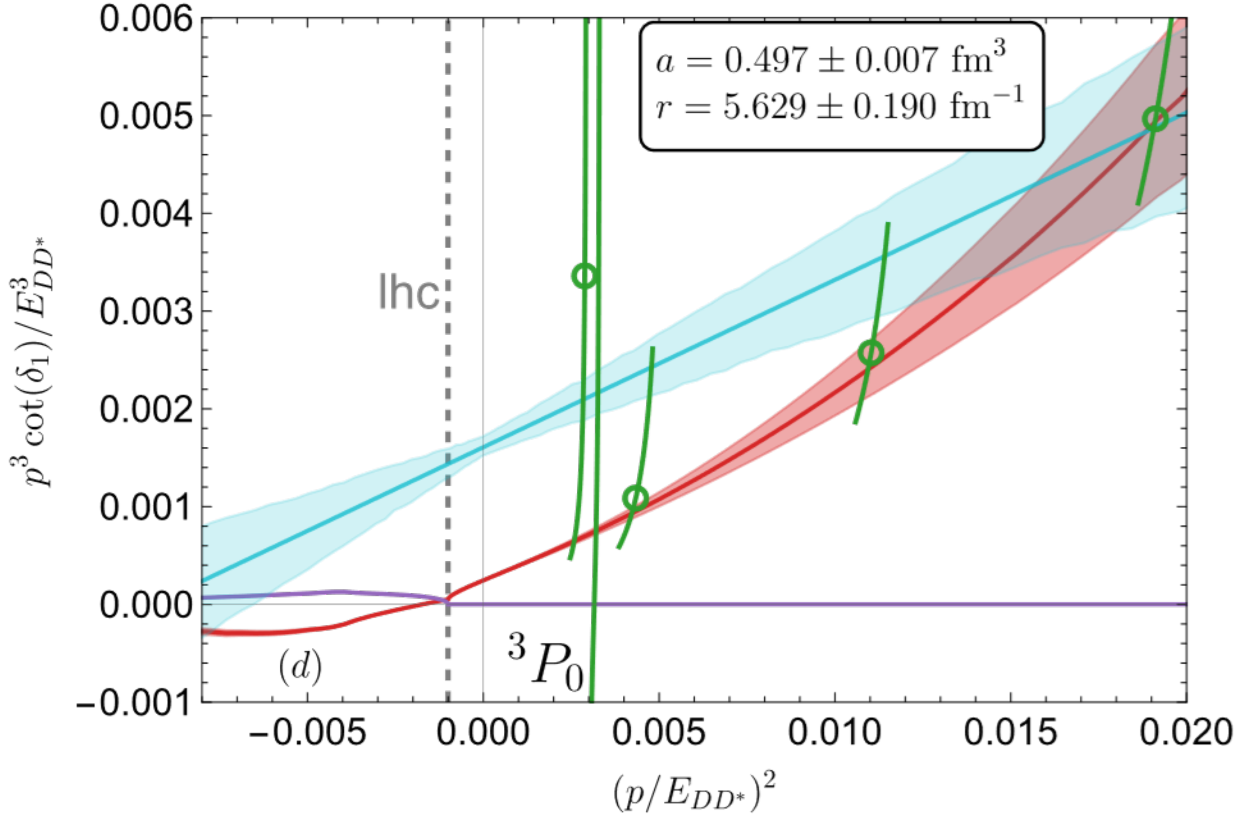
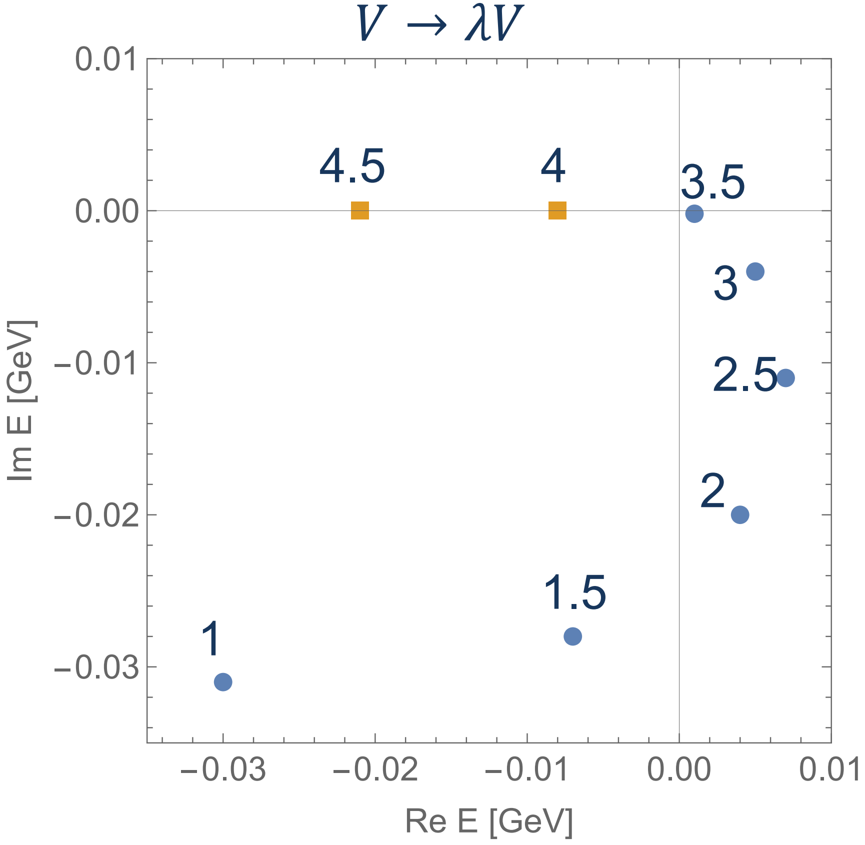
$^3P_0\ DD^*$ system





Pole: $-0.030 - i0.031$

华南师范大学 | 23.06.2024

Identify the P-wave $\bar{D}D^*/D\bar{D}^*$ resonance

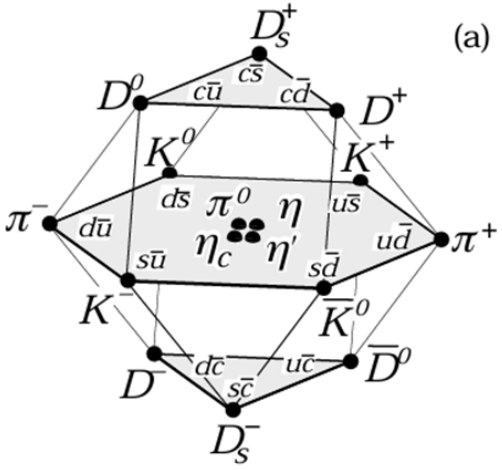
Lu Meng（孟璐） | RUHR-UNIVERSITÄT BOCHUM

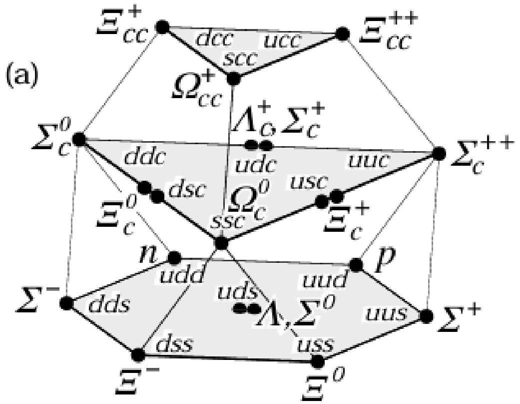
Base on arXiv:2403.01727
Together with Z-Y. Lin, J-Z. Wang, J-B.Cheng, S-L. Zhu (PKU)

Background

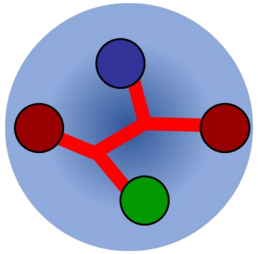
Hadron spectrum

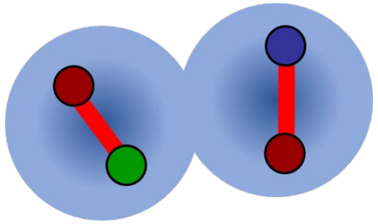
- Conventional hadrons: $(q\bar{q})$ or (qqq)

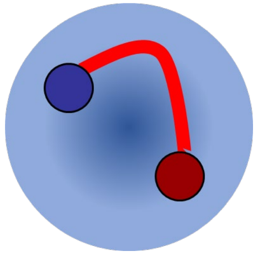


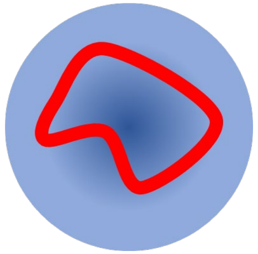


- Exotic hadrons









…

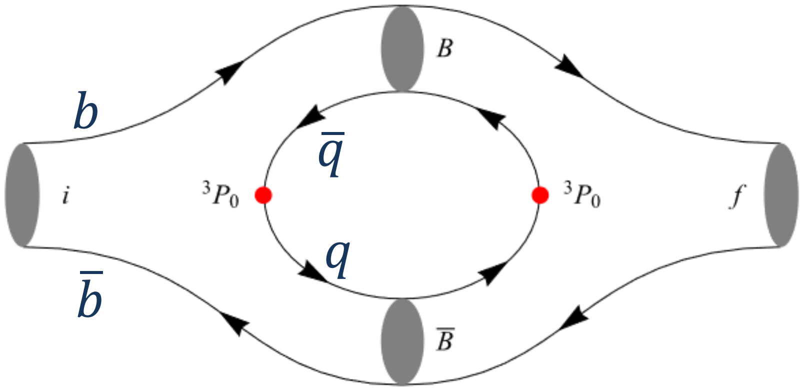
Compact type   Molecular type   Hybrid   Glueball

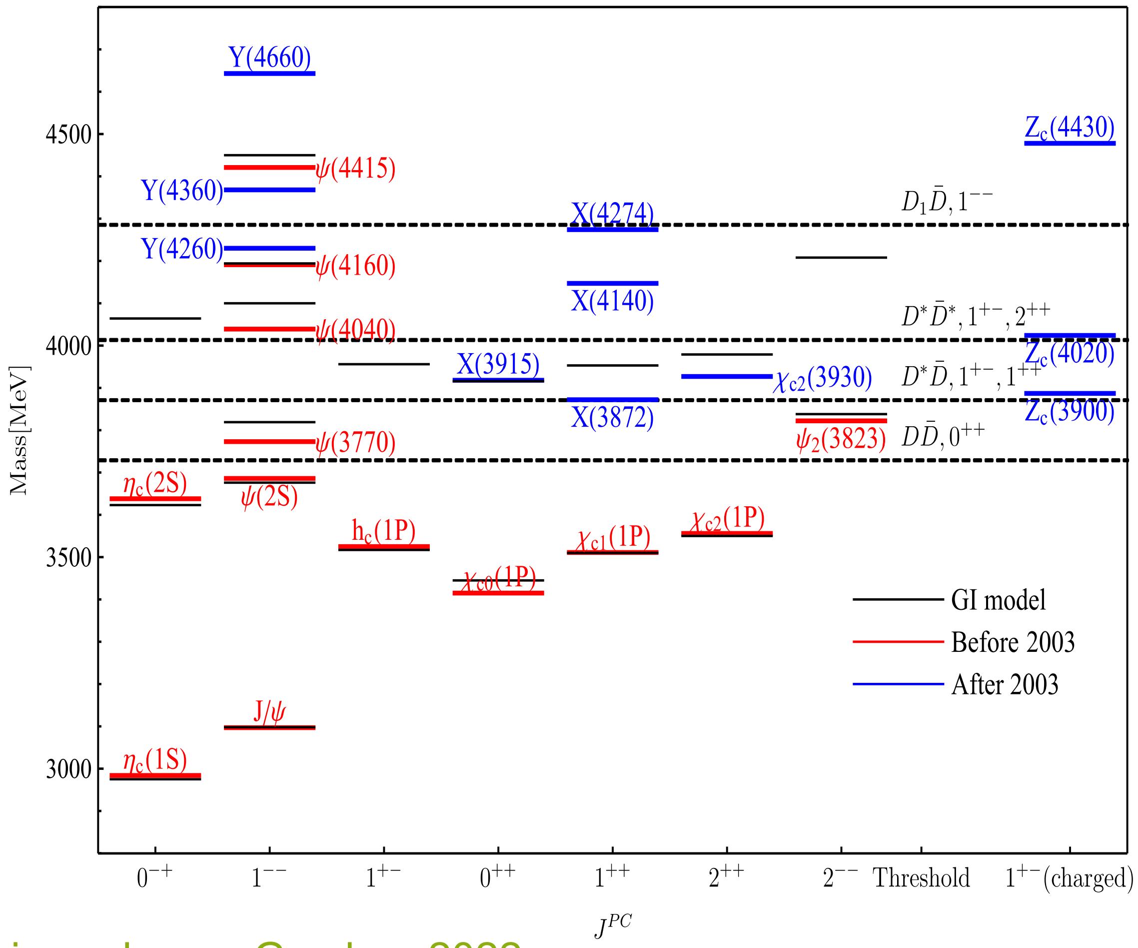
multiquark states

Review of Particle Physics, Chen:2022asf, Meng:2022ozq, Brambilla:2019esw, Guo:2017jvc, Esposito:2016noz, Lebed:2016hpi, Olsen:2014qna…

Exotic state candidates

- XYZ states (heavy-quarkonium-like states)
  - ▶ $Q\bar{Q}q\bar{q}/\bar{Q}Q$





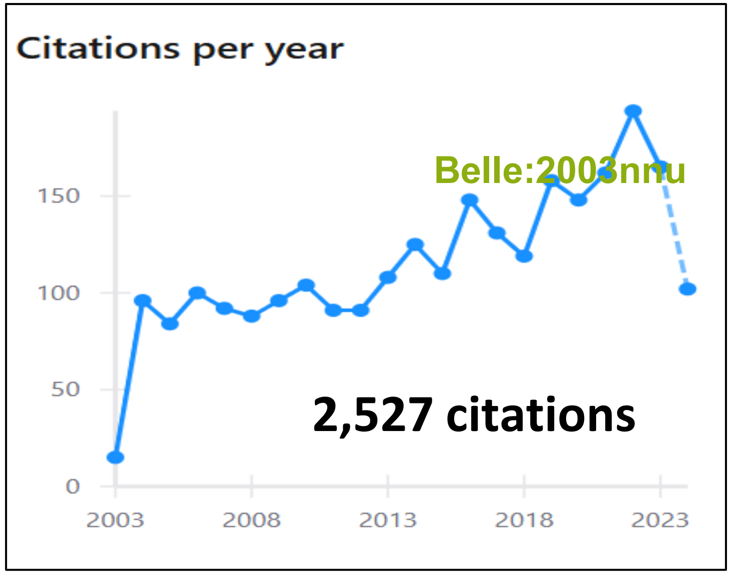
- Recently, more and more hadrons composed of at least four quarks (manifestly exotic states) were observed

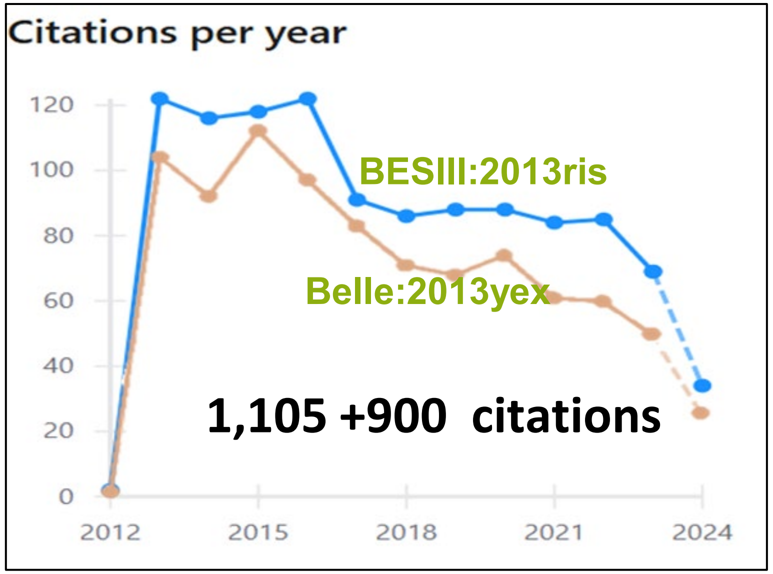
New naming scheme: Gershon:2022xnn

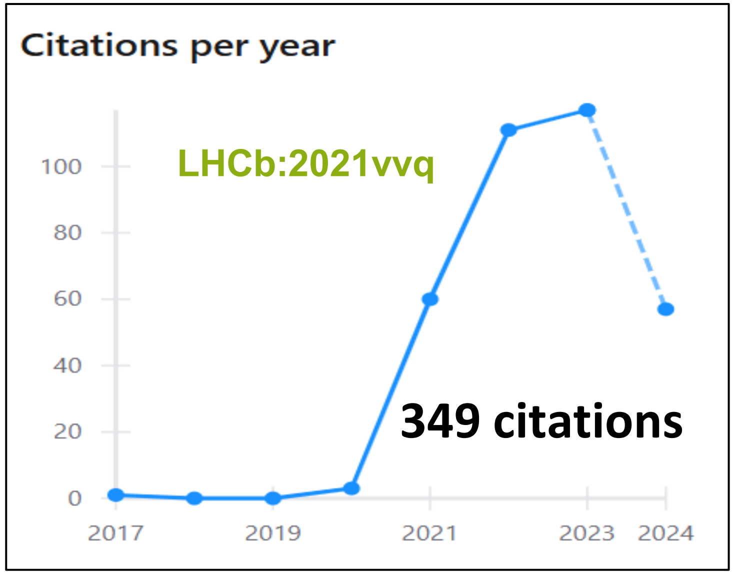
$[c\bar{c}qqq]$	$[cc\bar{c}\bar{c}]$	$[cs\bar{u}\bar{d}]$	$[cs\bar{u}\bar{d}]$	$[cc\bar{u}\bar{d}]$	$[c\bar{s}u\bar{d}][c\bar{s}\bar{u}d]$
P_c	$X(6900)$	$T_{cs1}(2900)$, $T_{cs0}(2900)$	$Z_{cs}(3985)$, $Z_{cs}(4000)$	$T_{cc}(3875)^+$	$T_{c\bar{s}0}(2900)^{++}$, $T_{c\bar{s}0}(2900)^0$
	2006.16957, 2304.08962, ...	2009.00025, 2009.00026	2011.07855, 2103.01803	2109.01038, 2109.01056	2212.02716, 2212.02717

Three super "stars"

	Quark contents	$I^G(J^{PC})$	Threshold	ΔM [MeV]	Γ [MeV]
$X(3872)$	$q\bar{q}c\bar{c}/c\bar{c}$	$0^+(1^{++})$	$D^0\bar{D}^{0*}$	$0.0068^{+0.1655}_{-0.17000}$	$0.380^{+0.412}_{-0.322}$
	The 1st charmonium-like state			BESIII:2023hml	
$Z_c(3900)$	$q\bar{q}c\bar{c}$	$1^+(1^{+-})$	$D\bar{D}^*$	11.3 ± 2.6	28.4 ± 2.6
	The 1st manifestly exotic charmonium-like state			PDG	
$T_{cc}(3985)$	$\bar{q}\bar{q}cc$	?	$D^{*+}D^0$	$-0.360^{+0.040}_{-0.040}$	$0.048^{+0.002}_{-0.014}$
	The 1st open double charm tetraquark state			LHCb:2021auc	

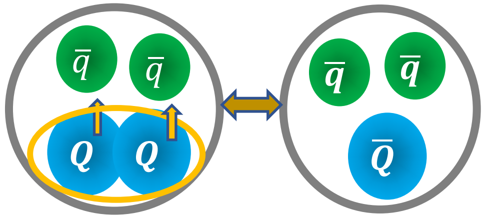






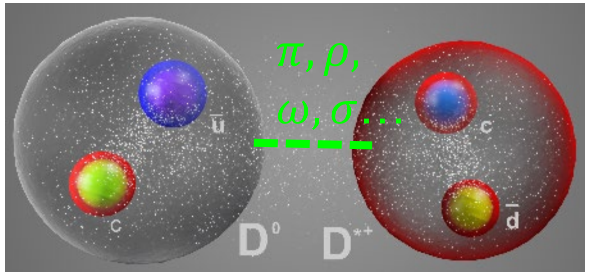
To the unified models

- More unified description: set up connection among/between different states
  - ▶ E.g. Heavy quark symmetry + EFT V. Baru, E. Epelbaum, et al., *Phys.Rev.D* 99 (2019) , 094013,…
  - ▶ E.g. Phenomenological model
- Compact $(QQ\bar{q}\bar{q})$ states
  - ▶ Been anticipated for several decades in quark models J.P. Ader, J.M. Richard, P. Taxi,PRD25(1982) 2370
  - ▶ Clustering behavior: (QQ) diquark Karliner et al,PRL119,202001; Eichten et al,PRL119,202002;



- Predicting the molecular-type T_{cc} N. Li, Z-F. Sun, X. Liu, S-L Zhu, *PRD*88(2013), 114008
  - ▶ One-boson-exchange interaction: $\pi, \eta, \rho, \omega, \sigma$
  - ▶ Para. are constrained by $X(3872)$
  - ▶ Results: a bound state with binding energy 470 keV
  - ▶ The picture is more consistent with the Exp. one: $\Delta M \approx -360$ keV

$\bar{D}D^* \Rightarrow DD^*$



- Including Z_c states? Other D^*D and $D^*\bar{D}$ states? Other quantum numbers? P-wave states?

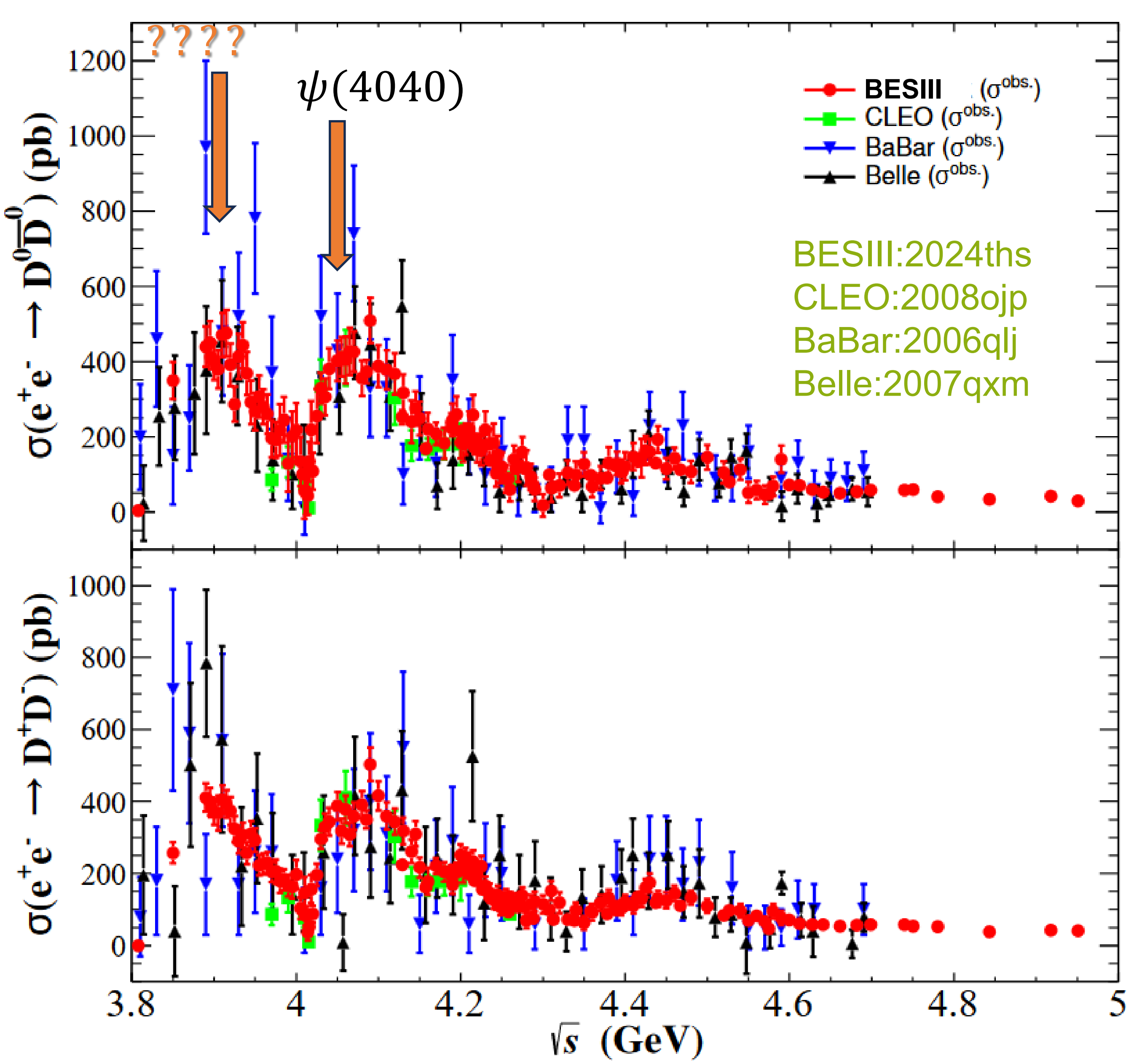
Experimental progress

- An enhancement close to 3.9 GeV
  - ▶ $D^*\bar{D}$ threshold
  - ▶ New $D^*\bar{D}$ resonance?
  - ▶ Quantum number $J^{PC} = 1^{--}$, from virtual photon
  - ▶ P-wave $D^*\bar{D}$ state?
- Cornell model: enhancement at 3.9 GeV

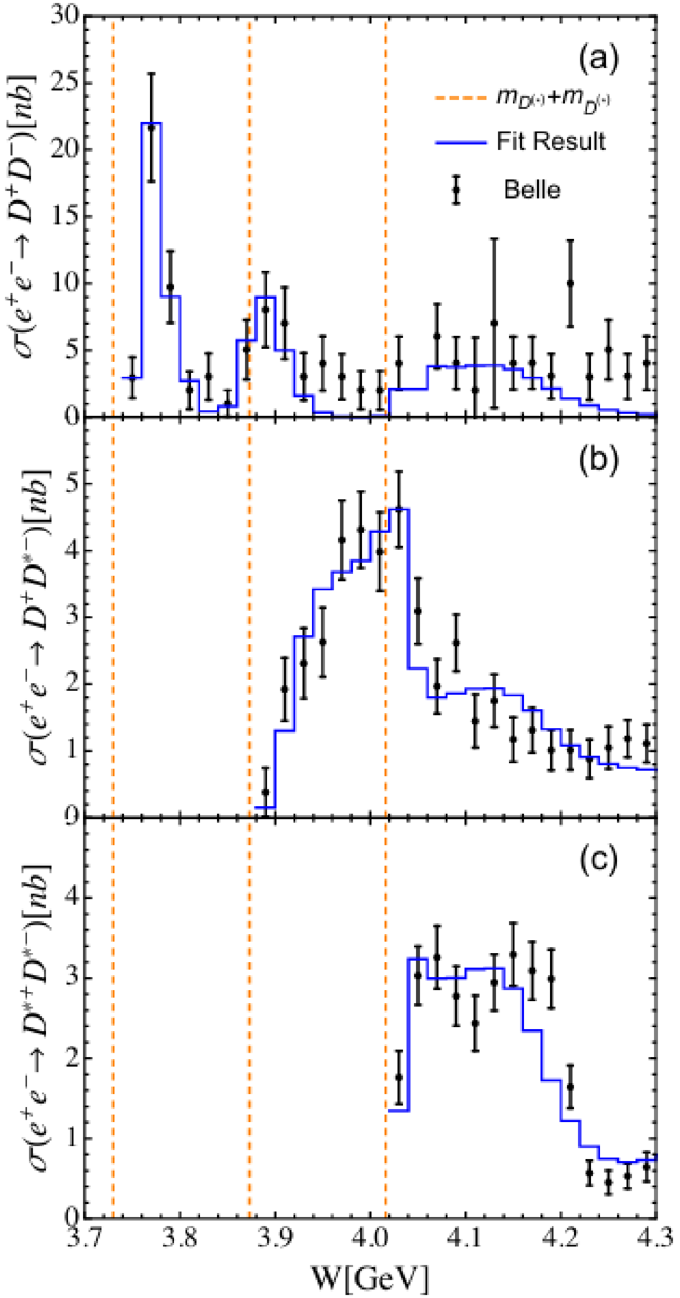
  Eichten:1978tg, Eichten:1979ms
- Belle and BaBar: $G(3900)$
  - ▶ Fit with a Gaussian function
  - ▶ Not regarded as a resonance

  BaBar:2006qlj, Belle:2007qxm
- More precise data from BESIII

  BESIII:2024ths



Lippmann-Schwinger equation formalism



Sheet	Poles (GeV)	$\lvert g_{D\bar{D}}\rvert$	$\lvert g_{D\bar{D}^*}\rvert$	$\lvert g_{D^*\bar{D}^*_{s=0}}\rvert$	$\lvert g_{D^*\bar{D}^*_{s=2}}\rvert$
II	$3.764 \pm i0.006$	13.53	9.48	5.88	16.78
III	$3.879 \pm i0.035$	4.40	10.96	7.63	18.15
IV	$4.034 \pm i0.014$	2.90	2.23	12.52	12.85

$$
\begin{array}{lllll}
\text{I} & \text{Im}\, q_{D\bar{D}} > 0, & \text{Im}\, q_{D\bar{D}^*} > 0, & \text{Im}\, q_{D^*\bar{D}^*} > 0, & \text{for } E < 2m_D\ , \\
\text{II} & \text{Im}\, q_{D\bar{D}} < 0, & \text{Im}\, q_{D\bar{D}^*} > 0, & \text{Im}\, q_{D^*\bar{D}^*} > 0, & \text{for } 2m_D < E < m_D + m_{D^*}\ , \\
\text{III} & \text{Im}\, q_{D\bar{D}} < 0, & \text{Im}\, q_{D\bar{D}^*} < 0, & \text{Im}\, q_{D^*\bar{D}^*} > 0, & \text{for } m_D + m_{D^*} < E < 2m_{D^*}\ , \\
\text{IV} & \text{Im}\, q_{D\bar{D}} < 0, & \text{Im}\, q_{D\bar{D}^*} < 0, & \text{Im}\, q_{D^*\bar{D}^*} < 0, & \text{for } E > 2m_{D^*}\ .
\end{array}
$$

- Lippmann-Schwinger equation formalism
  - ► Coupled channel effect
  - ► S-matrix unitarity and analyticity
  - ► Belle data
  - ► A resonance close to $\bar{D}D^*$ threshold

M.L. Du, U. G.Meissner and Q. Wang, Phys.Rev.D 94 (2016) 9, 096006

Breit-Wigner fit from BESIII

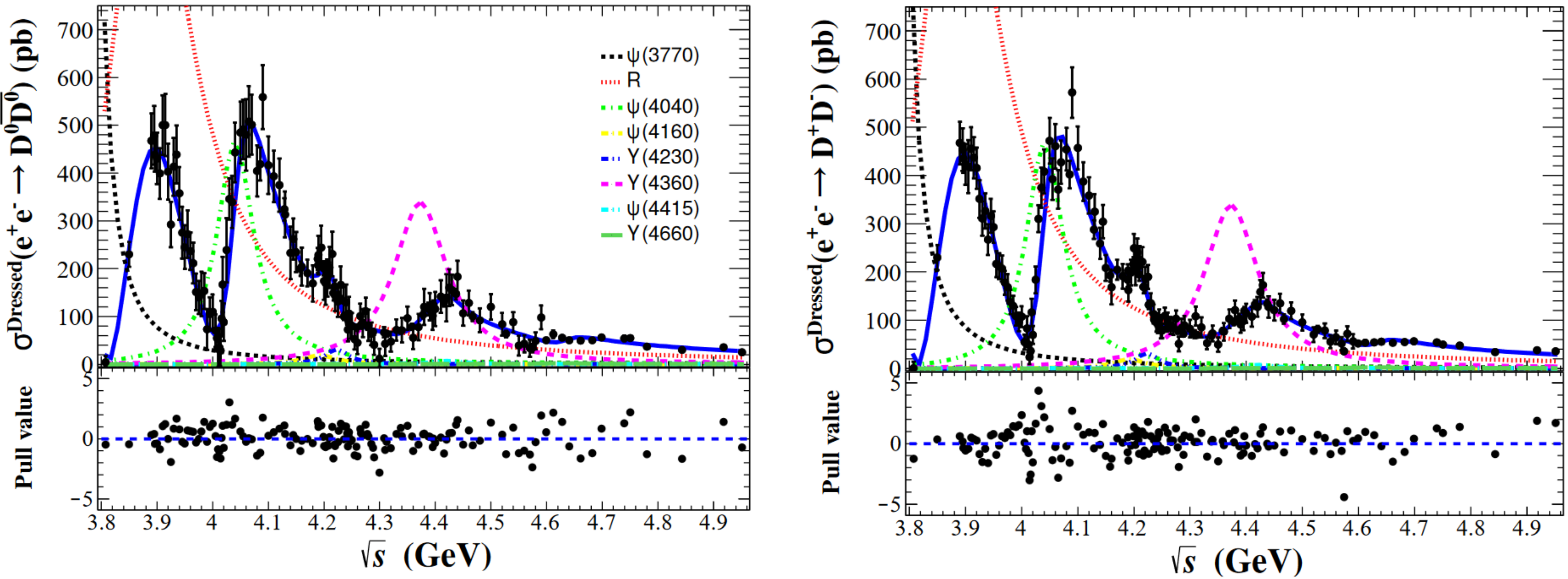


TABLE I. Fit results of the Born cross section, where the first uncertainties are the statistical and the second are systematic and S denotes the significance.

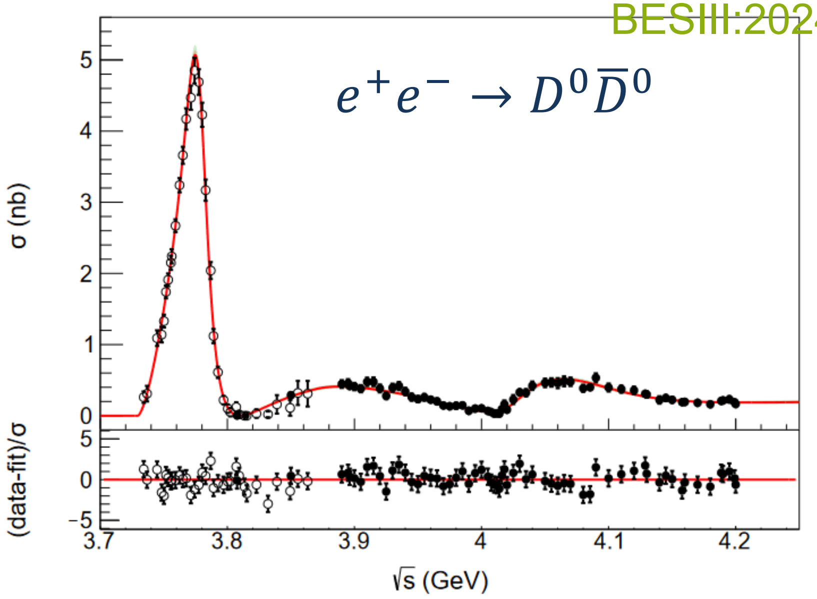
BESIII:2024ths

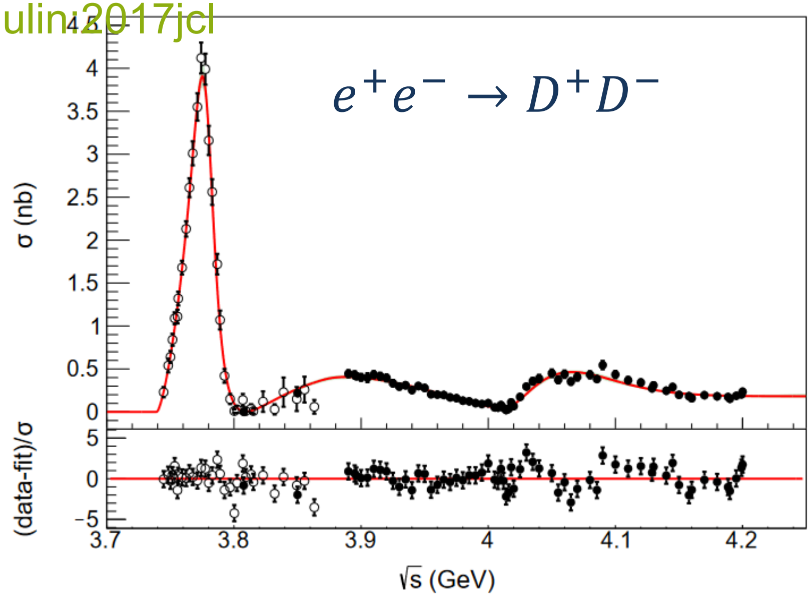
$e^+e^- \to D\bar{D}$								
Resonance	$\psi(3770)$	R	$\psi(4040)$	$\psi(4160)$	$Y(4230)$	$Y(4360)$	$\psi(4415)$	$Y(4660)$
Mass (MeV/c^2)	3773.7 (fixed)	3872.5±14.2±3.0	4039 (fixed)	4191 (fixed)	4222.5 (fixed)	4374 (fixed)	4421 (fixed)	4630 (fixed)
Width (MeV/c^2)	87.6 (fixed)	179.7±14.1±7.0	80 (fixed)	70 (fixed)	48 (fixed)	118 (fixed)	62 (fixed)	72 (fixed)
$\Gamma_{ee}\mathcal{B}$ (eV)	95-106	202-292	41-44	1-2	1-2	50-144	0-2	0-1
S(σ)	10	> 20	13	7	11	11	4	8
χ^2/d.o.f = 346/275				p-value = 0.002				

K-matrix formalism

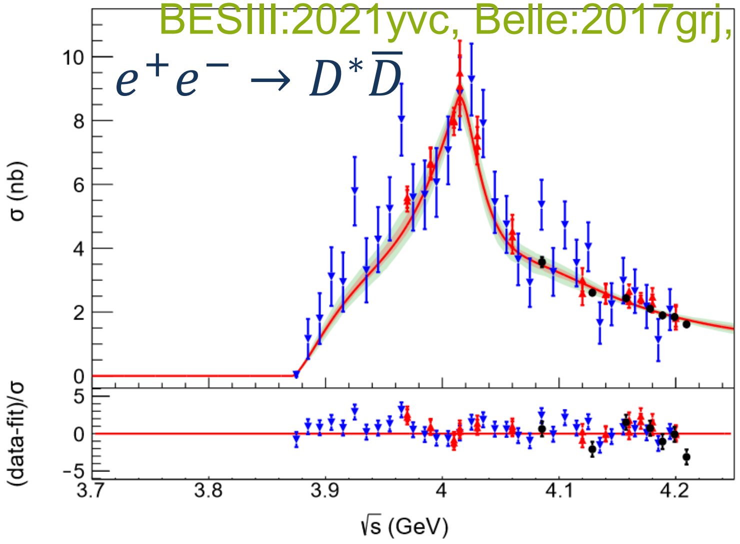
- K-matrix formalism
  - ▶ Two bare poles
  - ▶ $D^0\bar{D}^0$, D^+D^-, $D^*\bar{D}$, $D^*\bar{D}^*$, dummy channel

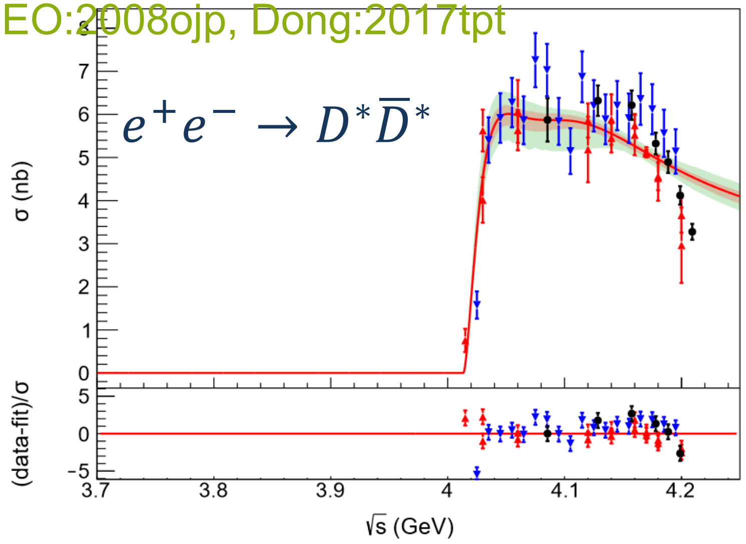
BESIII:2024ths, Julin:2017jcl



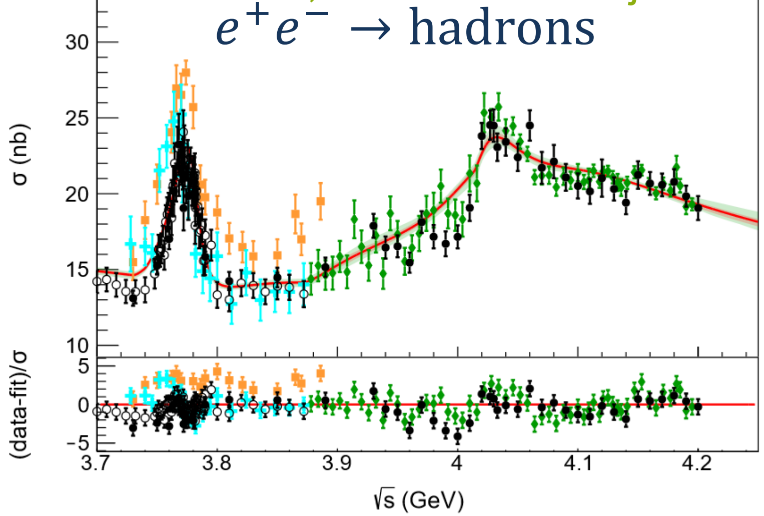


BESIII:2021yvc, Belle:2017grj, CLEO:2008ojp, Dong:2017tpt





BES:2001ckj, Ablikim:2006mb, Rapidis:1977cv, Osterheld:1986hw, Schindler:1979rj

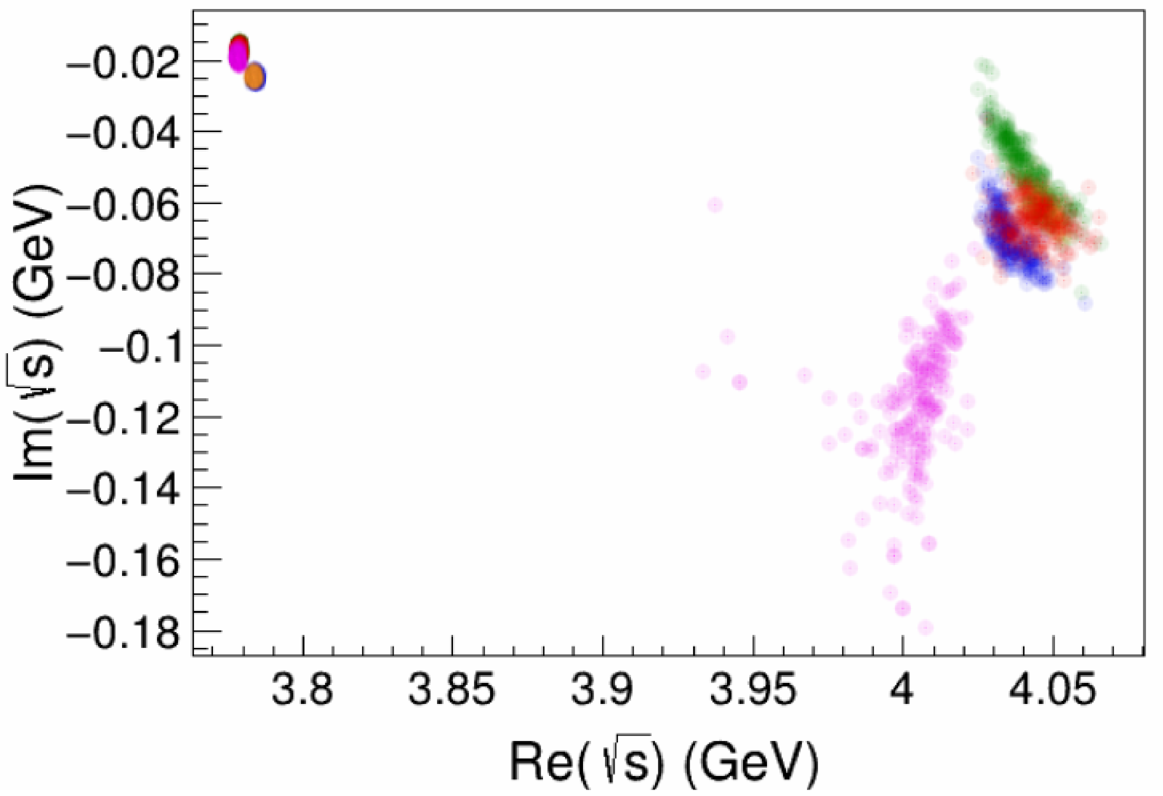


N. Hüsken, R. Lebed, R. Mitchell, E. Swanson, Y-Q Wang, C-Z Yuan, arXiv: 2404.03896

K-matrix formalism

Model	Channels	Constraints	# p
1	$D\bar{D}$, $\bar{D}D^*$, $D^*\bar{D}^*$, $[J/\psi(\pi\pi)]_{\text{dummy}}$	$g_{R:D^+D^-} = g_{R:D^0\bar{D}^0}$	29
2	$D\bar{D}$, $\bar{D}D^*$, $D^*\bar{D}^*$, $[J/\psi(\pi\pi)]_{\text{dummy}}$	$g_{R:D^+D^-} = g_{R:D^0\bar{D}^0}$ $f_{D^+D^-:\mu} = f_{D^0\bar{D}^0:\mu}$ $f_{D^+D^-:D^+D^-} = f_{D^0\bar{D}^0:D^0\bar{D}^0}$	24
3	$D\bar{D}$, $\bar{D}D^*$, $D^*\bar{D}^*$, $[D_s^+D_s^-]_{\text{dummy}}$	$g_{R:D^+D^-} = g_{R:D^0\bar{D}^0}$	29
4	$D\bar{D}$, $\bar{D}D^*$, $D^*\bar{D}^*$, $[D_s^+D_s^-]_{\text{dummy}}$	$g_{R:D^+D^-} = g_{R:D^0\bar{D}^0}$ $f_{D^+D^-:\mu} = f_{D^0\bar{D}^0:\mu}$ $f_{D^+D^-:D^+D^-} = f_{D^0\bar{D}^0:D^0\bar{D}^0}$	24
5	Model 1 plus node [Eq. (4)]		30

Model	Mass (MeV)	Γ (MeV)	Γ_{ee} (eV)	χ^2/ndf
1	3778.7(7)	34(4)	205(25)	2.20
2	3784.2(7)	49(4)	3000(1500)	2.48
3	3778.9(6)	33(4)	210(20)	2.39
4	3783.7(6)	49(4)	270(25)	2.45
5	3778.3(6)	38(5)	200(400)	1.88
fit summary	3778.7(7)(50)	34(4)(15)	205(25)(70)	
PDG	3773.7(4)	27.2(1)	261(21)	
1	4044.0(15)	130(30)	180(100)	
2	4036.0(10)	135(35)	15000(8000)	
3	4040.0(10)	95(30)	80(80)	
4	4046.0(10)	120(20)	10(50)	
5	4008.0(10)	220(80)	50000(40000)	
fit summary	4044(15)(36)	130(30)(125)	180(100)(170)	
PDG	4039(1)	80(10)	856(162)	



$G(3900)$ in the past, is **not in fact due to a new $c\bar{c}$ resonance**, but rather naturally emerges as a **threshold enhancement** due to the opening of the $D^*\bar{D}$ channel.

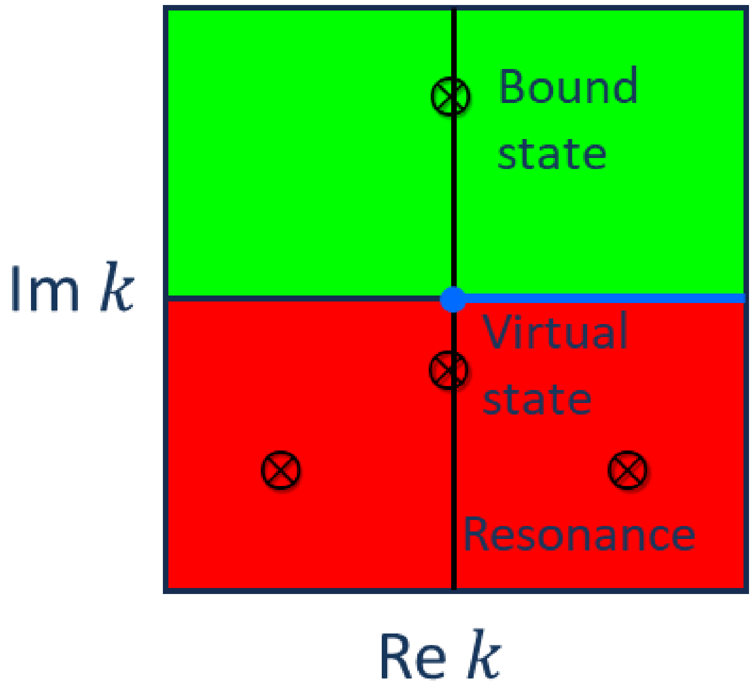
Our calculations

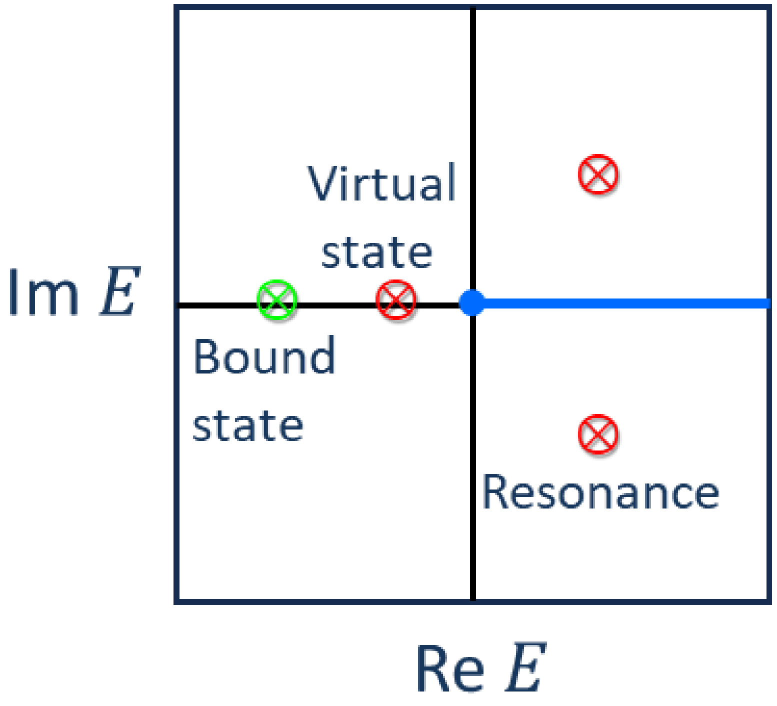
- Data: Does the pole close to 3900 exist?
- Dynamics: If so, could we embed the $X(3872), T_{cc}, Z_c(3900)$ and this state into a unified scheme?
- Uncertainties: If we can, the caveat of this scheme?

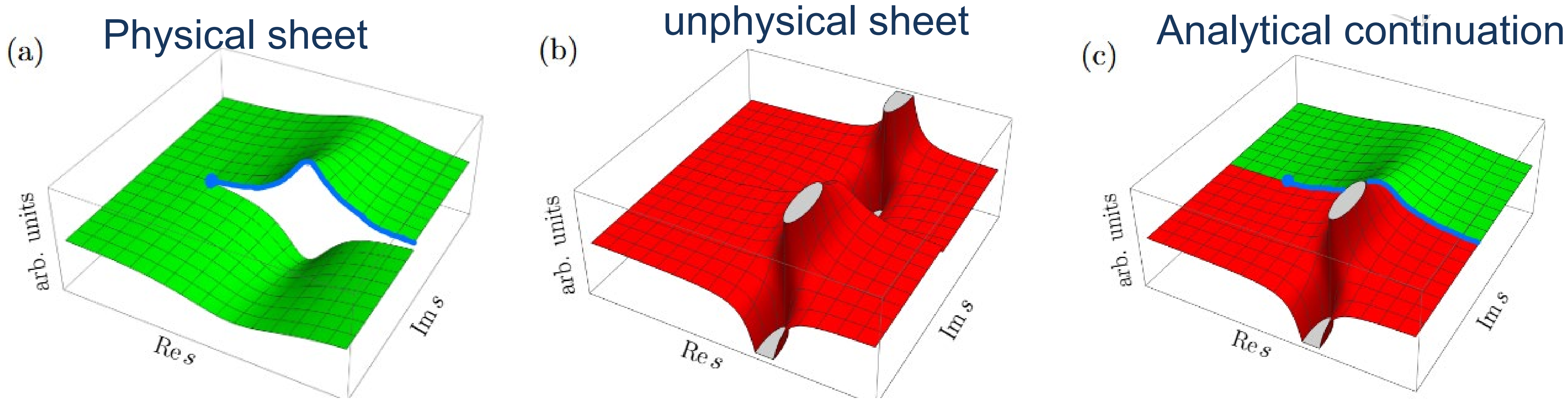
Poles of S-matrix

- "states" ⇔ S-matrix poles
- Causality⇒analyticity
- Conservation of probabilities ⇒unitarity
- Amplitude formalism to the correct pole position
  LSE, K-matrix

Mizera:2023tfe; PDG







K-matrix formalism

- In principle, K-matrix formalism could meet the requirement of the analyticity and unitarity

$$\mathrm{Im}\mathcal{M}^{-1} = -2\rho, \qquad k = \frac{\sqrt{[s-(m_1-m_2)^2][s-(m_1+m_2)^2]}}{2\sqrt{s}}, \qquad \rho(s) = \frac{k}{8\pi\sqrt{s}}$$

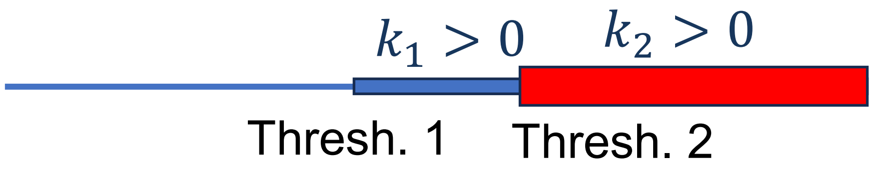
$$\mathcal{M}^{-1} = K^{-1} + C, \quad \mathrm{Im}\ C(s) = -2\rho,$$

$$K = \frac{g^2}{m_R^2 - s} + f, \qquad C(s) = C(s_0) - \frac{s-s_0}{\pi}\int_{s_0}^{\infty} ds' \frac{\mathrm{Im}C(s)}{(s'-s)(s'-s_0)}$$

- Three defects of K-matrix formalism in [arXiv: 2404.03896]

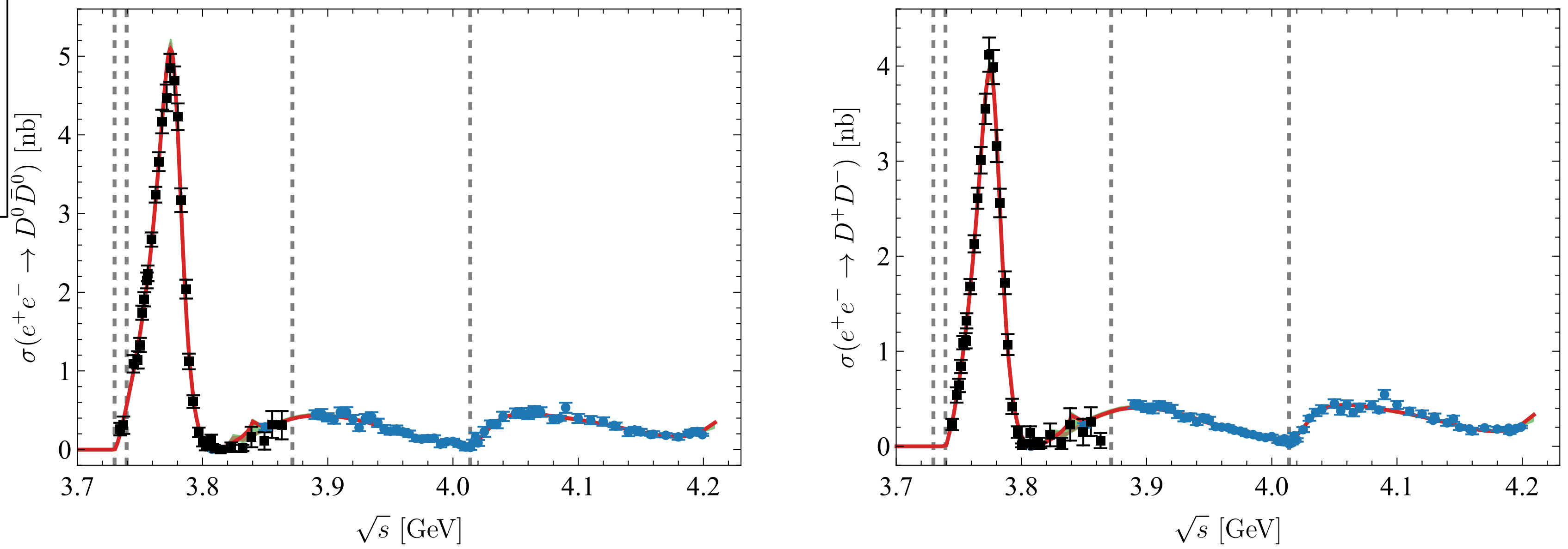
Different with that in PDG

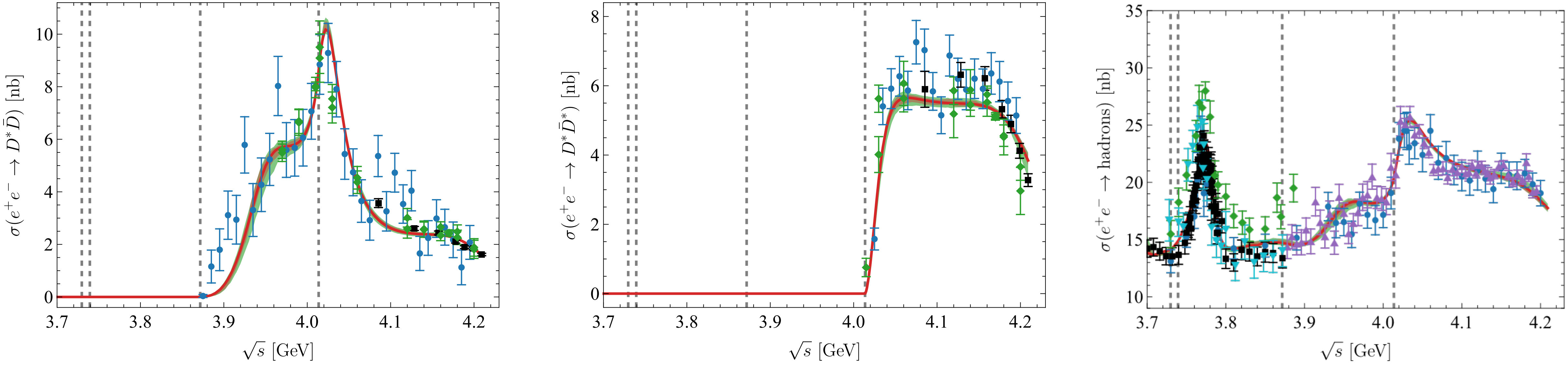
  - Analyticity is not kept: $\mathcal{M}(s) = \mathcal{M}(k_1, k_2, \ldots)$, e.g. For $M_{th1}^2 < s < M_{th2}^2$, k_2 is set to zero
  - Subtraction dependence / regularization dependence for P-wave
  - Did not search the pole in the unphysical sheets below the thresholds



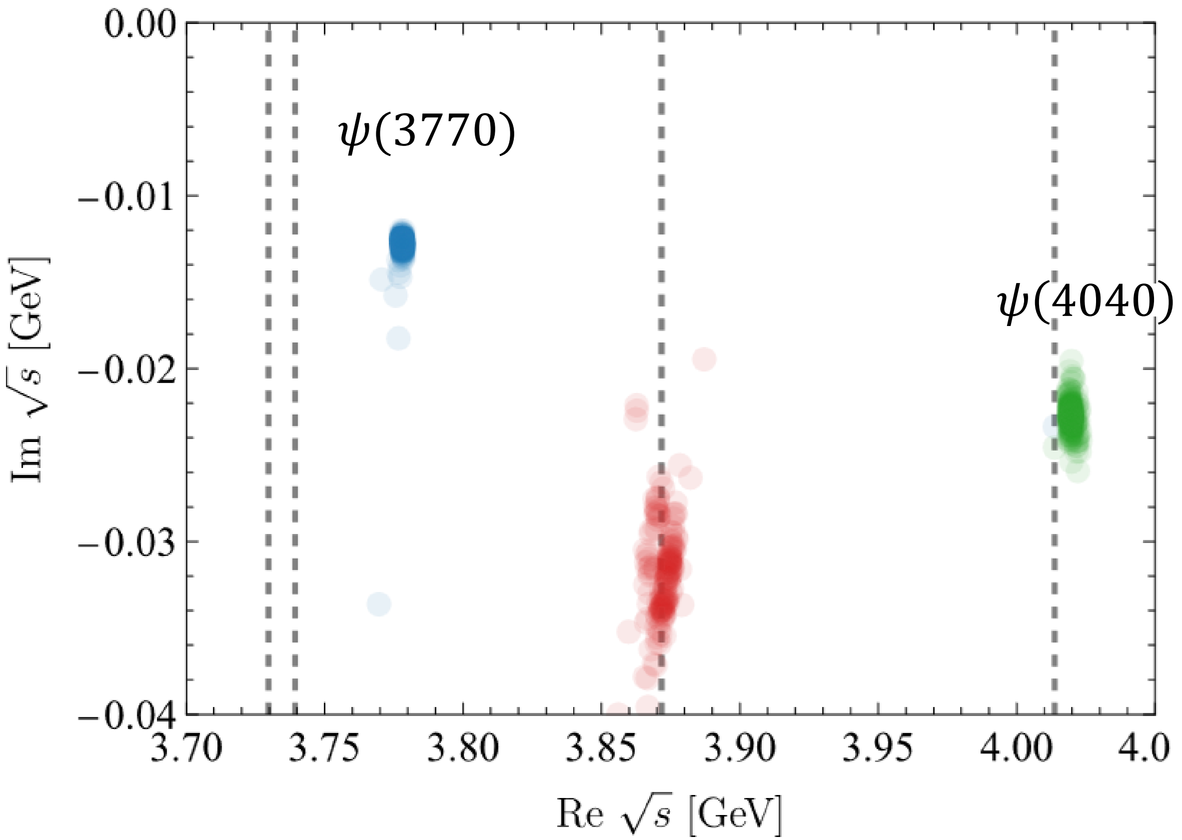
Our fit in the K-matrix formalism

- Repair the three defects
- Our refit results
  - ▶ $\chi^2/\mathrm{dof} = 2.07$





Our fit in the K-matrix formalism

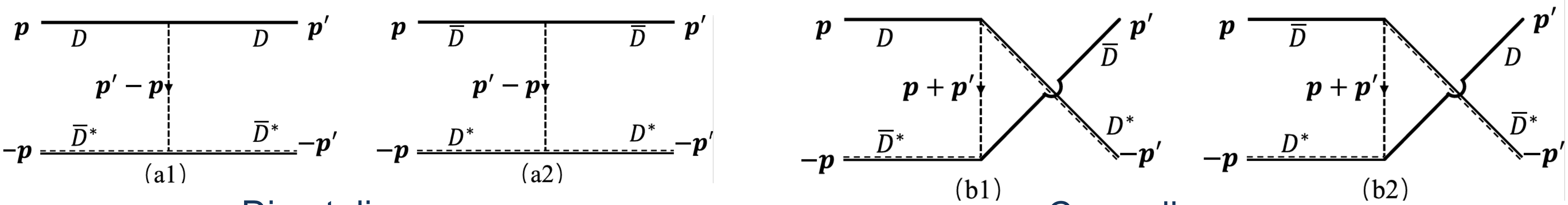


Extra poles

$$3869.2(67) - i29.0(52)$$

Dynamics model

- Interactions: exchange: $\pi, \eta, \rho, \omega, \sigma$
  - ▶ Success in high precision nuclear force, e.g. CD-Bonn potential — Machleidt:2000ge
  - ▶ S-wave and P-wave interactions are derived from the same Lagrangians



Direct diagram — Cross diagram

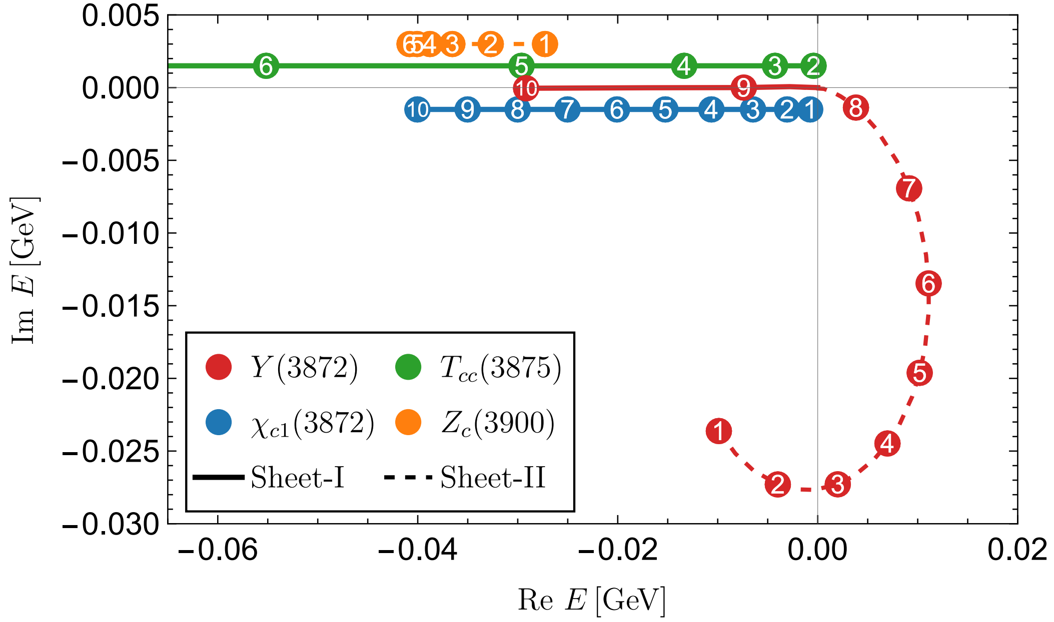
- G-parity rules (particle-particle ⇔ particle-antiparticle)
  - ▶ Direct diagram: $V^{D}_{D\bar{D}^*/\bar{D}D^*} = G_m V^{D}_{DD^*}$
  - ▶ Cross diagram: $V^{C}_{D\bar{D}^*/\bar{D}D^*} = -G_{D\bar{D}^*/\bar{D}D^*} G_m V^{C}_{DD^*}$ New!
- Three unknown coupling constants:
  - ▶ vector exchange for direct/cross diagrams, scalar exchange
  - ▶ pseudoscalar exchange is known
- Regulator and cutoff

$$V(\boldsymbol{p}',\boldsymbol{p}) \to V(\boldsymbol{p}',\boldsymbol{p}) \frac{\Lambda^2}{p'^2+\Lambda^2}\frac{\Lambda^2}{p^2+\Lambda^2}$$

$$V^D(\boldsymbol{q}) \to V^D(\boldsymbol{q}) \left(\frac{\Lambda^2 - m^2}{\Lambda^2 + \boldsymbol{q}^2}\right)^2$$

Scheme-I

- Fix parameters
  - vector meson dominance, Σ-model, light-cone sum rule, LQCD Li:2012cs, Li:2012ss
- Vary Λ from 0.4 to 1.3 GeV (1-10)
- $\Lambda \sim 0.5$ GeV: loosely bound $X(3872)$ and T_{cc}
  - Virtua state $Z_c(3900)$ Albaladejo:2015lob
  - $^3P_1$ resonance, reported by BESIII
  - Other prediction

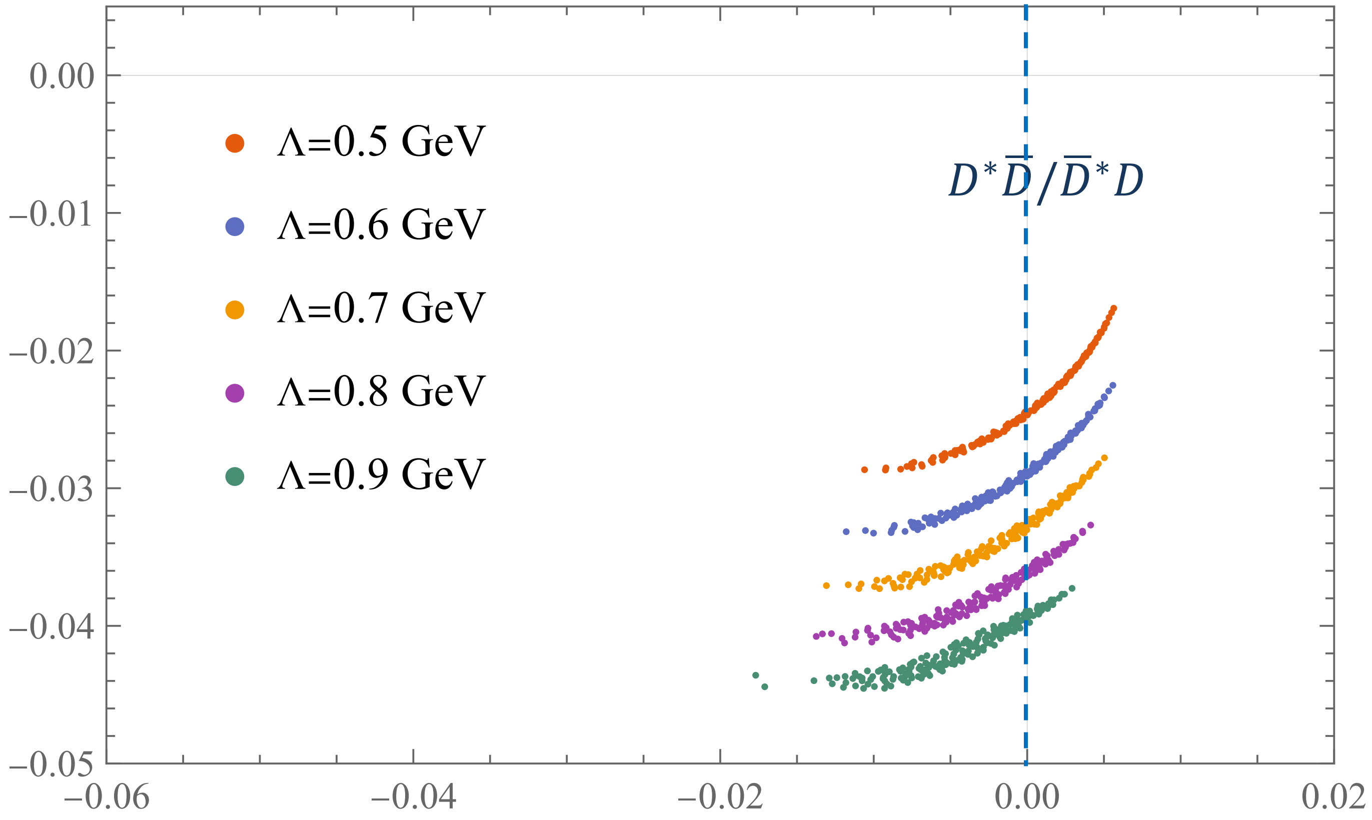


		$D\bar{D}^*, C=+$		$D\bar{D}^*, C=-$		DD^*	
		$I=0$	$I=1$	$I=0$	$I=1$	$I=0$	$I=1$
$\Lambda = 0.5$GeV	$1^+(^3S_1)$	-3.1^B, $\chi_{c1}(3872)$	-	-1.60^B	-35.6^V, $Z_c(3900)$	-0.41^B, $T_{cc}(3875)$	-
	$0^-(^3P_0)$	$-1.5-14.5i$	-	-	-	$-9.6-9.7i$	-
	$1^-(^3P_1)$	-	-	$-4.0-27.3i$, $Y(3872)$	-	$-31.7-70.6i$	-
	$2^-(^3P_2)$	$-42.6-39.4i$	-	$-21.3-50.7i$	-	$-37.8-40.9i$	-

B: bound state, V: virtual state

Scheme-II

- Fix parameters
  - ▶ Bound state $X(3872)$: $-4\sim0$ MeV
  - ▶ Bound state $T_{cc}(3875)$: $-4\sim0$ MeV
  - ▶ Virtual state $Z_c(3900)$: $-35\sim-15$ MeV
- For each cutoff, random pole positions in above ranges
- Numerically obtain three coupling constants
- Calculate the 1^{--}, ${}^3P_1$ $D^*\bar{D}/\bar{D}^*D$ poles



1^{--}, ${}^3P_1$ $D^*\bar{D}/\bar{D}^*D$ resonance pole

The para. constrained by $X(3872)$, $T_{cc}(3875)$, and $Z_c(3900)$ give rise to the P-wave pole almost unavoidably

Systemic uncertainties

- Coupled channel VS single channel

TABLE I: The pole of $Y(3872)$ in the coupled-channel calculations involving $D\bar{D}$, $D\bar{D}^*/\bar{D}D^*$ and $D^*\bar{D}^*$.

Λ (GeV)	0.5	0.6	0.7	0.8	0.9	1.0
Single channel	-4.0-27.3i	2.0-27.3i	7.0-24.4i	10.2-19.6i	11.1-13.5i	9.1-7.0i
Coupled channel	-3.5-27.2i	2.8-26.8i	7.9-23.2i	10.8-17.1i	10.1-9.4i	4.8-2.2i

- Three-body effect

TABLE II: The impact of the three-body $D\bar{D}\pi$ effect of the OPE potential of $D\bar{D}^* \to \bar{D}D^*$ on the pole of $Y(3872)$.

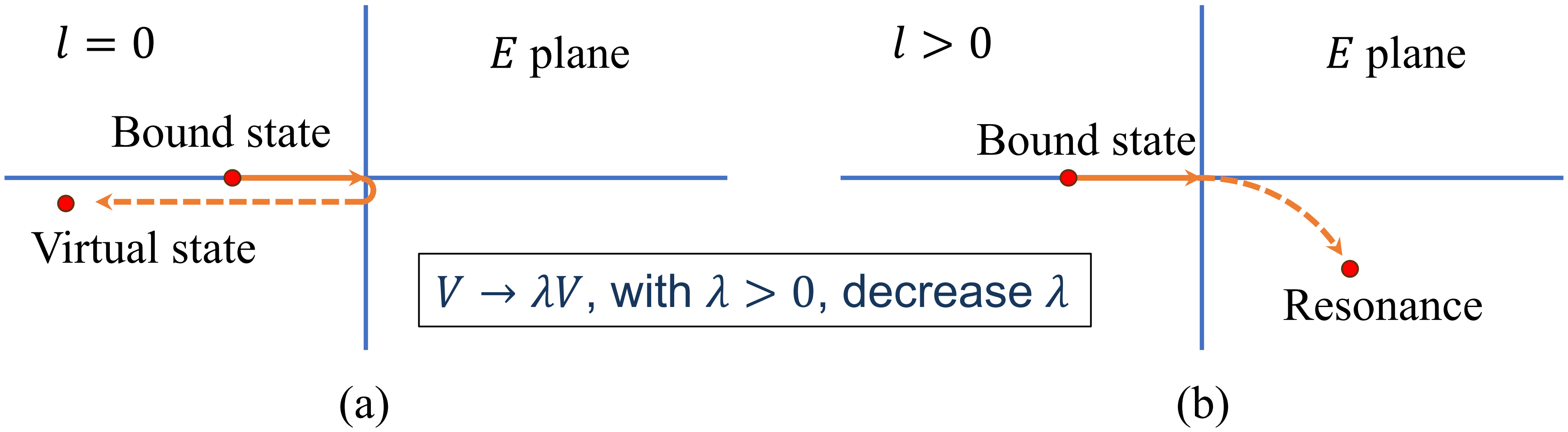
Λ (GeV)	0.5	0.6	0.7	0.8	0.9	1.0
Without three-body effect	-4.0-27.3i	2.0-27.3i	7.0-24.4i	10.2-19.6i	11.1-13.5i	9.1-7.0i
With three-body effect	-5.0-24.1i	-1.2-24.3i	6.3-21.7i	8.9-16.8i	9.4-11.1i	7.2-5.3i

- Regulator dependences: no qualifiedly changes

Why the existence of P-wave resonance is so robust?

From bound state to…

- A general statement: Taylor, scattering theory textbook sec. 13-b P245

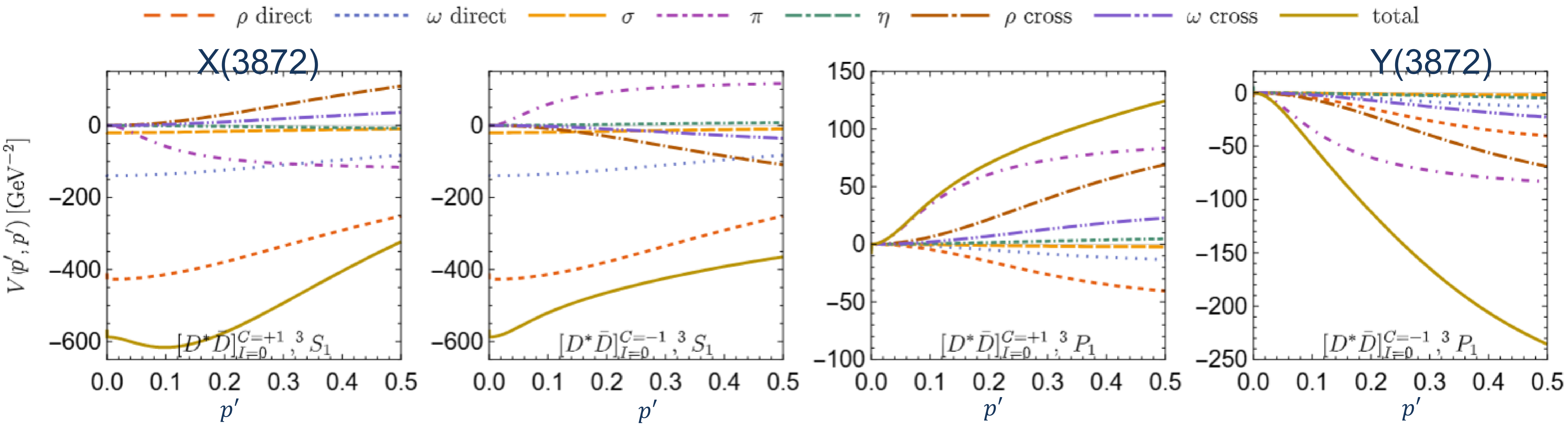


- The reasons of resonance: Weinberg QFT textbook
  - ▶ $H = H_s + V_w$, week perturbation allow the decaying
  - ▶ Potential barrier: centrifugal barrier for l>0
  - ▶ Unstable for statistic reason

Higher partial wave and long-range interaction

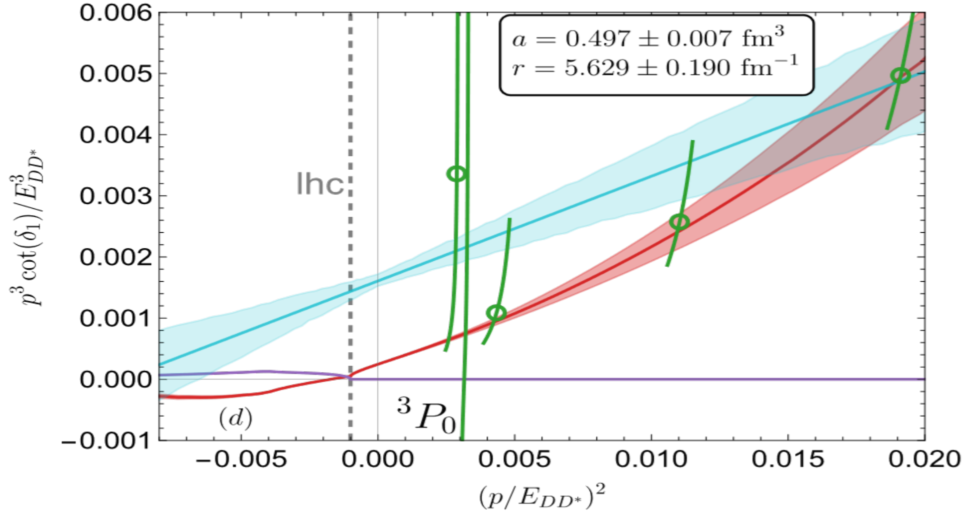
- The long-range interaction: OPE interaction (no free parameter)
  - ► $D^*D\pi$ coupling constant is determined by $D^* \to D\pi$ decay width

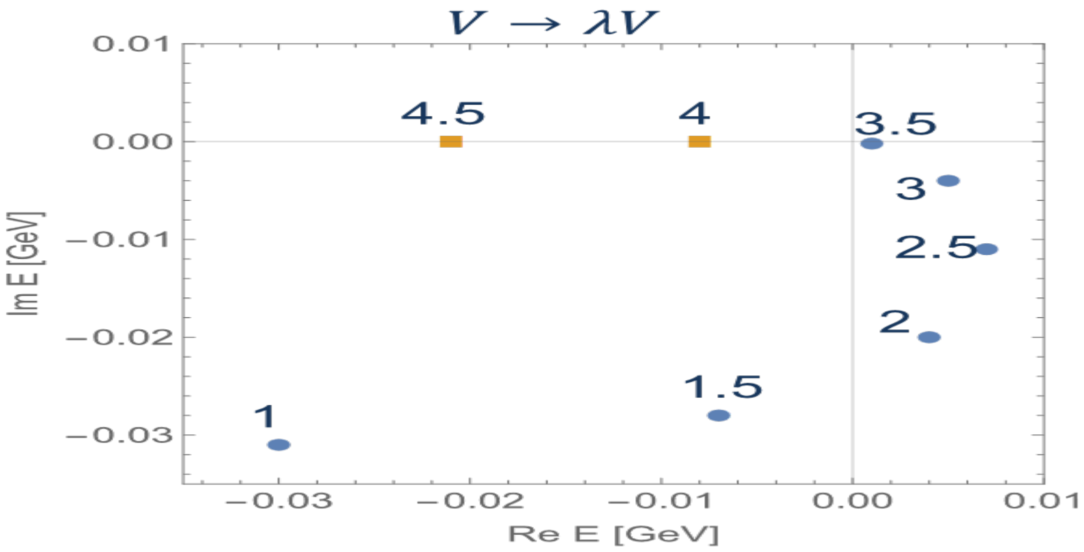
- The dynamics of high partial wave is sensitive to the long-range interaction due to centrifugal barrier



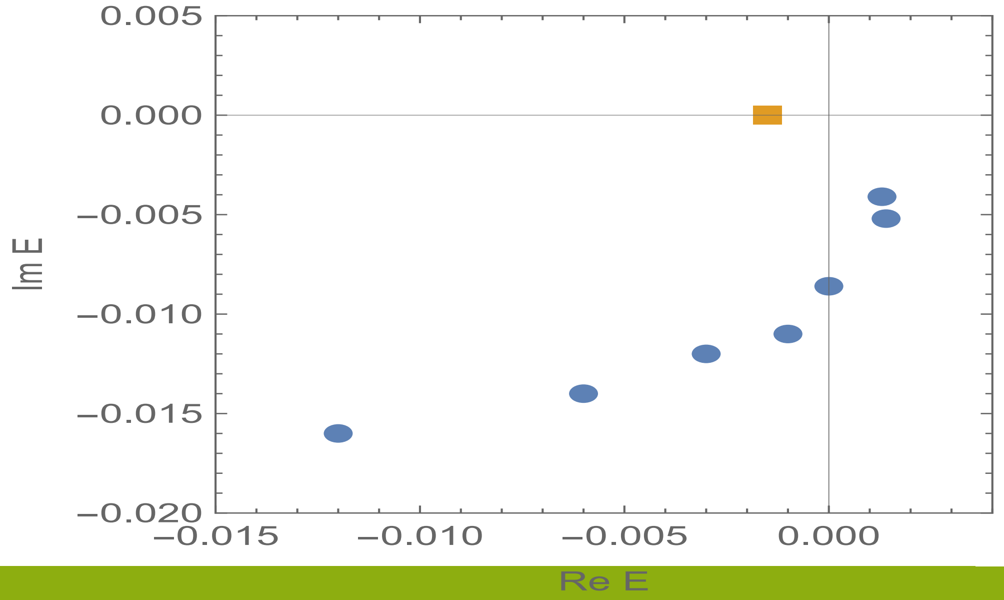
Populated P-wave resonance pole

- $^3P_0\ DD^*$ resonance pole: $-0.030 - i0.031$





- $^3P_0\ NN$ resonance pole: $-0.012 - i0.016$
  - ▶ Were seldom investigated
  - ▶ Hard to detect: no lower coupled-channel



- $1^{--}\ \bar{D}D^*$ pole can be tracked in $D\bar{D}$ final state

Experimental Search

		$D\bar{D}^*, C=+$		$D\bar{D}^*, C=-$		DD^*	
		$I=0$	$I=1$	$I=0$	$I=1$	$I=0$	$I=1$
$\Lambda=0.5\text{GeV}$	$1^+(^3S_1)$	-3.1^B, $\chi_{c1}(3872)$	-	-1.60^B ②	-35.6^V, $Z_c(3900)$	-0.41^B, $T_{cc}(3875)$	-
	$0^-(^3P_0)$	$-1.5-14.5i$ ③	-	-	-	$-9.6-9.7i$ ④	-
	$1^-(^3P_1)$	-	-	$-4.0-27.3i$, $Y(3872)$ ①	-	$-31.7-70.6i$	-
	$2^-(^3P_2)$	$-42.6-39.4i$	-	$-21.3-50.7i$	-	$-37.8-40.9i$	-

B: bound state, V: virtual state

① Precision measurement of $e^+e^- \to \bar{D}D^*$ close to threshold

② Hidden charm final state: $\eta_c\omega$, $J/\psi\eta$, $J/\psi\pi\pi$

③ Hidden charm final state: $J/\psi\omega$, $\eta_c\pi\pi$,$\chi_{c1}\pi\pi$

④ Final state $DD\pi$

Summary

- Concept: P-wave resonance
  - ▶ populated
  - ▶ more sensitive to the long-range interaction

} centrifugal barrier

- Data: A refined K-matrix fit support the existence of the 1^{--} pole close to $\bar{D}D^*/D\bar{D}^*$ threshold

- Dynamics: the meson-exchange model, $\bar{D}D^*/D\bar{D}^*$ $^3P_1$ resonance
  - ▶ Constrained by $X(3872)$, $Z_c(3900)$, and $T_{cc}(3875)$
  - ▶ G-parity rules, same Lagrangians for S-wave and P-wave
  - ▶ Robust
  - ▶ Unified framework

- Predictions: $[\bar{D}D^*]^{I=0}_{c=-1}$, $^3S_1$; $[\bar{D}D^*]^{I=0}_{c=+1}$, $^3P_0$; $[DD^*]^{I=0}$, $^3P_0$

Promising examples for studying the general few-body physics

Thanks for your attentions!

Backup

Cutoff and regulator dependence

		$D\bar{D}^*, C=+$		$D\bar{D}^*, C=-$		DD^*	
		$I=0$	$I=1$	$I=0$	$I=1$	$I=0$	$I=1$
$\Lambda=0.5$GeV	$1^+(^3S_1)$	-3.1^B, $\chi_{c1}(3872)$	-	-1.60^B	-35.6^V, $Z_c(3900)$	-0.41^B, $T_{cc}(3875)$	-
	$0^-(^3P_0)$	$-1.5-14.5i$	-	-	-	$-9.6-9.7i$	-
	$1^-(^3P_1)$	-	-	$-4.0-27.3i$, $Y(3872)$	-	$-31.7-70.6i$	-
	$2^-(^3P_2)$	$-42.6-39.4i$	-	$-21.3-50.7i$	-	$-37.8-40.9i$	-
$\Lambda=0.6$GeV	$1^+(^3S_1)$	-6.5^B, $\chi_{c1}(3872)$	-	-5.8^B	-34.6^V, $Z_c(3900)$	-4.3^B, $T_{cc}(3875)$	-
	$0^-(^3P_0)$	$3.2-13.7i$	-	-	-	$-10.2-12.1i$	-
	$1^-(^3P_1)$	-	-	$2.0-27.3i$, $Y(3872)$	-	$-33.7-84.8i$	-
	$2^-(^3P_2)$	$-44.2-49.0i$	-	$-19.3-58.8i$	-	$-37.8-49.3i$	-

		$D\bar{D}^*, C=+$		$D\bar{D}^*, C=-$		DD^*	
		$I=0$	$I=1$	$I=0$	$I=1$	$I=0$	$I=1$
$\Lambda=1.25$ GeV	$1^+(^3S_1)$	-0.40^B, $\chi_{c1}(3872)$	-	-25.0^V	-39.6^V, $Z_c(3900)$	-0.79^B, $T_{cc}(3875)$	-
	$0^-(^3P_0)$	$3.3-17.2i$	-	-	-	$-11.2-16.7i$	-
	$1^-(^3P_1)$	-	-	$4.4-39.9i$, $Y(3872)$	-	$-96.6-87.3i$	-
	$2^-(^3P_2)$	$-71.2-63.5i$	-	$-31.0-96.5i$	-	$-61.3-53.6i$	-
$\Lambda=1.35$ GeV	$1^+(^3S_1)$	-2.8^B, $\chi_{c1}(3872)$	-	-2.2^V	-38.5^V, $Z_c(3900)$	-8.8^B, $T_{cc}(3875)$	-
	$0^-(^3P_0)$	$6.6-11.6i$	-	-	-	$-10.2-18.0i$	-
	$1^-(^3P_1)$	-	-	$10.2-33.7i$, $Y(3872)$	-	$-92.9-97.7i$	-
	$2^-(^3P_2)$	$-68.0-75.4i$	-	$-23.3-97.2i$	-	$-58.4-59.6i$	-

K-matrix formalism

- The unitarity: $\mathrm{Im}\mathcal{M}^{-1} = -2\rho$

$$k = \frac{\sqrt{[s-(m_1-m_2)^2][s-(m_1+m_2)^2]}}{2\sqrt{s}}, \quad \rho(s) = \frac{k}{8\pi\sqrt{s}}$$

- K-matrix parameterization $\mathcal{M}^{-1} = K^{-1} + C, \quad \mathrm{Im}\ C(s) = -2\rho$
  - ► K is real, e.g. $K = \frac{g^2}{m_R^2 - s} + f$
  - ► $C(s)$ determined by once-subtracted dispersion relation

$$C(s) = C(s_0) - \frac{s-s_0}{\pi}\int_{s_0}^{\infty} ds' \frac{\mathrm{Im}C(s)}{(s'-s)(s'-s_0)}$$

  - ► The subtraction-dependence is absorbed by the K
- P-wave and higher partial wave: threshold effect $\mathcal{M} \to p^{2l}$
  - ► PDG:

$$p^l \mathcal{M}_l^{-1} p^l = K^{-1} + C_l, \quad \mathrm{Im}C_l \to -2\rho p^{2l}$$

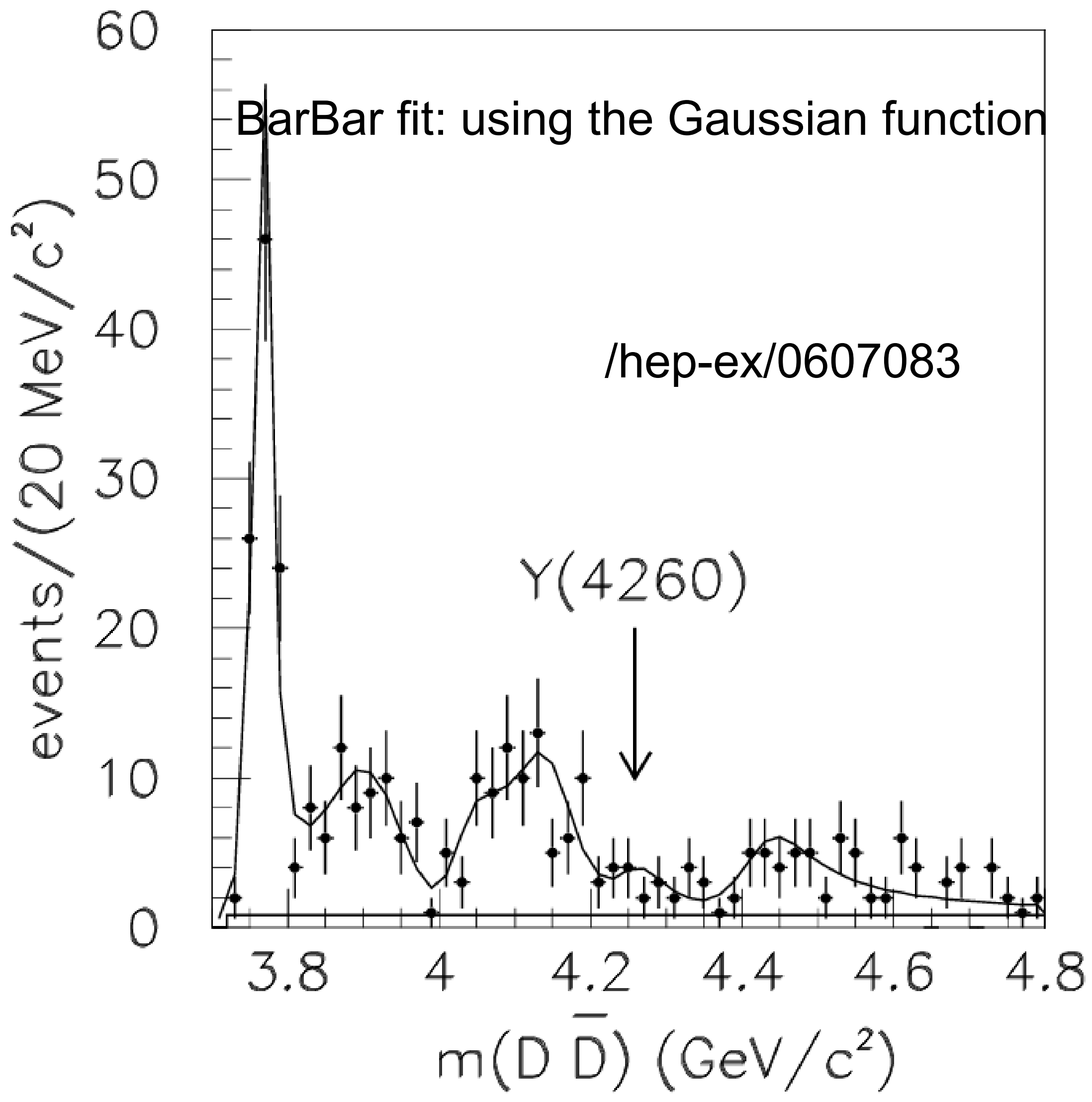
  - ► Swanson et al:

$$\mathcal{M}^{-1} = K^{-1} p^{-2l} + C_0(s)$$

- CM function VS one-loop diagram
  - ► Subtraction ⇔ regularization
  - ► In BSE, absorbed by the Kernal or coupling constant

$$C_0(s) \sim G(s) = \int \frac{d^4q}{(2\pi)^4} \frac{1}{q^2 - m_1^2 + i\epsilon} \frac{1}{(P-q)^2 - m_2^2 + i\epsilon},$$

G(3900) in BarBar and Belle



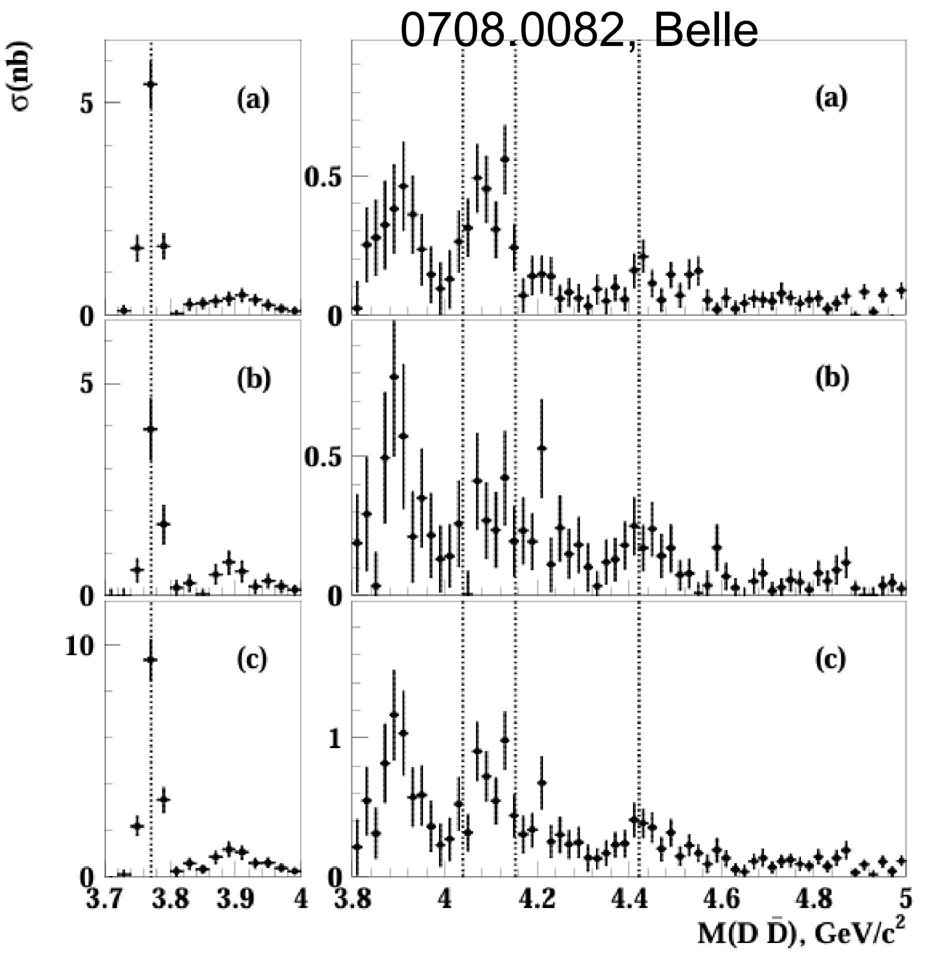
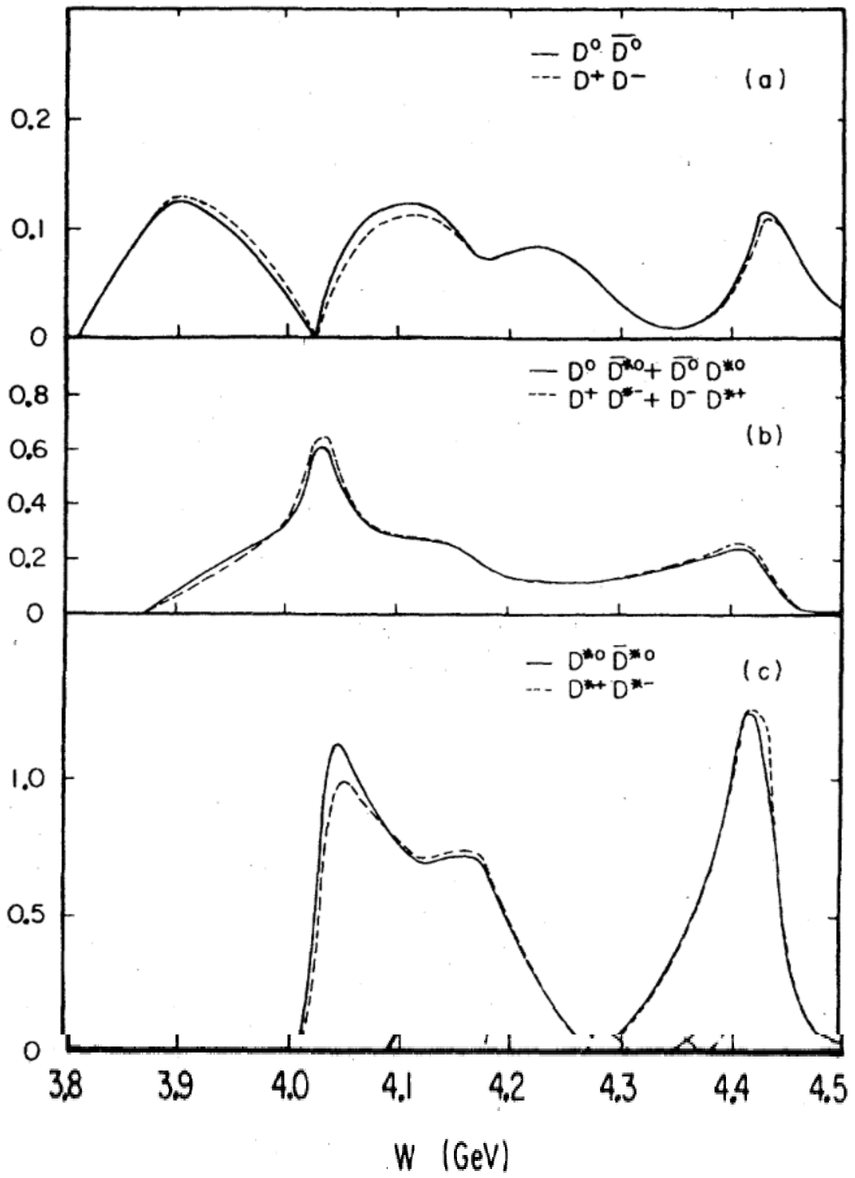
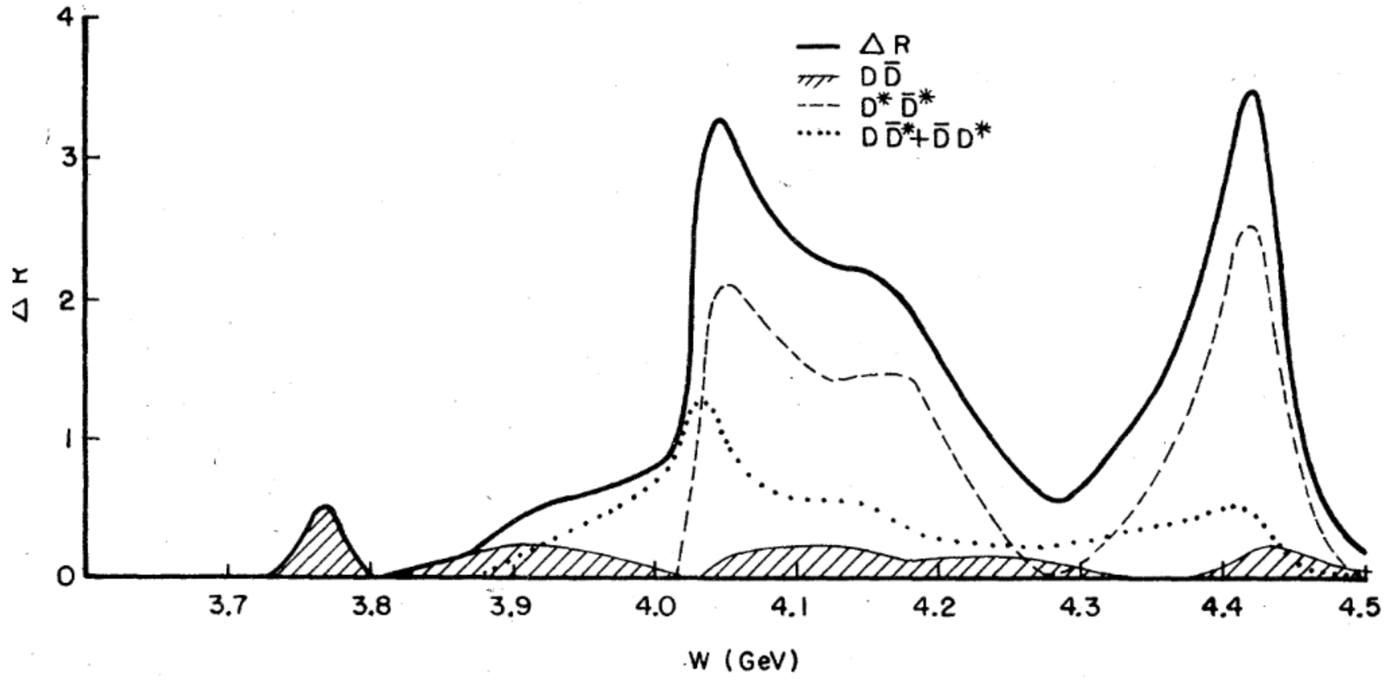


FIG. 3: The exclusive cross sections for: (a) $e^+e^- \to D^0\overline{D}^0$; (b)$e^+e^- \to D^+D^-$; (c) $e^+e^- \to D\overline{D}$. The dotted lines correspond to the $\psi(3770)$, $\psi(4040)$, $\psi(4160)$ and $\psi(4415)$

Cornell model- coupled-channel effect





S-wave virtual and P-wave resonance