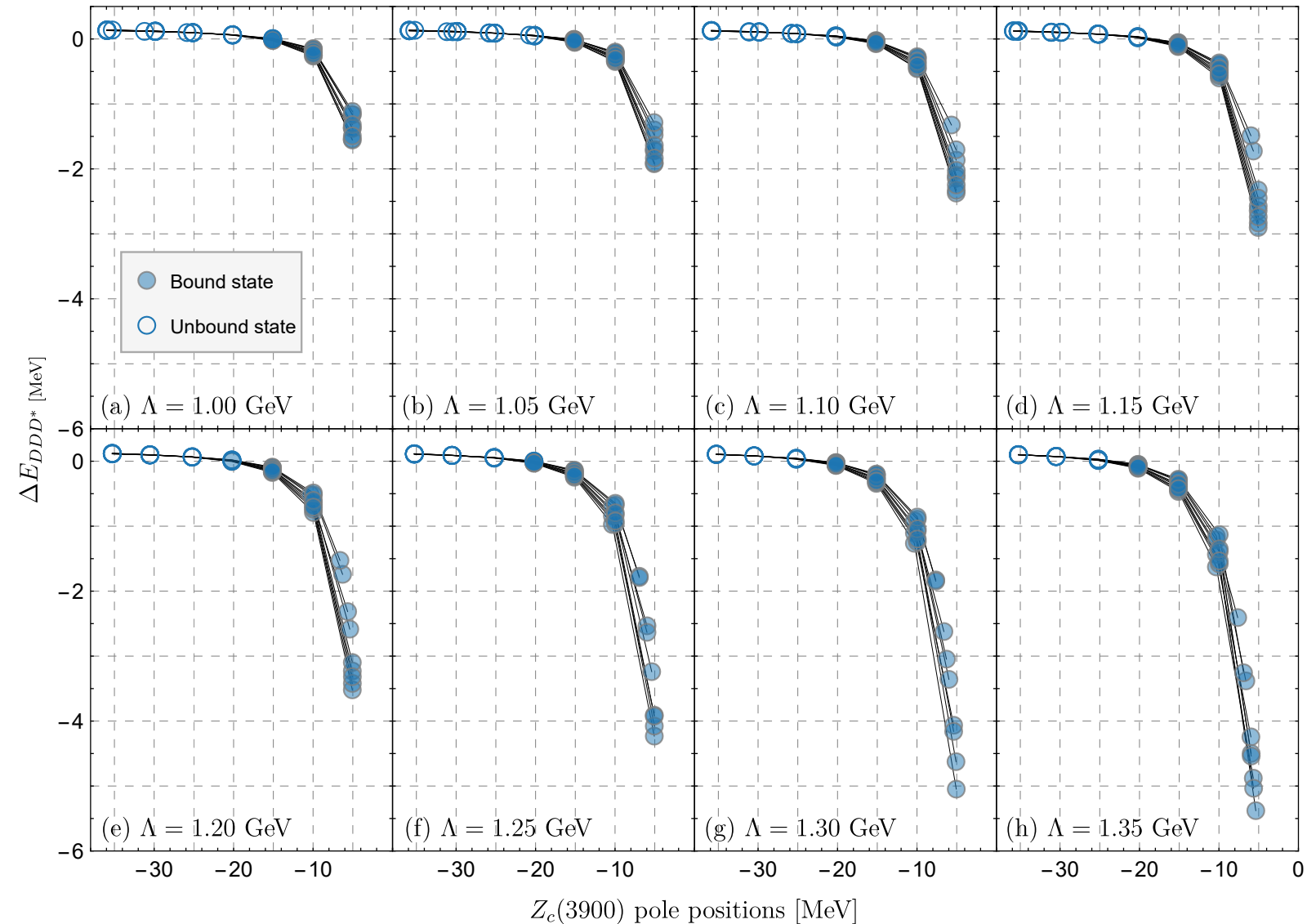
- σ -exchange:

- ▶ blind for the spin and isospin
- ▶ Attractive for all pairs
- ▶ 3-body: 3 different pairs
- ▶ 2-body: 1 pair

More easily show the effect

- $Z_c(3900)$ and DDD^* bound state

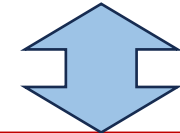




Stronger σ coupling, toward bound DDD*

DDD*

bound-unbound transition point



Z_c(3900)

virtual state pole -15 ~ -20 MeV

Conclusion

- Remain uncertain:
 - ▶ DDD* bind or not
 - ▶ Z_c(3900) pole position
- They are closely related
 - ▶ Z_c(3900) pole position
 - ▶ Progress in either direction shed light on the other
- Several sets of coupling



Short-range interaction of $P_{c\bar{c}}$



- Fail to determine 3 coupling constants by three states

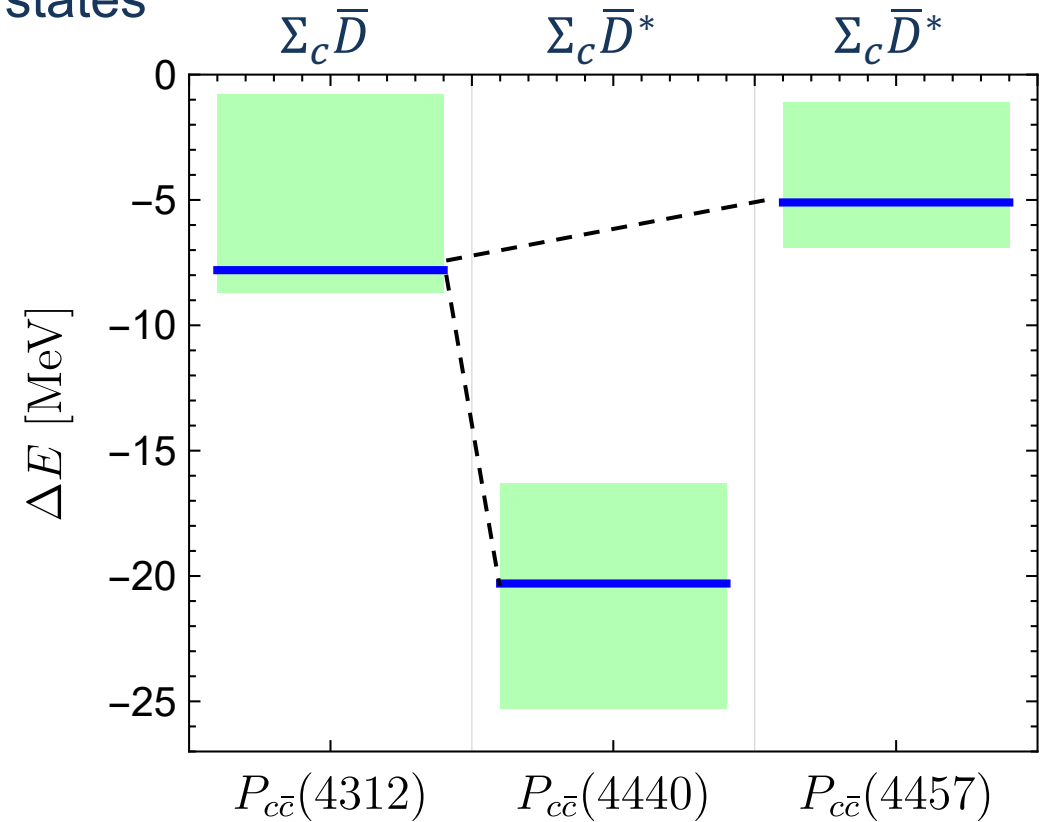
$$\beta\beta_s \rightarrow R_\beta\beta\beta_s, \quad l_sg_s \rightarrow R_sl_sg_s, \quad \lambda\lambda_s \rightarrow R_\lambda\lambda\lambda_s.$$

Why?

- S-wave Interaction: central + spin-depedent

$$V = V_C^{\rho,\omega,\sigma} + V_S^{\rho,\omega} + V_S^{\pi\eta},$$

- For $\Sigma_c\bar{D}$, only central part
- The hirachy of ΔE relect the V_S

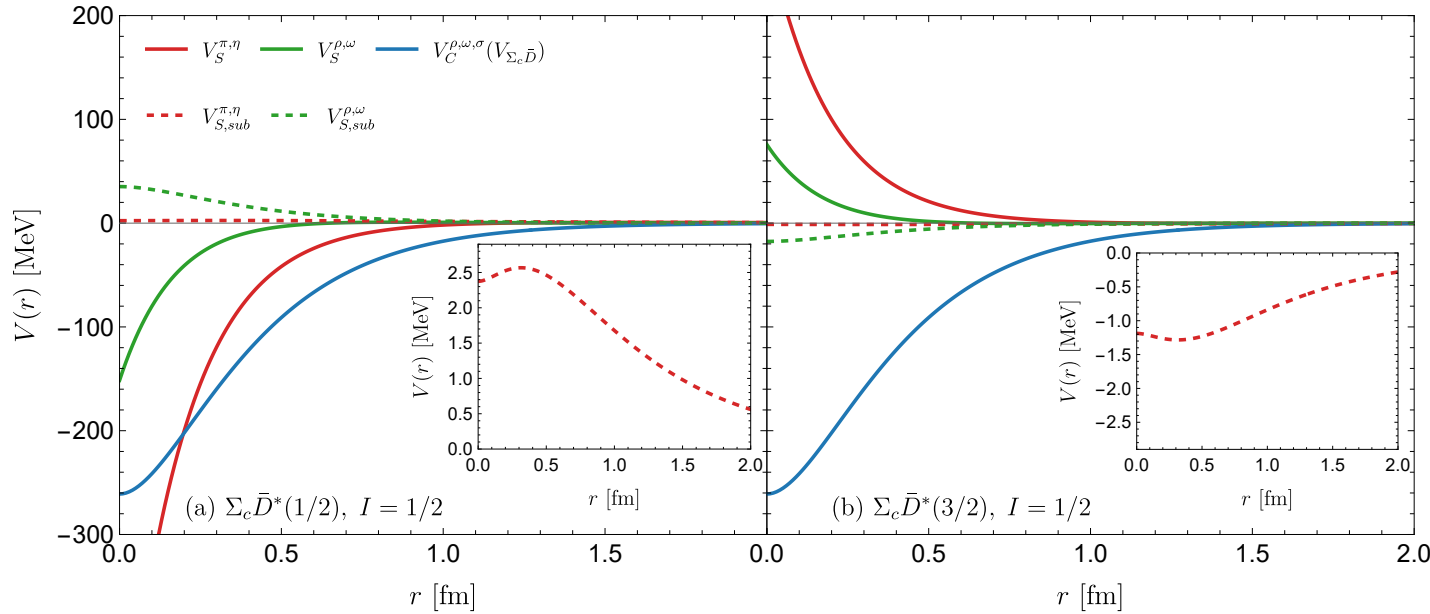


$$S = \frac{1}{2} \quad S = \frac{3}{2}$$

$$S = \frac{3}{2} \quad S = \frac{1}{2}$$

Spin-assigenment problem



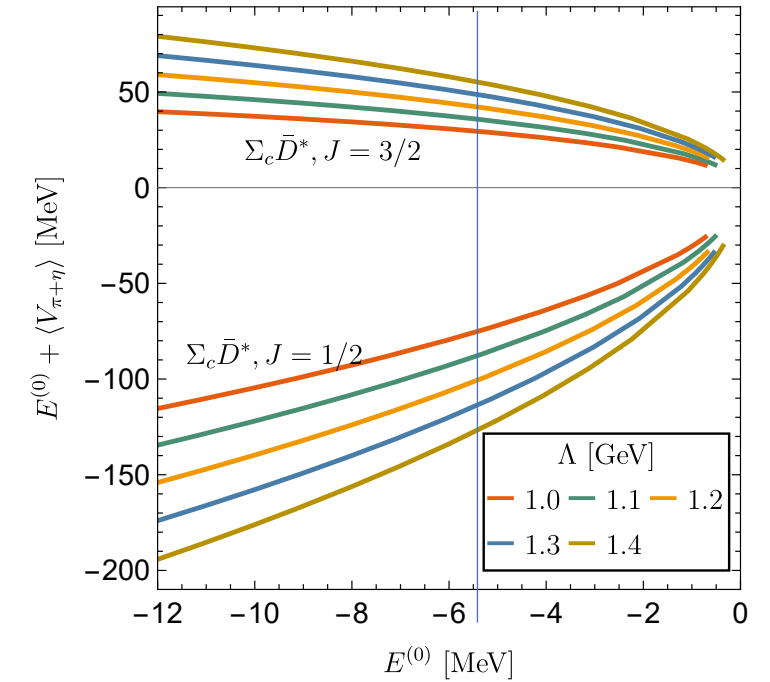


$$V = V_C^{\rho,\omega,\sigma} + V_S^{\rho,\omega} + V_S^{\pi,\eta},$$

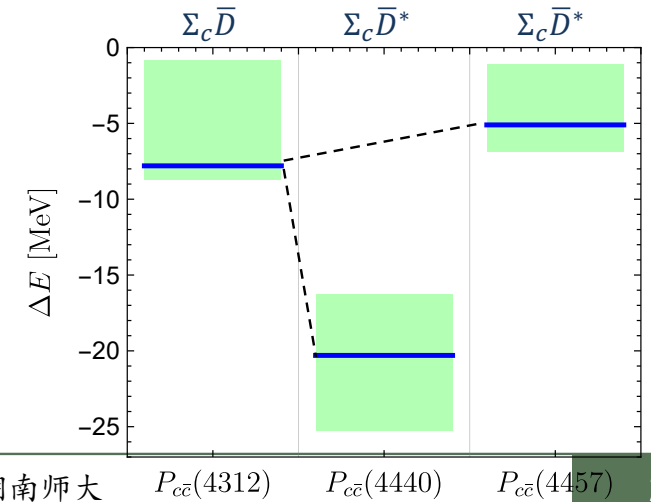
$$\text{sign}(V_S^{\rho,\omega}) = \text{sign}(V_S^{\pi,\eta})$$

The spin-depndent interaction is too strong to fit the Ex. spin splittiing

→ Adjust the short-range interaction



Lower limit of the V_S : $V_S^{\rho,\omega} = 0$



See also: M.-Z. Liu, T.-W. Wu, et al, PRD103, 054004 (2021); T. J. Burns and E. S. Swanson, PRD 100, 114033 (2019); N. Yalikhun, Y.-H. Lin, et al PRD104, 094039 (2021)

- Remove the $\delta^3(r)$ terms

$$\frac{\mathbf{q}^2}{u^2 + \mathbf{q}^2} = 1 - \frac{u^2}{u^2 + \mathbf{q}^2} \rightarrow \delta^3(\mathbf{r}) - \frac{u^2 e^{-ur}}{4\pi r},$$

However, some queries:

- The regulator has regularized the singularity, no reason to remove it
- Bonn potential: not remove it
- Some ambiguities:
 - ▶ Remove for all mesons or just pion
 - ▶ Remove it or soften it further
- G-parity rule: $V(\Sigma_c D^{(*)}) = (-1)^{G_{ex}} V(\Sigma_c D^{(*)})$
 - ▶ Do the removed part satisfy the G-parity rule?

$$\begin{aligned} \frac{q_i q_j}{u^2 + \mathbf{q}^2} F(u, \Lambda, q^2)^2 &\rightarrow - \left[H_3(u, \Lambda, r) T_{ij} + H_1(u, \Lambda, r) \frac{\delta_{ij}}{3} \right], \\ H_1(u, \Lambda, r) &= \frac{u^3}{4\pi} \left[\frac{e^{-ur} - e^{-\Lambda r}}{ur} - \frac{(\Lambda^2 - u^2)\Lambda^2}{2u^3\Lambda} e^{-\Lambda r} \right], \\ H_3(u, \Lambda, r) &\rightarrow \frac{\Lambda^4 + u^4 - 2\Lambda^2 u^2}{96\pi} r + \mathcal{O}(r^2). \end{aligned}$$

Physical reasonable Strategy: model the short-range interaction

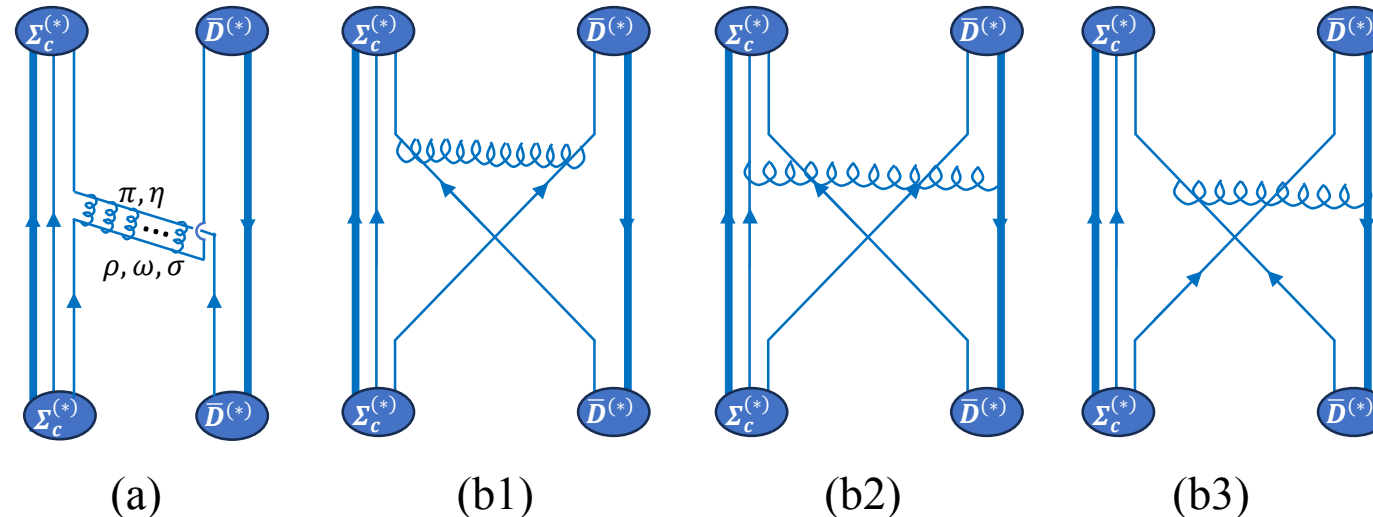


Our proposal: quark exchange effect

- Quark exchanged effect were used to model the repulsive core of the nuclear force

M. Oka and K. Yazaki, Prog. Theor. Phys. **66**, 556 (1981).

- Quark-exchange effect for $\Sigma_c^{(*)} \bar{D}^{(*)}$



Meson-exchange Subleading quark-exchange effect Leading quark-exchange effect

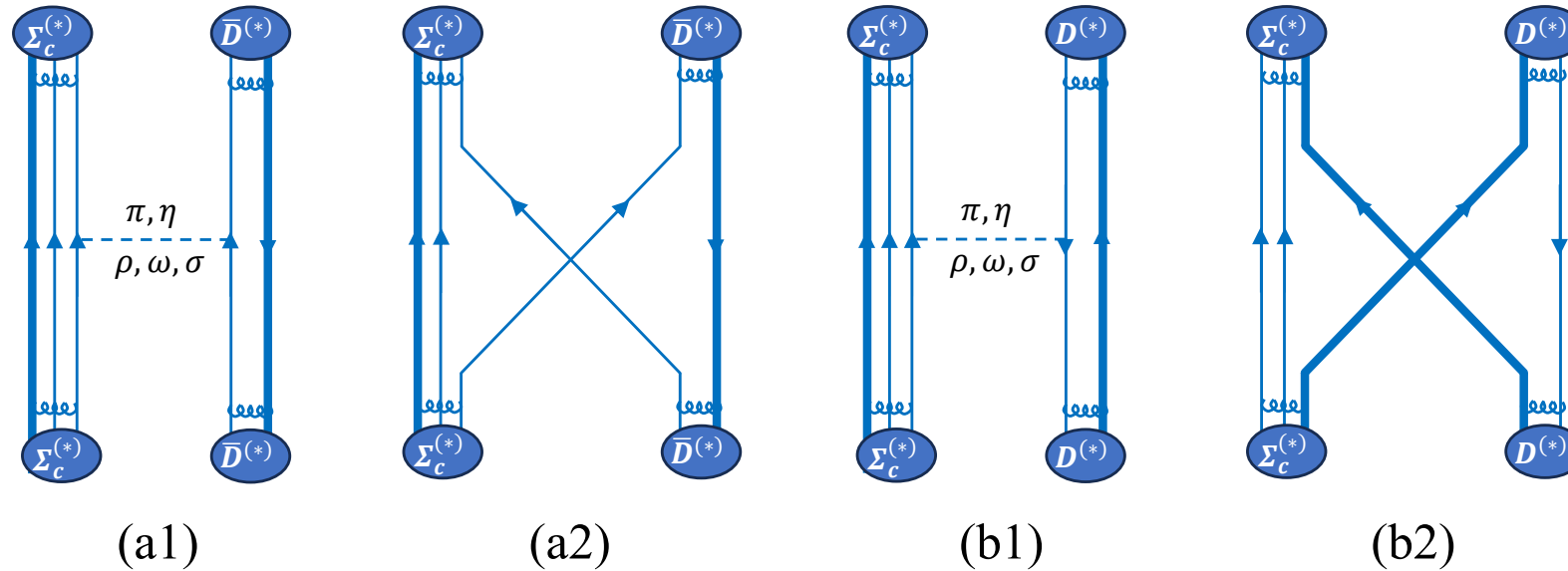
$$V_{qex}(s_1, s_2, J, I) = \frac{16}{27} \left(\mathbf{s}_1 \cdot \mathbf{s}_2 + \frac{3}{2} \right) \left(\mathbf{I}_1 \cdot \mathbf{I}_2 + \frac{1}{2} \right) \delta_v$$

$\delta_v > 0$: given no node from ground state wave functions

- The spin-isospin structure of V_{qex} is different from the OBE interaction
- For the $I = \frac{1}{2} \Sigma_c \bar{D}^{(*)}$ system, V_{qex} could cancel the spin-dependent interaction from OPE



- V_{ex} will not follow the G-parity rule
- Discern the short-range mechanism via $\Sigma_c^{(*)} D^{(*)}$ systems



- P-wave DD^* and $\bar{D}^*D/\bar{D}D^*$
 - ▶ centrifugal barrier for $l>0$ help form the shape resonance
 - ▶ High-partial wave dynamics sensitive to the long-range interaction
 - ▶ $G(3900)$ and related predictions
- DDD^* bound state and $Z_c(3900)$
 - ▶ Uncertainty of the coupling constants in OBE
 - ▶ Discrepancy of σ -exchange coupling constants
 - ▶ DDD^* bind or not and $Z_c(3900)$ pole position remain uncertain
 - ▶ **DDD^* bound state and $Z_c(3900)$ pole position are closely related**
 - ▶ Data sets **systematically** taking into account the uncertainty from the cutoff
- Short-range interaction of $P_{c\bar{c}}$
 - ▶ Spin-dependent part is too strong to fit the ex. spin splitting of binding energies
 - ▶ Adjust the short-range interaction: quark-exchange model
 - ▶ Different implications from the OBE in open charm systems
- General lessons I learned for **model calculation**:
 - ▶ Use models to test even the wildest ideas
 - ▶ Be cautious when drawing conclusions
 - ▶ Focus on qualitative picture, not decimal precision

**Thanks for
your attention!**



backup



- Cutoff-dependence? Regulator dependence?
 - Coupling constants?
 - Short-range interaction
 - Sigma mass why not physical one?
 - Width of the exchanged mesons
-
- Annihilation effect?
 - Coupled channel effect
 - Recoiling effect
 - Relativistic effect?



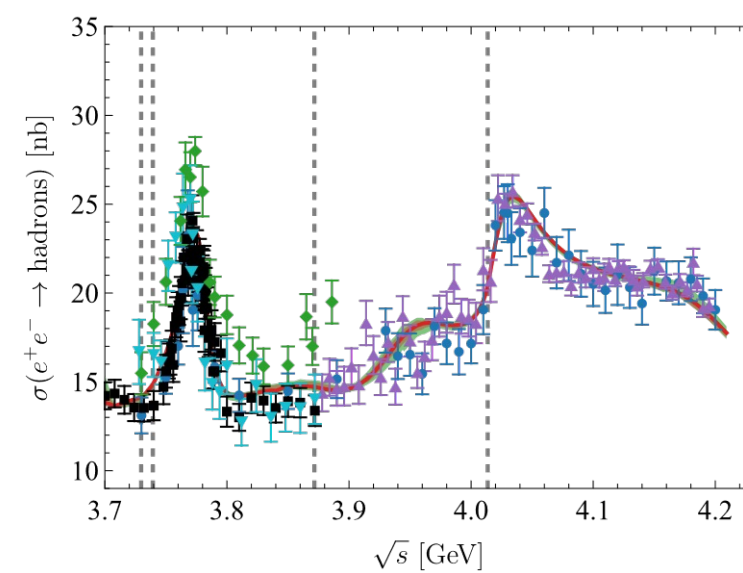
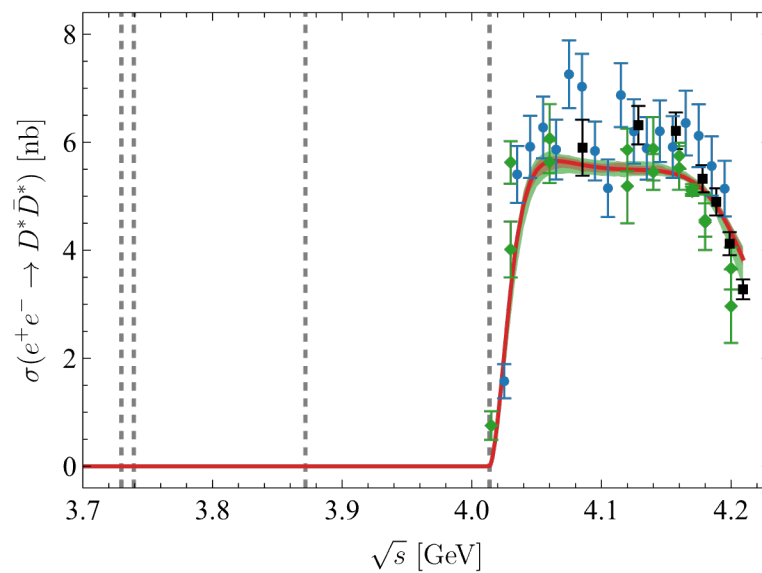
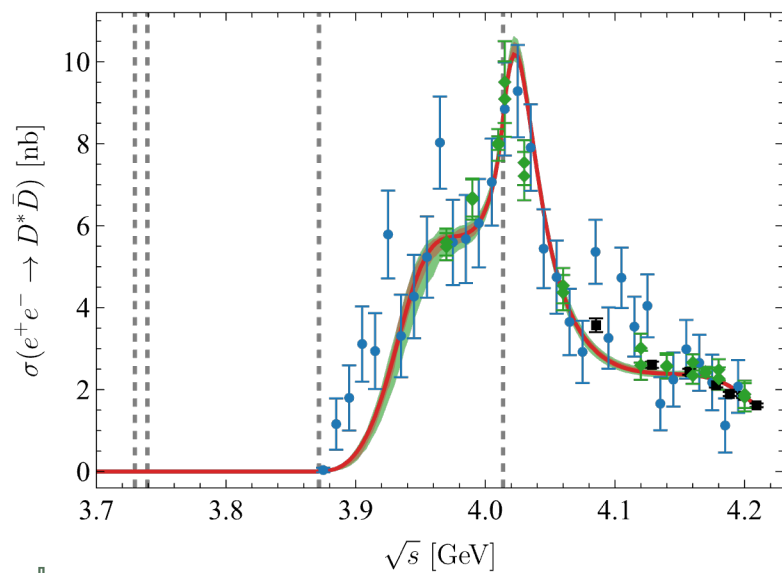
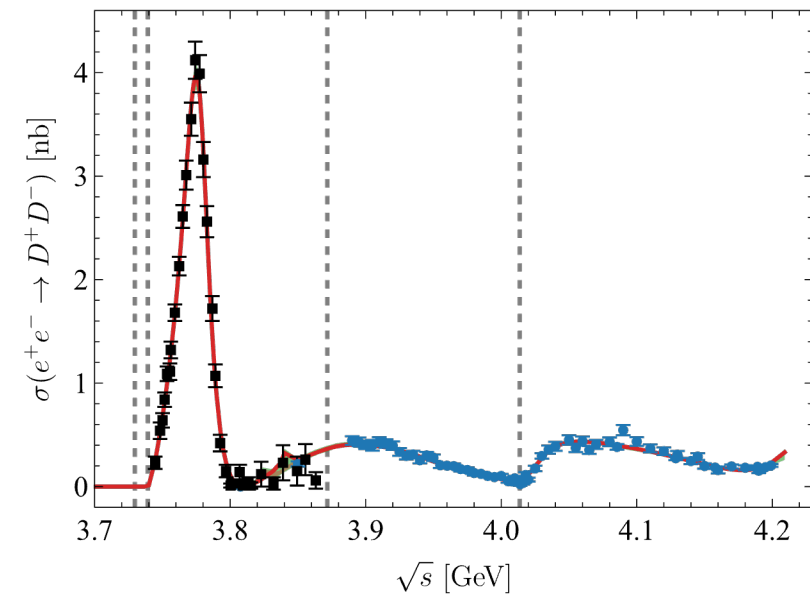
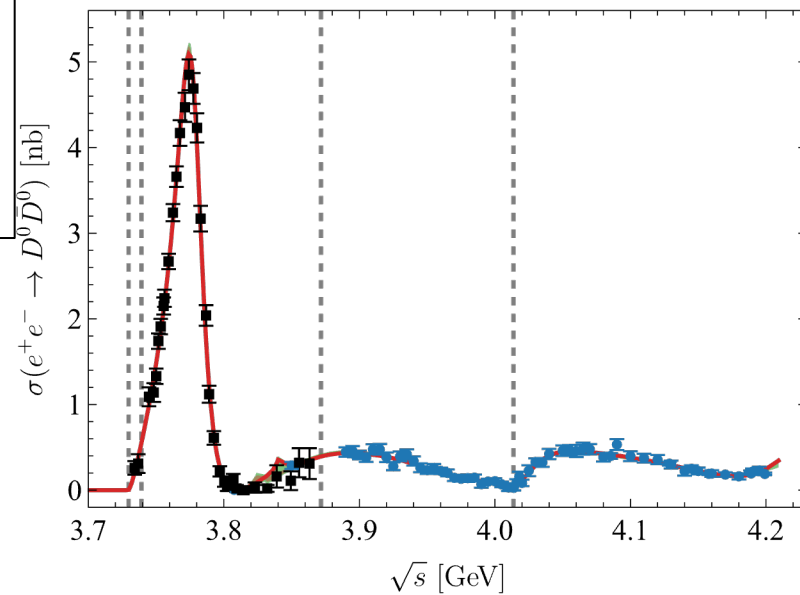
Meson parameters used in the relativistic (energy-independent) momentum space one-boson-exchange potential (OBE PQ)

	$g_\alpha^2/4\pi; [f_\alpha/g_\alpha]$	$g_\alpha^2/4\pi(k^2=0)$	m_α [MeV]	Λ_α [GeV]	n_α
π	14.6	14.27	138.03	1.3	1
ρ	0.81; [6.1]	0.43	769	2.0	2
η	5	3.75	548.8	1.5	1
ω	20; [0.0]	10.6	782.6	1.5	1
δ	1.1075	0.64	983	2.0	1
σ	8.2797 ^a	7.07	550 ^a	2.0	1

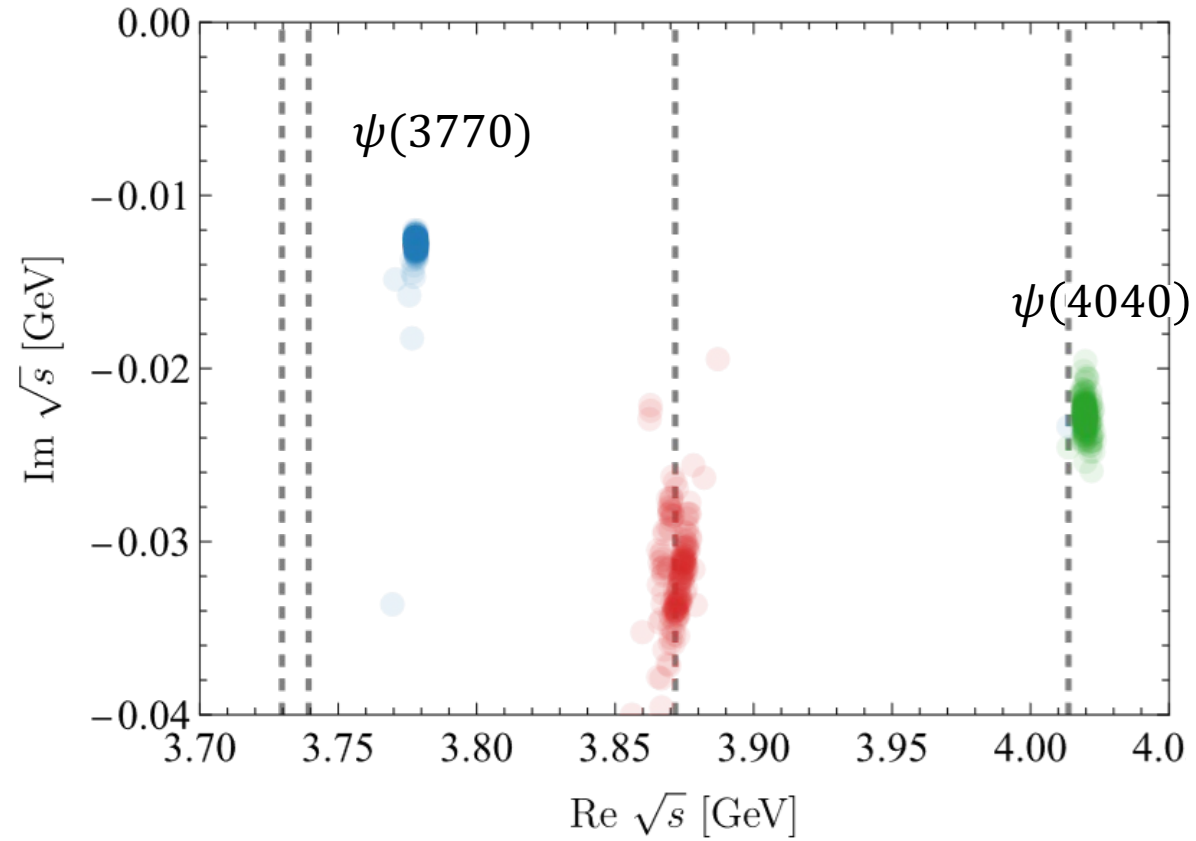


Our fit in the K-matrix formalism

- Repair the three defects
- Our refit results
 - ▶ $\chi^2/\text{dof} = 2.07$



Our fit in the K-matrix formalism



Extra poles

$$3869.2(67) - i29.0(52)$$



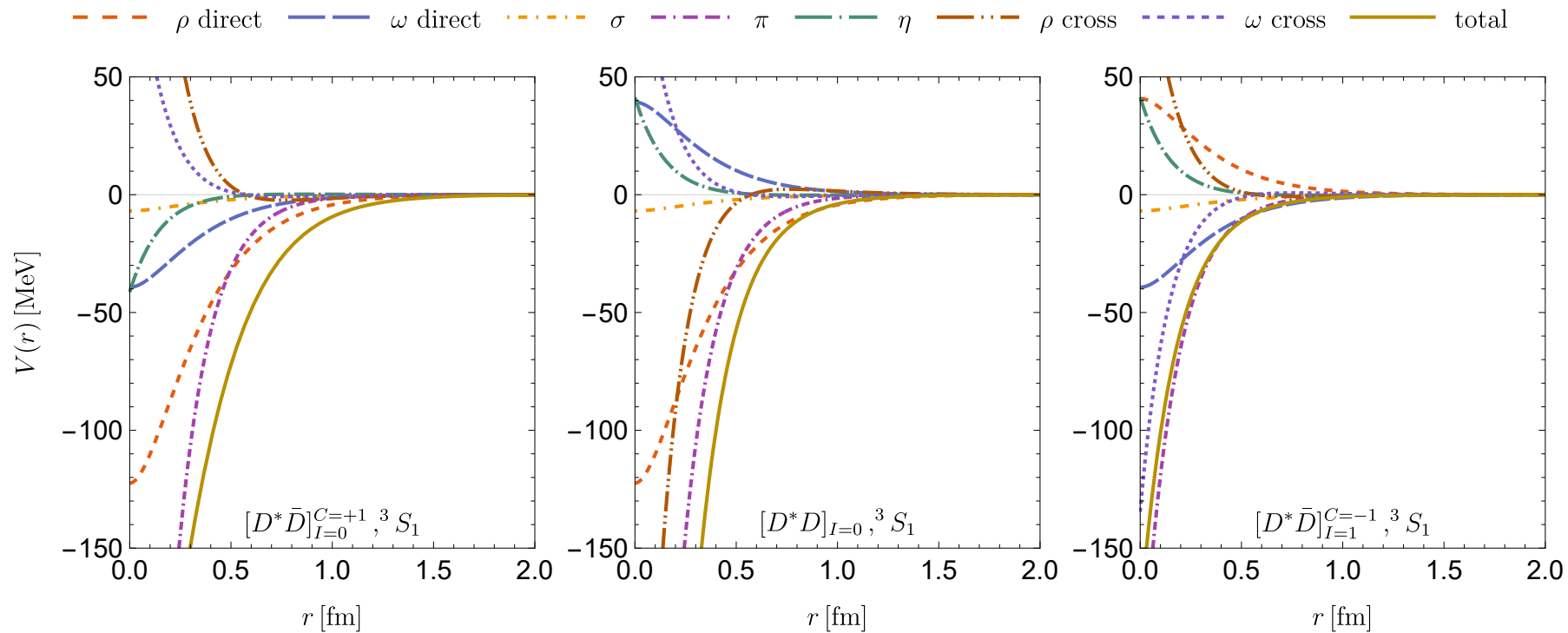
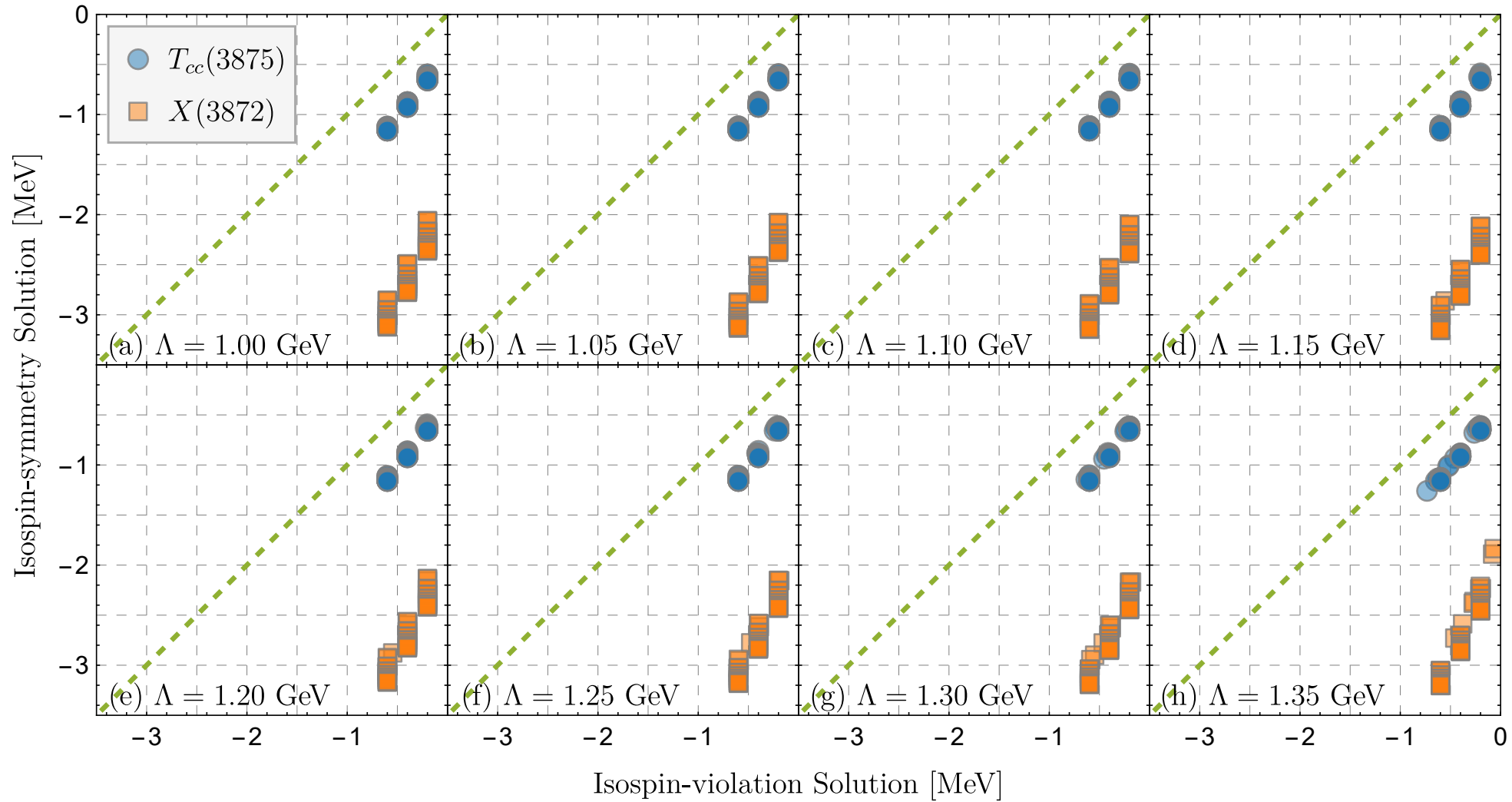
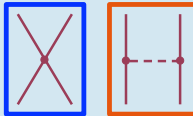
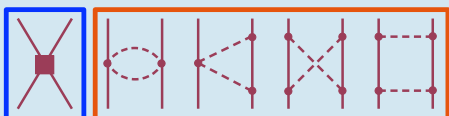
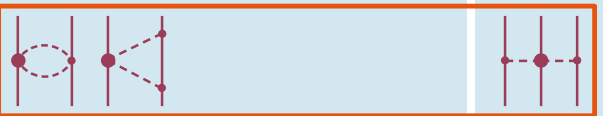
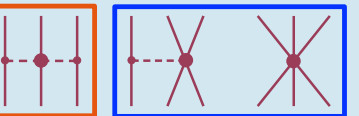
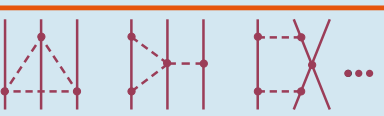
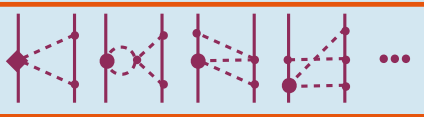
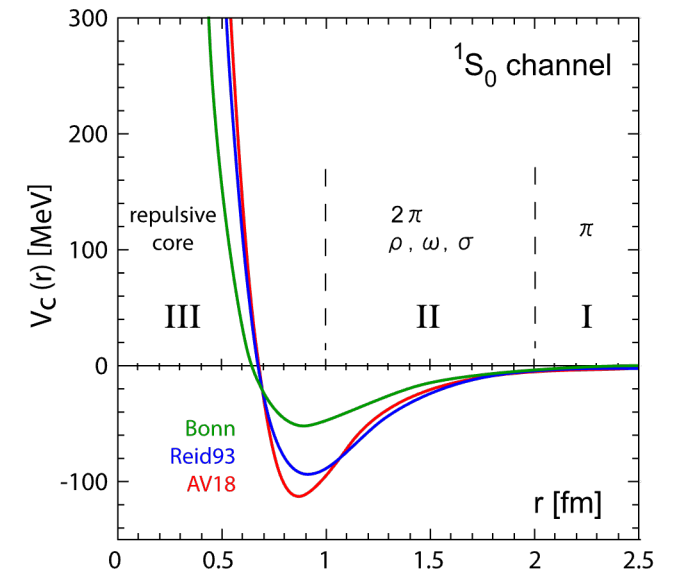


FIG. 2. The coordinate space potentials in the isospin limit for the systems corresponding to the $X(3872)$, $T_{cc}(3875)$, and $Z_c(3900)$ states. The parameter Λ is set to 1.20 GeV, with R_β , R_λ , and R_s fixed at 1.





	Two-nucleon force	Three-nucleon force	Four-nucleon force
LO:			
NLO:			
N ² LO:			
N ³ LO:			
N ⁴ LO:			



Chiral dynamics: Long-range interactions are predicted in terms of on-shell amplitudes 

Short-range few-N interactions are tuned to experimental data

Chiral nuclear force

Phenomenological
nuclear force

